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QDF: The Quasi-Direct Format program user guide with databases, examples and TABQDF outputs

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1. Purpose of QDF Program Calculations and of TABQDF Table Editions

This manual accompanies a utility program (Tran and Gaudry, 2009c; AJD-119) that derives demand elasticities and other statistics obtained from particular coupled QDF-type specifications combining two previously estimated models. The quasi-direct *rationale* and the *statistics* of interest are outlined in the report establishing the QDF structure (Tran and Gaudry, 2008c; AJD-117).

The QDF utility program makes it possible to produce TABQDF table editions of outputs and derived statistics inspired by the Tablex tables of results long used to compare LEVEL, SHARE and PROBABILITY regression variants produced either by stand-alone versions of these algorithms or by their versions implemented within TRIO, Version 1 (1993) and Version 2 (2001), software.

Derived results, presented in such Quasi-Direct-Format editions of a TABQDF table, are obtained from products of maximum likelihood estimates of selected pairs of models: they are consequently maximum likelihood estimates themselves, although not as efficient as estimates obtainable from the joint estimation of the model product pairs considered. Such joint estimation would be exceedingly complex[1] in our case because we allow ***20 QDF combinations among 12 model types***, each characterized by membership of a class and family, and requiring a distinct estimation procedure.

The LEVEL class contains 2 families distinguished by the kind of autocorrelation of residuals allowed; 5 families are found in the SHARE and PROBABILITY classes, including one LOGIT, two DOGIT and two INVERSE POWER TRANSFORMATION-LOGIT branches, as listed below.

Table 0. The twelve TRIO model types among which 20 paired combinations are allowed in QDF

	Value of the dependent variable in $y_m = f_m(.)$				Form of $f_m(.)$		
	in the regression		in the sample		in the fixed and stochastic parts		
TRIO Classes	$\sum_m y_m = 1$	Domain	Nature	Domain	**Root** and algorithmic branches		**TRIO Model Types**
LEVEL	No	$-\infty < y_m < \infty$	Continuous	$-\infty < y_m < \infty$	**OLS**	⊦ OLS-GAUHESEQ	**L-1.6**
		$0 < y_m < \infty$		$0 < y_m < \infty$		⊦ BC-GAUHESEQ	
		$-\infty < y_m < \infty$		$-\infty < y_m < \infty$		⊦ OLS-DAUHESEQ	**L-2.1**
		$0 < y_m < \infty$		$0 < y_m < \infty$		⊦ BC-DAUHESEQ	
SHARE	Yes	$0 \leq y_m \leq 1$	Continuous	$0 < y_m < 1$	**LIN-LOGIT**	⊦ BC-LOGIT	**S-1**
						⊦ S-DOGIT	**S-2**
						⊦ G-DOGIT	**S-3**
						⊦ LIN-IPT-LOGIT	**S-4**
						⊦ BT-IPT-LOGIT	**S-5**
PROBABILITY	Yes	$0 \leq y_m \leq 1$	Discrete	$y_m = 0 \; or \; 1$	**LIN-LOGIT**	⊦ BC-LOGIT	**P-2**
						⊦ S-DOGIT	**P-3**
						⊦ G-DOGIT	**P-4**
						⊦ LIN-IPT-LOGIT	**P-5**
						⊦ BT-IPT-LOGIT	**P-6**
	Defines 3 CLASSES			with 3 roots and 12 FAMILIES or Model Types			

The technical characteristics of algorithms (L-1.4; P-2 and S-1/S-5) implemented in TRIO, Version 2.0, or sef-standing (L-1.6; L-2.1; P-2/P-6; S-1/S-5 and QDF) for Version 6, are summarized in Appendix 5.

[1] Laferrière (1988) estimated a LEVEL and SHARE model pair jointly for the domestic Canadian air market.

A. Structure of a TABQDF edition or application

In the typical Quasi-Direct Format specification of transport demand analysis used here, T_{ijm}, the demand for a particular mode *m* from origin *i* to destination *j*, is obtained as a product of two models: a model of the total transport market by all modes T_{ij} and a model of the mode split P_{ijm}. This may be expressed simply as

$$T_{ijm} = T_{ij} \bullet P_{ijm}, \tag{0-1}$$

which can be written more explicitly for each component:

$$T_{ijm} = f\,(A_i\,, A_j\,, U_{ij}) \bullet \left\{ U_{ijm} \,/ \sum_m U_{ijm} \right\},\ m = 1, ..., M, \tag{0-2}$$

where the A terms denote activity variables (*e.g.* Population, Income…) and $\boldsymbol{U_{ij}}$, an index of the utility of travel by all modes called the coupling term, is defined by the denominator of the mode split model which can be of an aggregate or disaggregate (discrete) nature:

$$U_{ij} = \sum_m f\,(U_{ij_m})\,. \tag{0-3}$$

B. Core features of coupled models

The model of levels. More precisely, the model of total demand in (0-2) may come from either of two LEVEL families with common trunk L_0 but particular applications with autocorrelation belonging to type L_1 or L_2, depending on the nature of autocorrelation, namely, using *t* subscripts instead of *ij* O-D subscripts:

Regression equation:

$$T_t^{(\lambda_T)} = \sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + u_t \text{ , } t=1,\ldots,N \qquad \text{(i)}$$

where the variable U_t is the index of the utility of travel by all modes defined in (0-3) above. Note that this variable can be included or not in the equation.

Heteroskedasticity:

$$u_t = \left[exp\left(\sum_m \delta_m \cdot Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet \ v_t \qquad \text{(ii)}$$

(L_0)

Serial autocorrelation:

$$v_t = \sum_{\ell=1}^{r} \rho_l \cdot v_{t-l} + w_t \qquad \text{(iii)}$$

(L_1)

Regression equation:

$$T_t^{(\lambda_T)} = \sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + u_t \text{ , } t=1,\ldots,N \qquad \text{(i)}$$

where the variable U_t is the index of the utility of travel by all modes defined in (0-3) above. Note that this variable can be included or not in the equation.

Heteroskedasticity:

$$u_t = \left[exp\left(\sum_m \delta_m \cdot Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet \ v_t \qquad \text{(ii)}$$

(L_0)

Directed autocorrelation:

$$v_t = \sum_{l=1}^{2} \rho_l \cdot \left(\sum_{n=1}^{N} \tilde{r}_{l,tn} v_n \right) + w_t \qquad \text{(iv)}$$

where $\tilde{r}_{l,tn}$ is the typical element of the matrix $\tilde{R}_l$ or, in matrix form:

$$v \ = \ \sum_{l=1}^{2} \rho_l \tilde{R}_l v \ + \ w$$

and $\tilde{R}_l$ is the matrix obtained by the R-Koyck or R-ARMA (1,1) process applied to the row normalized Boolean neighbourhood matrix $\bar{R}_\ell$ expressing an hypothesis concerning the presence of directed correlation, ***including spatial autocorrelation***:

$$\tilde{R}_l = \pi_l \left[I - (1-\pi_l)\bar{R}_l \right]^{-1} \bar{R}_l, \quad (0 < \pi_l \leq 1)\,,$$

where the proximity parameter π_l allows endogenisation of the relative importance of near and distant effects considered in the rule of construction of each distinct R_l.

(L_2)

The model of splits. The other component of (0-2), the model of the mode split, comes from:

$P_{ijm} = \left\{ U_{ijm} / \sum_m U_{ijm} \right\},\ m = 1, \ldots, M.$ *where* P_{ijm} *is a PROBABILITY or a SHARE*	(S or P)

and five families are available within SHARE and PROBABILITY classes (neglecting *ij* subscripts):

Family	**U_m function,** $m = 1, \ldots, i, j, \ldots, M$	**Model type**
LOGIT: Logit	$exp(V_i),$	(S_1) or (P_2)
S-DOGIT: Standard Dogit	$exp(V_i) + \theta_i \sum_j exp(V_j),$ with $\theta_i \geq 0$	(S_2) or (P_3)
G-DOGIT: Generalized Dogit	$exp(V_i) + \theta_{ij} \sum_{j \neq i} exp(V_j),$ with $\theta_{ii} = 0,\ \theta_{ij} \geq 0$	(S_3) or (P_4)
LIN-IPT-LOGIT: Linear Inverse Power Transformation-Logit	$\left[\varphi_i\ exp(V_i) + 1 \right]^{1/\varphi_i} - \mu_i.$ with $\varphi_i \geq 0,\ \mu_i \leq 1$	(S_4) or (P_5)
BT-IPT-LOGIT: Box-Tukey Inverse Power Transformation-Logit	$\exp\left\{ \left[\left[\exp(V_i) + \tilde{\mu}_i \right]^{\tilde{\phi}_i} - 1 \right] / \tilde{\phi}_i \right\},$ with $\tilde{\mu}_i \geq 0$	(S_5) or (P_6)

and the ***Utility function V_i*** can be of Generalized Linear *type* or of Generalized Box-Cox *type*

G-LIN

$$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^i + \sum_{j \neq i} \sum_n \beta_{in}^j X_n^j + \sum_s \beta_{is} X_s \qquad \text{(v)}$$

G-BC

$$V_i = \beta_{i0} + \sum_n \beta_{in}^i X_n^{(\lambda_{in}^i)} + \sum_{j \neq i} \sum_n \beta_{in}^j X_n^{(\lambda_{in}^j)} + \sum_s \beta_{is} X_s^{(\lambda_{is})} \qquad \text{(vi)}$$

where n and s are indices of network and socioeconomic variables and the upper index refers to the mode. But, as G-LIN parameters (including all M constants) are all identified only in LIN-IPT-L and BT-IPT-L families, it is practical to use instead two forms of G-BC:

Standard Box-Cox utility

the first case occurring if a network variable X_{mn} appears only in its own mode m; the form of V_m is as follows:

$$V_m = \beta_{m0} + \sum_n \beta_{mn} X_{mn}^{(\lambda_{mn})} + \sum_s \beta_{ms} X_{ms}^{(\lambda_{ms})} \qquad \text{(vii)}$$

and the second case occurring if a network variable X_{mn} appears not only in its own mode m but also as X_{jn} across all the other modes $j \neq m$; the corresponding V_m and V_j can be written as:

Generalized Box-Cox utility

$$V_m = \beta_{m0} + \sum_n \beta_{mn} X_{mn}^{(\lambda_{mn})} + \sum_s \beta_{ms} X_{ms}^{(\lambda_{ms})}$$
$$V_j = \beta_{j0} + \sum_n \beta_{jn} X_{jn}^{(\lambda_{jn})} + \sum_s \beta_{js} X_{js}^{(\lambda_{js})}\ . \qquad \text{(viii)}$$

The twelve model types to be considered are therefore [L_1 & L_2], [S_1 & P_2], [S_2 & P_3], [S_3 & P_4] and [S_5 & P_6]. But TABQDF editions have restrictive features that do not make it possible to use all of the information produced by the twelve estimation procedures or algorithms. Note that:

1. ***Null-value and negative-value observations: the OLS model type case.*** Although observations on the dependent variable must, as written above, be strictly positive for (L_1) and (L_2) applications, in fact the L-1.6 and L-2.1 algorithms also allow null-value and negative-value observations, as in Classical Least Squares, if the dependent or explained variable appears linearly in the variant. The root OLS case indicated in Table 0 is therefore also a model type covered by these procedures (see AJD-105 and AJD-108).

2. ***Multinomial and binomial formulations.*** Although the mode split model expressions S_s and P_p as written above pertain to multinomial cases, binomial applications can also be estimated, including cases where all but one of the modal quantities $\exp(V_i)$ equal 1, *i.e.* cases where, except for an arbitrary "first" function V_1, all other V_i representative utility functions are null (see AJD-111 and AJD-114).

3. ***Only "sample" elasticities evaluated at sample means.*** For in this QDF program, we only use so-called standard "sample" elasticity measures *evaluated at sample means* for both (L_1) and (L_2) formulations, as implied by the expressions found Table 3 below. As we will remind the reader later on, we neglect elasticities of the first three moments of the dependent variable also calculated by the LEVEL L-1.6 algorithm (see AJD-105) and the average value of point elasticities of the expected value of y_t also calculated by the LEVEL L-2.1 algorithm (see AJD-108).

Similarly, we only use, for aggregate models of shares (S) and discrete probability models of individual choices (P), standard elasticities *evaluated at sample means*, as we will also remind the reader later on. For this reason, analytical expressions for elasticities collected below in Table 5 and Table 6 hold for both (S) and (P) formulations. In QDF, we make no use of the Percentage point elasticity measure in (S) cases, as computed by the SHARE S-1/S-5 algorithm (see AJD-111), or of the Probability point elasticity measure in (P) cases, for which we also neglect for *reasons of consistency* (not for theoretical reasons) the weighted aggregate elasticities computed by the PROBABILITY algorithm P-2/P6 (see AJD-114). The latter coud be considered in a further version of QDF.

4. ***Transformation of variables containing zero-value observations***. In expressions (i) and (ii) of (L_1) and of (L_2), as well as in the explicitations (vii) and (viii) of the U_m terms of (S) or (P), the commonly defined Box-Cox transformation (without a shif parameter) is applicable to any strictly positive T, X_k or Z_m variable, namely for instance:

$$X_k^{(\lambda)} = \begin{cases} \dfrac{X_k^{\lambda}-1}{\lambda} & \text{if } \lambda \neq 0, \\ \ln X_k & \text{if } \lambda \to 0. \end{cases} \tag{BCT}$$

However, all LEVEL, SHARE and PROBABILITY model family type options also allow, under certain conditions requiring the creation of "associated" dummy variables, for the transformation of X_k or Z_m variables containing some null-value observations. These associated dummy variables are also treated and appear in the TABQDF table, as they appeared in Tablex tables editions for the coupled models considered independently (see AJD-106, AJD-109, AJD-112 and AJD-115).

5. ***Regression intercepts.*** All LEVEL, SHARE and PROBABILITY model families and model type options require the presence of a regression intercept, in accordance with Schlesselman's (1971) invariance condition for Box-Cox parameters. These constants do not appear in TABQDF tables.

6. ***Presence of coupling term U.*** Coupling is not necessary in QDF model specifications used here. Coupling is often effected in transport (and communication) demand models in order to explicitly distinguish, following a change in the properties of one or more of the modes, the demand "generation" or "induction" effect on the total market from their "diversion" effect on mode choice. For an extensive summary of mostly coupled model results in passenger and freight transport, the reader might consult Gaudry *et al.* (1998) or Gaudry (2010).

C. Derived elasticities

Because (0-1) is a product of models, the elasticity of the modal demand (the left-hand-side variable in (0-2)), can be expressed as a sum of their elasticities:

$$[\,\eta \text{ of Demand for Mode } m\,] = [\,\eta \text{ of Total Demand for all modes}] + [\,\eta \text{ of Modal Split of Mode } m], \qquad \text{(0-4)}$$

but it is important to note that, in coupled paired model products, a given explanatory variable X_k, such as consumer Income or the Price of a mode, might well appear in all three terms (A), (C) and (D) distinguishable by rewriting (0-4) in a more general and explicit fashion as:

$$\boxed{\underset{\text{(F)}}{\eta(T_m, X_k)} = \underbrace{\underset{\text{(A)}}{\tilde{\eta}(T, X_k)} + \underset{\text{(B)}}{\eta(T,U)}\bullet\underset{\text{(C)}}{\eta(U, X_k)}}_{\text{(E)}} + \underset{\text{(D)}}{\eta(P_m, X_k)}} \qquad (1)$$

where we have neglected all *ij* indices, so that $U = \sum_{m=1}^{M} U_m$ simply denotes (0-3), the sum of all M *modal utilities* appearing in P_m, *i.e.* the denominator of each probability or share expression for the m-th mode.

In order to define further these various elasticities, we document the contents of TABQDF tables generated by the QDF program. The modal demand elasticity result (F) actually requires computing all elasticity components defined in (1) and, for this reason, QDF tables show all of them explicitly in addition to other statistics such as the Diversion Rate (see AJD-117 for the derivation and explanations of this concept).

1.1 Description of the Contents of a TABQDF Table Edition

Each column of a TABQDF table lists, for a given pair of regression variants (one obtained from LEVEL and one obtained from SHARE or from PROBABILITY), the different components of the modal elasticity (F) ≡ $\eta(T_m, X_k)$ exactly in the same order as that found in (1), from left to right.

The first part of the table, associated with the LEVEL variant used (for instance a Generation-Distribution model), consists in elasticities (A) and (B) explained in Table 1.A. These two elasticity components (A) and (B) are computed in QDF as $\eta_{\bar{Y},\bar{X}}$, the elasticity of T with respect to the independent variables X, *i.e.* $(X_k \text{ or } U)$, evaluated at the sample means.

Table 1.A. Total demand part of a TABQDF table

(A)	EL(T,X) ≡ $\tilde{\eta}(T, X_k)$ (t)	elasticity of T with respect to X_k, given a fixed U ; *t*-value of the associated regression coefficient.
(B)	EL(T,U) ≡ $\eta(T,U)$ (t)	elasticity of T with respect to U : *t*-value of the associated regression coefficient.

We note in passing that *we do not use* the other measure of elasticity found in Table B of AJD-117, $\bar{\eta}_{E(Y_t),X_t}$, the average value[2] of the point elasticities of the expected value of T with respect to the independent variables computed for every observation in the sample.

The second part of the table, associated with a SHARE or a PROBABILITY modal split model variant, presents, for each and all explanatory variables X_k appearing in the representative utility function for mode *m* (variables and dummies included), all the elements listed in Table 1.B, considering the modes successively in M subsections of varying length depending on the number of regressors in the actual utility function of the mode in question.

Table 1.B. Modal section part of a TABQDF table edition

(C)	EL(U) ≡ $\eta(U, X_k)$	elasticity of U with respect to X_k.
(D)	EL(Pm) ≡ $\eta(P_m, X_k)$ (t)	(*split*) elasticity of P_m with respect to X_k ; *t*-value of the associated regression coefficient.
(E)	EL(T) ≡ $\eta(T, X_k)$	*total* elasticity given in (1) ≡ [$\tilde{\eta}(T, X_k) + \eta(T,U) \bullet \eta(U, X_k)$].
(F)	EL(Tm) ≡ $\eta(T_m, X_k)$	*modal* elasticity given in (1) ≡ [$\eta(T, X_k) + \eta(P_m, X_k)$].
	D.R. ≡$[(\text{E})/(P_m(\text{F}))] - 1$	*Diversion Rate* associated with X_k (Equation 18 in AJD-117).

In each such modal section, the two elasticity components (C) and (D) are computed, for a SHARE or a PROBABILITY model type, as $\eta_{\bar{X}}$, an elasticity evaluated at the sample means of the independent variables. And we note again in passing that, for consistency reasons, we *do not use* that other measure of elasticity found in Table B of AJD-117, η_w, the weighted aggregate elasticity.

[2] The average value of (point) elasticities of $E(y_t)$ can be calculated by LEVEL 2.1 algorithm but not by LEVEL 1.6 algorithm.

In each subsection pertaining to a given mode, the list (taking variables in turn) presents the (C), (D) and (E) components, followed by the resulting (F) elasticity defined in Equation (1), as well as by the modal Diversion Rate for the variable in question.

Table 2 presents the combinations of elasticities actually used by the QDF program. Note that Column 1 and Column 3 in Part 1 are identical but belong in the table because they are ***calculated*** by different LEVEL programs (L-1.6 and L-2.1), a distinction naturally ignored in Table B of report AJD-117 where only ***theoretically feasible*** elasticity combinations matter, not ***actually computed*** ones.

Table 2. Model pairs for which elasticities are computed in QDF output tables

Part 1	**Products of the 2 LEVEL family model types 1.6 & 2.1 with any SHARE or PROBABILITY family model type**					
LEVEL	**1.6**		**2.1**			
	Autocorrelation		***Autocorrelation***			
Hetero-skedasticity	***None*** (L_0)	***Serial*** (L_1)	***None*** (L_0)	***Directed*** (L_2)	Model types	
No	L_0•[S_s/P_p]	L_1•[S_s/P_p]	L_0•[S_s/P_p]	L_2•[S_s/P_p]	[S_s or P_p]	**SHARE S-1/S-5 or PROBABILITY P-2/P-6**
Yes	L_0•[S_s/P_p]	L_1•[S_s/P_p]	L_0•[S_s/P_p]	L_2•[S_s/P_p]	[S_s or P_p]	
Column	**1**	**2**	**3**	**4**		

and there will be two categories of L_0, L_1 and L_2 elasticities, for (A) and (B) terms, depending on whether one is interested in an X_k variable or in the U variable found in the regression equation.

Part 2	**Products of the 5 SHARE family model types S-1 to S-5 with any LEVEL family model type**					
	SHARE	**S-1**	**S-2**	**S-3**	**S-4**	**S-5**
	Model types	***Logit & BC-Logit***	***Dogit*** ***Standard***	***Dogit*** ***Generalized***	***Inverse Power Transformation-Logit*** ***Linear***	***Box-Tukey***
LEVEL 1.6 or 2.1	[L_L]	[L_L]•S_1	[L_L]•S_2	[L_L]•S_3	[L_L]•S_4	[L_L]•S_5
	Column	**1**	**2**	**3**	**4**	**5**

Part 3	**Products of the 5 PROBABILITY family model types P-2 to P-6 with any LEVEL family model type**					
	PROBABILITY	**P-2**	**P-3**	**P-4**	**P-5**	**P-6**
	Model types	***Logit & BC-Logit***	***Dogit*** ***Standard***	***Dogit*** ***Generalized***	***Inverse Power Transformation-Logit*** ***Linear***	***Box-Tukey***
LEVEL 1.6 or 2.1	[L_L]	[L_L]•P_2	[L_L]•P_3	[L_L]•P_4	[L_L]•P_5	[L_L]•P_6
	Column	**1**	**2**	**3**	**4**	**5**

and there will be two categories of S_s and P_p elasticities, depending on whether the characteristic of a mode appears only in the representative utility function of that mode (the Standard Box-Cox utility function case) or appears also in the representative utility function of another mode (the Generalized Box-Cox utility function case).

1.2 Detailed Explicitation of all Elasticity Terms adding up to (F) in Formula (1)

The precisions found in Table 2 imply that the resulting elasticity value (F) in Equation (1) can be calculated for 20 combinations of models selected from 12 model types, each pair involving components of reasonably complex structure.

We will now state the analytical expressions for these elasticities, taking each part of (F) in turn. When derivations of explicit analytical elasticity expressions are found in the general econometric literature, as for terms (A), (B) and (D), we will repeat them for memory and clarity. When they are hard to find in the literature or are not sufficiently explicit there, as for terms (C) and (D), we will provide them.

This explicitation will be effected in four sub-sections. The first three pertain successively to terms (A)-(B), (C), and (D); the fourth explains how all of these elasticity expressions are modified when the X_k variable of interest is not a continuous variable but is a dummy variable. In these four sub-sections, we change the numbering of successive equations used above by adding a prefix (*e.g.* A, B, or C) to the equations numbers, all reset to 1 at the beginning of a sub-section, to remind the reader of the term of (1) being documented.

1.2.1 Explicitation of Elasticity Terms (A) and (B)

The sample elasticities (A) and (B) evaluated at the means are found in the documents describing the LEVEL 1.6 (Tran *et al*., 2008; AJD-105) or LEVEL 2.1 (Tran and Gaudry, 2008a; AJD-108) estimation procedures yielding the L_0 , L_1 and L_2 terms of Table 2. We now rederive them.

In L_0, the regression equation for total demand T_t can be written as:

$$T_t^{(\lambda_T)} = \sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + u_t \ , \ t = 1,\dots,N \qquad \text{(AB-1)}$$

where the residuals u_t ’s are assumed to be heteroskedastic with zero mean:

$$u_t = \left[f(Z_t) \right]^{1/2} \bullet \ v_t = \left[exp\left(\sum_m \delta_m \cdot Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet \ v_t \, . \qquad \text{(AB-2)}$$

<u>*Derivation of elasticity terms (A) and (B) for L_1*</u>

Assume further that the residuals v_t ’s follow a stationary autoregressive ***serial*** process of order r :

$$v_t = \sum_{\ell=1}^{r} \rho_l \cdot v_{t-l} + w_t \, . \qquad \text{(AB-3-S)}$$

where the new residuals w_t ’s are assumed to have a zero mean. In this case, the total demand equation (AB-1) can be rewritten with u_t replaced by its expression in (AB-2) as:

$$T_t = \left[1 + \lambda_T \left(\sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + u_t \right) \right]^{1/\lambda_T}$$

$$T_t = \left\{ 1 + \lambda_T \left(\sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + \left[exp\left(\sum_m \delta_m \cdot Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet \; v_t \right) \right\}^{1/\lambda_T}$$

$$= E_t^{1/\lambda_T} \qquad \text{(AB-4)}$$

where $E_t = 1 + \lambda_T \left(\sum_{k=1}^{K} \beta_k \cdot X_{kt}^{(\lambda_{X_k})} + \beta_U \cdot U_t^{(\lambda_U)} + \left[exp\left(\sum_m \delta_m \cdot Z_{mt}^{(\lambda_{z_m})} \right) \right]^{1/2} \bullet \; v_t \right)$.

but there is, for our purposes, no need to replace v_t by its expression given in (AB-3-S) since v_t is neither a function of X_{kt} with respect to which the elasticity term (A) is presently derived, nor a function of U_t with respect to which the elasticity term (B) is also presently derived.

a) Elasticity term (A)

The elasticity $\tilde{\eta}(T_t, X_{kt})$ is given by:

$$\tilde{\eta}(T_t, X_{kt}) = \frac{\partial T_t}{\partial X_{kt}} \frac{X_{kt}}{T_t} \qquad \text{(AB-5)}$$

where

$$\frac{\partial T_t}{\partial X_{kt}} = \frac{1}{\lambda_T} E_t^{\frac{1}{\lambda_T}-1} \frac{\partial E_t}{\partial X_{kt}} = \frac{1}{\lambda_T} \frac{E_t^{\frac{1}{\lambda_T}}}{E_t} \frac{\partial E_t}{\partial X_{kt}} = \frac{1}{\lambda_T} \frac{T_t}{T_t^{\lambda_T}} \frac{\partial E_t}{\partial X_{kt}}$$

and

$$\frac{\partial E_t}{\partial X_{kt}} = \begin{cases} \lambda_T \beta_k X_{kt}^{\lambda_{X_k}-1} & \text{, if } f(Z_t) \text{ is not a function of } X_{kt}. \\ \lambda_T \beta_k X_{kt}^{\lambda_{X_k}-1} + \lambda_T \frac{1}{2} \delta_m Z_{mt}^{\lambda_{Z_m}-1} (T^{(\lambda_T)} - \sum_{k=1}^{K} \beta_k X_{kt}^{(\lambda_{X_k})} - \beta_U U_t^{(\lambda_U)}) & \text{, if } f(Z_t) \text{ contains one } Z_{mt} = X_{kt}. \end{cases}$$

Hence

$$\tilde{\eta}(T_t, X_{kt}) = \frac{\partial T_t}{\partial X_{kt}} \frac{X_{kt}}{T_t} = \frac{1}{\lambda_T} \frac{T_t}{T_t^{\lambda_T}} \frac{\partial E_t}{\partial X_{kt}} \frac{X_{kt}}{T_t} = \frac{1}{\lambda_T T_t^{\lambda_T}} \frac{\partial E_t}{\partial X_{kt}} X_{kt}$$

$$= \begin{cases} \frac{1}{T_t^{\lambda_T}} \beta_k X_{kt}^{\lambda_{X_k}} & \text{, if } f(Z_t) \text{ is not a function of } X_{kt}. \\ \frac{1}{T_t^{\lambda_T}} \left[\beta_k X_{kt}^{\lambda_{X_k}} + \frac{1}{2} \delta_m Z_{mt}^{\lambda_{Z_m}} (T^{(\lambda_T)} - \sum_{k=1}^{K} \beta_k X_{kt}^{(\lambda_{X_k})} - \beta_U U_t^{(\lambda_U)}) \right] & \text{, if } f(Z_t) \text{ contains one } Z_{mt} = X_{kt}. \end{cases}$$

b) Elasticity term (B)

The elasticity $\eta(T_t, U_t)$ is given by:

$$\eta(T_t, U_t) = \frac{\partial T_t}{\partial U_t} \frac{U_t}{T_t} \qquad \text{(AB-6)}$$

where

$$\frac{\partial T_t}{\partial U_t} = \frac{1}{\lambda_T} E_t^{\frac{1}{\lambda_T}-1} \frac{\partial E_t}{\partial U_t} = \frac{1}{\lambda_T} \frac{E_t^{\frac{1}{\lambda_T}}}{E_t} \frac{\partial E_t}{\partial U_t} = \frac{1}{\lambda_T} \frac{T_t}{T_t^{\lambda_T}} \frac{\partial E_t}{\partial U_t} = \frac{1}{\lambda_T} \frac{T_t}{T_t^{\lambda_T}} \lambda_T \beta_U U_t^{\lambda_U - 1}.$$

Hence

$$\eta(T_t,U_t)=\frac{\partial T_t}{\partial U_t}\frac{U_t}{T_t}=\beta_U\,\frac{U_t^{\lambda_U}}{T_t^{\lambda_T}}.$$

It follows that, in the presence of heteroskedasticity and ***serial autocorrelation*** (L_1), the analytical expressions for the two elasticity terms (A) and (B) can be summarized as in the following table.

(A case)	(B case)
$\tilde{\eta}(T_t,X_{kt})=\frac{1}{T_t^{\lambda_T}}(\beta_k X_{kt}^{\lambda_{X_k}}+G_{X_kt})$	$\eta(T_t,U_t)=\beta_U\,\frac{U_t^{\lambda_U}}{T_t^{\lambda_T}}$
where $G_{X_kt}=\begin{cases}0 & \text{, if } f(Z_t) \text{ is not a function of } X_{kt}.\\ \frac{1}{2}\delta_m Z_{mt}^{\lambda_{Z_m}}(T^{(\lambda_T)}-\sum_{k=1}^{K}\beta_k X_{kt}^{(\lambda_{X_k})}-\beta_U U_t^{(\lambda_U)}), & \text{if } f(Z_t) \text{ contains one } Z_{mt}=X_{kt}.\end{cases}$ and $f(Z_t)=\exp\left(\sum_m \delta_m Z_{mt}^{(\lambda_{Z_m})}\right)$.	

<u>*Derivation of elasticity terms (A) and (B) for L_2*</u>

Given again (AB-1) and (AB-2), now assume further that the residual v_t's, instead of following the ***serial*** autoregressive process (AB-3-S), follow a stationary autoregressive ***directed*** process of first or second order with zero mean:

$$v_t=\sum_{l=1}^{2}\rho_l\cdot\left(\sum_{n=1}^{N}\tilde{r}_{l,tn}v_n\right)+w_t\,. \qquad \text{(AB-3-D)}$$

where the residuals w_t's are assumed to have a zero mean.

The total demand equation (AB-1), rewritten with u_t replaced by its expression in (AB-2), will again yield (AB-4) because there is no need to replace v_t by its expression given in (AB-3-D) since v_t is neither function of X_{kt} with respect to which the elasticity term (A) needs to be derived, nor a function of U_t with respect to which the elasticity term (B) also needs to be derived.

Because (AB-4) remains unchanged, the (A) elasticity $\tilde{\eta}(T_t,X_{kt})$ and the (B) elasticity $\eta(T_t,U_t)$, defined again respectively as

$$\tilde{\eta}(T_t,X_{kt})=\frac{\partial T_t}{\partial X_{kt}}\frac{X_{kt}}{T_t} \quad \text{and} \quad \eta(T_t,U_t)=\frac{\partial T_t}{\partial U_t}\frac{U_t}{T_t} \qquad \text{(AB-7)}$$

will, in the presence of heteroskedasticity and ***directed autocorrelation*** (L_2), be identical to those found in the previous (L_1) case, and the summary table just constructed will equally hold for (L_2).

In fact, autocorrelation has no effect on the analytical expressions for (AB-7); and heteroskedasticity matters only in the rare case in which $f(Z_t)$ contains one $Z_{mt} = X_{kt}$. If heteroskedasticity depends on variables that are not regressors, (A) and (B) derivatives with respect to regressors are identically structured.

Level model summary table

For those reasons, the analytical expressions for (A) and (B) elasticities listed in Table 3 for the different combinations of heteroskedasticity and autocorrelation allowed by L_0 , L_1 and L_2 exhibit a simple repetitive pattern.

Table 3. Elasticity terms (A) and (B) for heteroskedasticity and autocorrelation combinations

LEVEL	**1.6**		**2.1**	
Column	**1**	**2**	**3**	**4**
	(L_0)	(L_1)	(L_0)	(L_2)
	Autocorrelation		***Autocorrelation***	
Heteroskedasticity	***None***	***Serial***	***None***	***Directed***
No	(A-1), (B)	(A-1), (B)	(A-1), (B)	(A-1), (B)
Yes	(A-2), (B)	(A-2), (B)	(A-2), (B)	(A-2), (B)

and the combinations of bracketed terms denote

(A-1)

$$\tilde{\eta}(T_t, X_{kt}) = \beta_k \frac{X_{kt}^{\lambda_{X_k}}}{T_t^{\lambda_T}}$$

(B)

$$\eta(T_t, U_t) = \beta_U \frac{U_t^{\lambda_U}}{T_t^{\lambda_T}}$$

(A-2)

$$\tilde{\eta}(T_t, X_{kt}) = \frac{1}{T_t^{\lambda_T}} (\beta_k X_{kt}^{\lambda_{X_k}} + G_{X_k t})$$

where

$$G_{X_k t} = \begin{cases} 0 & \text{, if } f(Z_t) \text{ is not a function of } X_{kt}. \\ \frac{1}{2} \delta_m Z_{mt}^{\lambda_{Z_m}} (T^{(\lambda_T)} - \sum_{k=1}^{K} \beta_k X_{kt}^{(\lambda_{X_k})} - \beta_U U_t^{(\lambda_U)}), & \text{if } f(Z_t) \text{ contains one } Z_{mt} = X_{kt}. \end{cases}$$

and

$$f(Z_t) = \exp\left(\sum_m \delta_m Z_{mt}^{(\lambda_{Z_m})} \right) .$$

1.2.2 Explicitation of Utility Elasticity Term (C)

Unfortunately, similarly explicit expressions of elasticity corresponding to term (C) in Equation (1) are not easily found in the literature, except perhaps on page 3 of Tran and Gaudry (2008c; AJD-117) where the formulas may well be written too compactly for an intuitive grasp of their contents. There is therefore a particular need to explicate what (C) is actually made of because, in the transport literature, the induced or "generated" transport demand elasticity is commonly defined in two different ways: (i) as (B) only, usually called the *induction* elasticity; (ii) as the product (B)*(C), typically called the *modal induction* elasticity.

It is important to reiterate that, because TABQDF only uses elasticities evaluated at sample means, the forthcoming expressions apply equally to SHARE and to PROBABILITY elasticities for corresponding S_s and in P_p model types used in Table 2 and listed in Table 4.

For this explication and clarification of the contents of (C), we must first document the mode split model families of interest and then specify the two kinds of utility functions allowed in all 5 families of the PROBABILITY and SHARE estimation procedures and programs, namely the Standard and Generalized representative utility function formulation.

Modal utilities. In (0-3), the *modal utility* U_m specification actually defines the 5 model families allowed for in the PROBABILITY and SHARE estimations procedures. Given that V_m (to be specified in more detail below) denotes the *representative utility* of mode m, these 5 families are listed in Table 4 with slight changes in indices from the corresponding expressions found on page 5.

Table 4. The five modal utilities specified in SHARE and PROBABILITY procedures

MODEL FAMILY	Modal utility $U_m \geq 0$
LOGIT	$\exp(V_m)$
STANDARD DOGIT	$\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j)$, $\theta_m \geq 0$
GENERALIZED DOGIT	$\exp(V_m) + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j)$, $\theta_{mm} = 0$ and $\theta_{mj} \geq 0$
LIN-IPT-LOGIT	$\left[\phi_m \exp(V_m) + 1\right]^{1/\phi_m} - \mu_m$, $\phi_m \geq 0$ and $\mu_m \leq 1$
BT-IPT-LOGIT	$\exp\left\{\dfrac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m}\right\}$, $\tilde{\mu}_m \geq 0$

Representative utility functions. The representative utility components V_m to be distinguished in all modal utilities found in Table 4 belong to the two groups of specifications also found on page 4:

i) **Standard representative utility formulation**

In this case, a network variable X_{mn} appears only in its own mode m. The form of V_m is as follows:

$$V_m = \beta_{m0} + \sum_n \beta_{mn} X_{mn}^{(\lambda_{mn})} + \sum_s \beta_{ms} X_{ms}^{(\lambda_{ms})} \tag{C-1}$$

where n and s are the indices for the network and socioeconomic variables respectively.

ii) **Generalized representative utility formulation**

In this case, a network variable X_{mn} appears not only in its own mode m but also as X_{jn} across all the other modes $j \neq m$. The corresponding V_m and V_j can be written as:

$$\begin{aligned} V_m &= \beta_{m0} + \sum_n \beta_{mn} X_{mn}^{(\lambda_{mn})} + \sum_s \beta_{ms} X_{ms}^{(\lambda_{ms})} \\ V_j &= \beta_{j0} + \sum_n \beta_{jn} X_{jn}^{(\lambda_{jn})} + \sum_s \beta_{js} X_{js}^{(\lambda_{js})} \ . \end{aligned} \tag{C-2}$$

If we assume for expository purposes that the particular variable X_{mn} of interest is the price of the rail mode transformed by the Box-Cox parameter λ_{mn} in the own rail utility function and by λ_{jn} in the utility functions pertaining to other modes, we can document the impact of changes in this price X_{mn} on the utility term U defined as the denominator of the given choice model. For additional simplicity, we also relabel the X_{mn} price variable as X .

We are now in a position to explicate the (C) term of Equation (1), systematically distinguishing between Standard and Generalized representative utility function categories for all modal utility families. We therefore consider in turn each of the (indifferently aggregate or disaggregate) mode split model families: Logit, Standard Dogit, Generalized Dogit, Lin-IPT-Logit and BT-IPT-Logit.

LOGIT

Standard utility case. As, in (C-1), X appears only in the rail mode utility function with a Box-Cox parameter $\lambda_{rail,X}$, the elasticity of U with respect to X can be expressed as:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X}\frac{X}{U} = \frac{\partial \exp(V_{rail})}{\partial X}\frac{X}{U}$$

where

$$\frac{\partial \exp(V_{rail})}{\partial X} = \exp(V_{rail})\frac{\partial V_{rail}}{\partial X} = \beta_{rail,X}\frac{\partial X^{(\lambda_{rail,X})}}{\partial X}\exp(V_{rail}) = \beta_{rail,X} X^{\lambda_{rail,X}-1}\exp(V_{rail}),$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \beta_{rail,X} X^{\lambda_{rail,X}} \frac{\exp(V_{rail})}{\sum_{m=1}^{M}\exp(V_m)} = \beta_{rail,X} X^{\lambda_{rail,X}} P_{rail} \quad \text{(C-3)}$$

where $P_{rail} = \exp(V_{rail}) / \sum_{m=1}^{M}\exp(V_m)$ is the rail mode share or choice probability.

Generalized utility case. In (C-2), the rail price variable X appears in all utility functions with a Box-Cox parameter λ_{mX} specific to each modal function. We therefore have in this case:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X}\frac{X}{U} = \frac{\partial \sum_{m=1}^{M}\exp(V_m)}{\partial X}\frac{X}{U}$$

where

$$\frac{\partial \sum_{m=1}^{M}\exp(V_m)}{\partial X} = \sum_{m=1}^{M}\exp(V_m)\frac{\partial V_m}{\partial X} = \sum_{m=1}^{M}\beta_{mX}\frac{\partial X^{(\lambda_{mX})}}{\partial X}\exp(V_m) = \sum_{m=1}^{M}\beta_{mX} X^{\lambda_{mX}-1}\exp(V_m),$$

so that, if P_m is the market share or choice probability of mode *m*, the elasticity can be rewritten:

$$\eta_{U.X} = \sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} \frac{\exp(V_m)}{\sum_{m=1}^{M} \exp(V_m)} = \sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} P_m \tag{C-4}$$

where $P_m = \exp(V_m) / \sum_{m=1}^{M} \exp(V_m)$ is the m-th mode share or choice probability.

STANDARD DOGIT

Standard utility case. As, in (C-1), X appears only in the rail mode utility function with a Box-Cox parameter $\lambda_{rail,X}$, the elasticity of U with respect to X can be expressed as:

$$\eta_{U.X} = \frac{\partial U}{\partial X} \frac{X}{U}$$

where

$$\begin{aligned}
\frac{\partial U}{\partial X} &= \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\partial \sum_{m=1}^{M} \left[\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j) \right]}{\partial X} \\
&= \sum_{m=1}^{M} \frac{\partial \exp(V_m)}{\partial X} + \sum_{m=1}^{M} \theta_m \sum_{j=1}^{M} \frac{\partial \exp(V_j)}{\partial X} \\
&= \frac{\partial \exp(V_{rail})}{\partial X} + \sum_{m=1}^{M} \theta_m \frac{\partial \exp(V_{rail})}{\partial X} \\
&= (1 + \sum_{m=1}^{M} \theta_m) \frac{\partial \exp(V_{rail})}{\partial X} \\
&= (1 + \sum_{m=1}^{M} \theta_m) \exp(V_{rail}) \frac{\partial V_{rail}}{\partial X} \\
&= (1 + \sum_{m=1}^{M} \theta_m) \exp(V_{rail}) \beta_{rail,X} \frac{\partial X^{(\lambda_{rail,X})}}{\partial X} \\
&= (1 + \sum_{m=1}^{M} \theta_m) \exp(V_{rail}) \beta_{rail,X} X^{\lambda_{rail,X} - 1}
\end{aligned}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{(1 + \sum_{m=1}^{M} \theta_m) \beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})}{\sum_{m=1}^{M} \left[\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j) \right]} \ . \tag{C-5}$$

Generalized utility case. In (C-2), the rail price variable X appears in all utility functions with a Box-Cox parameter λ_{mX} specific to each modal function. We therefore have in this case:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U}$$

where

$$\begin{aligned}
\frac{\partial U}{\partial X} &= \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\partial \sum_{m=1}^{M}\left[\exp(V_m) + \theta_m \sum_{j=1}^{M}\exp(V_j)\right]}{\partial X} \\
&= \sum_{m=1}^{M}\frac{\partial \exp(V_m)}{\partial X} + \sum_{m=1}^{M}\theta_m \sum_{j=1}^{M}\frac{\partial \exp(V_j)}{\partial X} \\
&= \sum_{m=1}^{M}\exp(V_m)\frac{\partial V_m}{\partial X} + \sum_{m=1}^{M}\theta_m \sum_{j=1}^{M}\exp(V_j)\frac{\partial V_j}{\partial X} \\
&= \sum_{m=1}^{M}\exp(V_m)\beta_{mX}\,\frac{\partial X^{(\lambda_{mX})}}{\partial X} + \sum_{m=1}^{M}\theta_m \sum_{j=1}^{M}\exp(V_j)\beta_{jX}\,\frac{\partial X^{(\lambda_{jX})}}{\partial X} \\
&= \sum_{m=1}^{M}\exp(V_m)\beta_{mX}X^{\lambda_{mX}-1} + \sum_{m=1}^{M}\theta_m \sum_{j=1}^{M}\exp(V_j)\beta_{jX}X^{\lambda_{jX}-1} \\
&= (1+\sum_{m=1}^{M}\theta_m)\sum_{m=1}^{M}\exp(V_m)\beta_{mX}X^{\lambda_{mX}-1}
\end{aligned}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{(1+\sum_{m=1}^{M}\theta_m)\sum_{m=1}^{M}\beta_{mX}X^{\lambda_{mX}}\exp(V_m)}{\sum_{m=1}^{M}\left[\exp(V_m)+\theta_m\sum_{j=1}^{M}\exp(V_j)\right]}\ . \qquad \text{(C-6)}$$

GENERALIZED DOGIT

Standard utility case. As, in (C-1), X appears only in the rail mode utility function with a Box-Cox parameter $\lambda_{rail,X}$, the elasticity of U with respect to X can be expressed as:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U}$$

where

$$\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\partial \sum_{m=1}^{M} \left[\exp(V_m) + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j) \right]}{\partial X}$$
$$= \sum_{m=1}^{M} \frac{\partial \exp(V_m)}{\partial X} + \sum_{m=1}^{M} \sum_{j \neq m}^{M} \theta_{mj} \frac{\partial \exp(V_j)}{\partial X}$$
$$= \exp(V_{rail}) \frac{\partial V_{rail}}{\partial X} + \sum_{m \neq rail}^{M} \theta_{m,rail} \exp(V_{rail}) \frac{\partial V_{rail}}{\partial X}$$
$$= (1 + \sum_{m \neq rail}^{M} \theta_{m,rail}) \exp(V_{rail}) \frac{\partial V_{rail}}{\partial X}$$
$$= (1 + \sum_{m \neq rail}^{M} \theta_{m,rail}) \exp(V_{rail}) \beta_{rail,X} \frac{\partial X^{(\lambda_{rail,X})}}{\partial X}$$
$$= (1 + \sum_{m \neq rail}^{M} \theta_{m,rail}) \exp(V_{rail}) \beta_{rail,X} X^{\lambda_{rail,X} - 1}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{(1 + \sum_{m \neq rail}^{M} \theta_{m,rail}) \beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})}{\sum_{m=1}^{M} \left[\exp(V_m) + \sum_{j = m}^{M} \theta_{mj} \exp(V_j) \right]} . \quad \text{(C-7)}$$

Generalized utility case. In (C-2), the rail price variable X appears in all utility functions with a Box-Cox parameter λ_{mX} specific to each modal function. We therefore have in this case:

$$\eta_{U.X} = \frac{\partial U}{\partial X} \frac{X}{U}$$

where

$$\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\sum_{m=1}^{M} \left[\exp(V_m) + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j) \right]}{\partial X}$$
$$= \sum_{m=1}^{M} \left[\frac{\partial \exp(V_m)}{\partial X} + \sum_{j \neq m}^{M} \theta_{mj} \frac{\partial \exp(V_j)}{\partial X} \right]$$
$$= \sum_{m=1}^{M} \left[\exp(V_m) \frac{\partial V_m}{\partial X} + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j) \frac{\partial V_j}{\partial X} \right]$$
$$= \sum_{m=1}^{M} \left[\exp(V_m) \beta_{mX} \frac{\partial X^{(\lambda_{mX})}}{\partial X} + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j) \beta_{jX} \frac{\partial X^{(\lambda_{jX})}}{\partial X} \right]$$
$$= \sum_{m=1}^{M} \left[\exp(V_m) \beta_{mX} X^{\lambda_{mX} - 1} + \sum_{j \neq m}^{M} \theta_{mj} \exp(V_j) \beta_{jX} X^{\lambda_{jX} - 1} \right]$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{\sum_{m=1}^{M}\left[\beta_{mX} X^{\lambda_{mX}} \exp(V_m) + \sum_{j\neq m}^{M} \theta_{mj}\beta_{jX} X^{\lambda_{jX}} \exp(V_j)\right]}{\sum_{m=1}^{M}\left[\exp(V_m) + \sum_{j\neq m}^{M} \theta_{mj} \exp(V_j)\right]} . \tag{C-8}$$

LIN-IPT-LOGIT

Standard utility case. As, in (C-1), X appears only in the rail mode utility function with a Box-Cox parameter $\lambda_{rail,X}$, the elasticity of U with respect to X can be expressed as:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U}$$

where

$$\begin{aligned}
\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} &= \frac{\partial \sum_{m=1}^{M}\left\{\left[\phi_m \exp(V_m)+1\right]^{\frac{1}{\phi_m}} - \mu_m\right\}}{\partial X} \\
&= \left[\phi_{rail} \exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1} \frac{\partial \exp(V_{rail})}{\partial X} \\
&= \left[\phi_{rail} \exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1} \exp(V_{rail}) \frac{\partial V_{rail}}{\partial X} \\
&= \left[\phi_{rail} \exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1} \exp(V_{rail}) \beta_{rail,X} \frac{\partial X^{(\lambda_{rail,X})}}{\partial X} \\
&= \left[\phi_{rail} \exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1} \exp(V_{rail}) \beta_{rail,X} X^{\lambda_{rail,X}-1}
\end{aligned}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{\beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})\left[\phi_{rail} \exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1}}{\sum_{m=1}^{M}\left\{\left[\phi_m \exp(V_m)+1\right]^{\frac{1}{\phi_m}} - \mu_m\right\}} . \tag{C-9}$$

Generalized utility case. In (C-2), the rail price variable X appears in all utility functions with a Box-Cox parameter λ_{mX} specific to each modal function. We therefore have in this case:

$$\eta_{U.X} = \frac{\partial U}{\partial X}\frac{X}{U}$$

where

$$\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\partial \sum_{m=1}^{M} \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m}} - \mu_m \right\}}{\partial X}$$

$$= \sum_{m=1}^{M} \frac{\partial \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m}} - \mu_m \right\}}{\partial X}$$

$$= \sum_{m=1}^{M} \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m} - 1} \exp(V_m) \frac{\partial V_m}{\partial X} \right\}$$

$$= \sum_{m=1}^{M} \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m} - 1} \exp(V_m) \beta_{mX} \frac{\partial X^{(\lambda_{mX})}}{\partial X} \right\}$$

$$= \sum_{m=1}^{M} \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m} - 1} \exp(V_m) \beta_{mX} X^{\lambda_{mX} - 1} \right\}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{\sum_{m=1}^{M} \left\{ \beta_{mX} X^{\lambda_{mX}} \exp(V_m) \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m} - 1} \right\}}{\sum_{m=1}^{M} \left\{ \left[\phi_m \exp(V_m) + 1 \right]^{\frac{1}{\phi_m}} - \mu_m \right\}} . \qquad \text{(C-10)}$$

BT-IPT-LOGIT

Standard utility case. As, in (C-1), X appears only in the rail mode utility function with a Box-Cox parameter $\lambda_{rail,X}$, the elasticity of U with respect to X can be expressed as:

$$\eta_{U.X} = \frac{\partial U}{\partial X} \frac{X}{U}$$

where

$$\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} = \frac{\partial \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}{\partial X}$$

$$= \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\} \frac{\partial \left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\}}{\partial X}$$

$$= \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail} - 1} \frac{\partial \exp(V_{rail})}{\partial X}$$

$$= \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail} - 1} \beta_{rail,X} \frac{\partial X^{(\lambda_{rail,X})}}{\partial X}$$

$$= \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail} - 1} \beta_{rail,X} X^{\lambda_{rail,X} - 1}$$

so that the elasticity can be rewritten as:

$$\eta_{U.X} = \frac{\beta_{rail,X} X^{\lambda_{rail,X}} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail} - 1} \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\}}{\sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}$$

$$= \beta_{rail,X} X^{\lambda_{rail,X}} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail} - 1} P_{rail} \qquad \text{(C-11)}$$

where

$$P_{rail} = \exp\left\{ \frac{\left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}} - 1}{\tilde{\phi}_{rail}} \right\} / \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}$$

is the rail mode share or choice probability.

Generalized utility case. In (C-2), the rail price variable X appears in all utility functions with a Box-Cox parameter λ_{mX} specific to each modal function. We therefore have in this case:

$$\eta_{U.X} = \frac{\partial U}{\partial X} \frac{X}{U}$$

where

$$
\begin{aligned}
\frac{\partial U}{\partial X} = \frac{\partial \sum_{m=1}^{M} U_m}{\partial X} &= \frac{\partial \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}{\partial X} \\
&= \frac{\sum_{m=1}^{M} \partial \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}{\partial X} \\
&= \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \frac{\partial \left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}{\partial X} \\
&= \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} \frac{\partial \exp(V_m)}{\partial X} \\
&= \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} \exp(V_m) \frac{\partial V_m}{\partial X} \\
&= \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} \exp(V_m) \beta_{mX} \frac{\partial X^{(\lambda_{mX})}}{\partial X} \\
&= \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} \exp(V_m) \beta_{mX} X^{\lambda_{mX} - 1}
\end{aligned}
$$

so that the elasticity can be rewritten as:

$$
\begin{aligned}
\eta_{U.X} &= \frac{\sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} \exp(V_m) \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}}{\sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}} \\
&= \sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} \exp(V_m) \left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m - 1} P_m \qquad \text{(C-12)}
\end{aligned}
$$

where

$$
P_m = \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\} \Big/ \sum_{m=1}^{M} \exp\left\{ \frac{\left[\exp(V_m) + \tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m} \right\}
$$

is the m-th mode share or choice probability.

Utility Elasticity Term (C): a Summary of Standard and Generalized cases. The 10 analytical expressions of elasticity term (C) just derived above are collected in Table 5.

Table 5. Elasticity term (C), or $\eta(U, X_k)$, when X_{rail} is transformed by $\lambda_{rail,X}$, λ_{jX} or λ_{mX}

FAMILY	**Modal utility** $U_m \geq 0$
LOGIT	$\exp(V_m)$
$\beta_{rail,X} X^{\lambda_{rail,X}} P_{rail}$	$\sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} P_m$
STANDARD DOGIT	$\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j)$, $\theta_m \geq 0$
$\dfrac{(1+\sum_{m=1}^{M}\theta_m)\beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})}{\sum_{m=1}^{M}\left[\exp(V_m) + \theta_m \sum_{j=1}^{M}\exp(V_j)\right]}$	$\dfrac{(1+\sum_{m=1}^{M}\theta_m)\sum_{m=1}^{M}\beta_{mX} X^{\lambda_{mX}} \exp(V_m)}{\sum_{m=1}^{M}\left[\exp(V_m) + \theta_m \sum_{j=1}^{M}\exp(V_j)\right]}$
GENERALIZED DOGIT	$\exp(V_m) + \sum_{j\neq m}^{M} \theta_{mj} \exp(V_j)$, $\theta_{mm} = 0$ and $\theta_{mj} \geq 0$
$\dfrac{(1+\sum_{m\neq rail}^{M}\theta_{m,rail})\beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})}{\sum_{m=1}^{M}\left[\exp(V_m) + \sum_{j=m}^{M}\theta_{mj}\exp(V_j)\right]}$	$\dfrac{\sum_{m=1}^{M}\left[\beta_{mX} X^{\lambda_{mX}} \exp(V_m) + \sum_{j\neq m}^{M}\theta_{mj}\beta_{jX} X^{\lambda_{jX}} \exp(V_j)\right]}{\sum_{m=1}^{M}\left[\exp(V_m) + \sum_{j\neq m}^{M}\theta_{mj}\exp(V_j)\right]}$
LIN-IPT-LOGIT	$\left[\phi_m \exp(V_m) + 1\right]^{1/\phi_m} - \mu_m$, $\phi_m \geq 0$ and $\mu_m \leq 1$
$\dfrac{\beta_{rail,X} X^{\lambda_{rail,X}} \exp(V_{rail})\left[\phi_{rail}\exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1}}{\sum_{m=1}^{M}\left\{\left[\phi_m \exp(V_m)+1\right]^{\frac{1}{\phi_m}} - \mu_m\right\}}$	$\dfrac{\sum_{m=1}^{M}\left\{\beta_{mX} X^{\lambda_{mX}} \exp(V_m)\left[\phi_m \exp(V_m)+1\right]^{\frac{1}{\phi_m}-1}\right\}}{\sum_{m=1}^{M}\left\{\left[\phi_m \exp(V_m)+1\right]^{\frac{1}{\phi_m}} - \mu_m\right\}}$
BT-IPT-LOGIT	$\exp\left\{\dfrac{\left[\exp(V_m)+\tilde{\mu}_m\right]^{\tilde{\phi}_m} - 1}{\tilde{\phi}_m}\right\}$, $\tilde{\mu}_m \geq 0$
$\beta_{rail,X} X^{\lambda_{rail,X}} \left[\exp(V_{rail}) + \tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}-1} P_{rail}$	$\sum_{m=1}^{M}\beta_{mX} X^{\lambda_{mX}} \exp(V_m)\left[\exp(V_m)+\tilde{\mu}_m\right]^{\tilde{\phi}_m-1} P_m$
Standard representative utility function	**Generalized representative utility function**

1.2.3 Explicitation of Elasticity Term (D)

Analytical expressions for term (D) are found in the PROBABILITY P-2/P-6 (Tran and Gaudry, 2009a; AJD-111) and SHARE S-1/S-5 (Tran and Gaudry, 2008b; AJD-114) documents. We repeat them for convenience and make them as explicit as those just provided for term (C), distinguishing for each of the 5 families listed in Table 4 between Standard and Generalized representative utility function specifications.

It is again important to note that the forthcoming expressions apply equally to SHARE and to PROBABILITY elasticities P and S in Table 2 because TABQDF solely uses elasticities evaluated at sample means.

Table 6. Elasticity term (D), or $\eta(P_m, X_k)$, when X_{rail} is transformed by $\lambda_{rail,X}$, λ_{jX} or λ_{mX}

Part 1	
FAMILY	**Probability or Share P_m**
LOGIT	$\exp(V_m) / \sum_{m=1}^{M} \exp(V_m)$
$\beta_{rail,X} X^{\lambda_{rail,X}} \left(1-P_{rail}\right)$	$\beta_{rail,X} X^{\lambda_{rail,X}} \left(1-P_{rail}\right) - P_{rail} \sum_{m \neq rail}^{M} \beta_{mX} X^{\lambda_{mX}}$
STANDARD DOGIT	$\left[\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j) \right] \Big/ \sum_{m=1}^{M} \left[\exp(V_m) + \theta_m \sum_{j=1}^{M} \exp(V_j) \right], \ \theta_m \geq 0$
$\dfrac{\beta_{rail,X} X^{\lambda_{rail,X}} (1-P_{rail})(1+\theta_{rail})}{1+\theta_{rail} \sum_{j=1}^{M} \exp(V_j - V_{rail})}$	$\dfrac{A_{rail}(1-P_{rail}) - P_{rail} \sum_{m \neq rail}^{M} A_m}{1+\theta_{rail} \sum_{j=1}^{M} \exp(V_j - V_{rail})}$ where $A_{rail} = \left[\beta_{rail,X} X^{\lambda_{rail,X}} + \theta_{rail} \sum_{m=1}^{M} \beta_{mX} X^{\lambda_{mX}} \exp(V_m - V_{rail}) \right]$ $A_m = \left[\beta_{mX} X^{\lambda_{mX}} \exp(V_m - V_{rail}) + \theta_m \sum_{j=1}^{M} \beta_{jX} X^{\lambda_{jX}} \exp(V_j - V_{rail}) \right]$
Standard utility function	**Generalized utility function**

Part 2	
FAMILY	**Probability or Share** P_m
GENERALIZED DOGIT	$\left[\exp(V_m)+\sum_{j\neq m}^{M}\theta_{mj}\exp(V_j)\right]\Big/\sum_{m=1}^{M}\left[\exp(V_m)+\sum_{j\neq m}^{M}\theta_{mj}\exp(V_j)\right],$ $\theta_{mm}=0$ and $\theta_{mj}\geq 0$
$\dfrac{\beta_{rail,X}X^{\lambda_{rail,X}}(1-P_{rail})}{1+\sum_{j\neq rail}^{M}\theta_{rail,j}\exp(V_j-V_{rail})}$	$\dfrac{B_{rail}(1-P_{rail})-P_{rail}\sum_{m\neq rail}^{M}B_m}{1+\sum_{j\neq rail}^{M}\theta_{rail,j}\exp(V_j-V_{rail})}$ where $B_{rail}=\left[\beta_{rail,X}X^{\lambda_{rail,X}}+\sum_{m\neq rail}^{M}\theta_{rail,m}\beta_{mX}X^{\lambda_{mX}}\exp(V_m-V_{rail})\right]$ $B_m=\left[\beta_{mX}X^{\lambda_{mX}}\exp(V_m-V_{rail})+\sum_{j\neq m}^{M}\theta_{mj}\beta_{jX}X^{\lambda_{jX}}\exp(V_j-V_{rail})\right]$
LIN-IPT-LOGIT	$\left\{\left[\phi_m\exp(V_m)+1\right]^{1/\phi_m}-\mu_m\right\}\Big/\sum_{m=1}^{M}\left\{\left[\phi_m\exp(V_m)+1\right]^{1/\phi_m}-\mu_m\right\},$ $\phi_m\geq 0$ and $\mu_m\leq 1$
$\dfrac{C_{rail}(1-P_{rail})}{\left[\phi_{rail}\exp(V_{rail})+1\right]^{1/\phi_{rail}}-\mu_{rail}}$	$\dfrac{C_{rail}(1-P_{rail})-P_{rail}\sum_{m\neq rail}^{M}C_m}{\left[\phi_{rail}\exp(V_{rail})+1\right]^{1/\phi_{rail}}-\mu_{rail}}$ where $C_{rail}=\left[\phi_{rail}\exp(V_{rail})+1\right]^{\frac{1}{\phi_{rail}}-1}\exp(V_{rail})\beta_{rail,X}X^{\lambda_{rail,X}}$ $C_m=\left[\phi_m\exp(V_m)+1\right]^{\frac{1}{\phi_m}-1}\exp(V_m)\beta_{mX}X^{\lambda_{mX}}$
BT-IPT-LOGIT	$\exp\left\{\dfrac{\left[\exp(V_m)+\tilde{\mu}_m\right]^{\tilde{\phi}_m}-1}{\tilde{\phi}_m}\right\}\Big/\sum_{m=1}^{M}\exp\left\{\dfrac{\left[\exp(V_m)+\tilde{\mu}_m\right]^{\tilde{\phi}_m}-1}{\tilde{\phi}_m}\right\},\ \tilde{\mu}_m\geq 0$
$D_{rail}(1-P_{rail})$	$D_{rail}(1-P_{rail})-P_{rail}\sum_{m\neq rail}^{M}D_m$ where $D_{rail}=\left[\exp(V_{rail})+\tilde{\mu}_{rail}\right]^{\tilde{\phi}_{rail}-1}\exp(V_{rail})\beta_{rail,X}X^{\lambda_{rail,X}}$ $D_m=\left[\exp(V_m)+\tilde{\mu}_m\right]^{\tilde{\phi}_m-1}\exp(V_m)\beta_{mX}X^{\lambda_{mX}}\exp(E_m)$ $E_m=\left[\exp(V_m)+\tilde{\mu}_m\right]^{(\tilde{\phi}_m)}-\left[\exp(V_{rail})+\tilde{\mu}_{rail}\right]^{(\tilde{\phi}_{rail})}$
Standard utility function	**Generalized utility function**

1.2.4 Modified Elasticity Expressions when the explanatory variable is a Dummy

In (1), the elasticity result (F), $\eta(T_m, X_k)$, is modified when the variable X_k is a dummy variable since the three elasticity terms (A), (C) and (D) are themselves corrected to take into account only the positive observations of X_k. As the U term is never a dummy, the issue does not arise for (B).

Table 7. Corrected Elasticity terms (A*), (C*) and (D*) when X_k is a Dummy variable

LEVEL 1.6 or LEVEL 2.1	**(A*)**	$$\tilde{\eta}(T, X_k)\Big\vert_{\bar{X}_k^+} = \tilde{\eta}(T, X_k)\Big\vert_{\bar{X}_k} \frac{N_k}{N_k^+}$$ where $\tilde{\eta}(T, X_k)\Big\vert_{\bar{X}_k}$ is the elasticity of T with respect to X_k given in case (A-1) or (A-2) of Table 3, evaluated at the sample mean of X_k over N_k observations of the dummy; $\tilde{\eta}(T, X_k)\Big\vert_{\bar{X}_k^+}$ is the elasticity of T with respect to X_k after the correction for the positive observations of the dummy, evaluated at the sample mean of X_k over N_k^+ positive observations of X_k.
PROBABILITY P-2 to P-6 or SHARE S-1 to S-5	**(C*)**	$$\eta(U, X_k)\Big\vert_{\bar{X}_k^+} = \eta(U, X_k)\Big\vert_{\bar{X}_k} \frac{N_k}{N_k^+}$$ where $\eta(U, X_k)\Big\vert_{\bar{X}_k}$ is the elasticity of U with respect to X_k given in Table 5, evaluated at the sample mean of X_k over N_k observations of the dummy; $\eta(U, X_k)\Big\vert_{\bar{X}_k^+}$ is the elasticity of U with respect to X_k after the correction for the positive observations of the dummy, evaluated at the sample mean of X_k over N_k^+ positive observations of X_k.
	(D*)	$$\eta(P_m, X_k)\Big\vert_{\bar{X}_k^+} = \eta(P_m, X_k)\Big\vert_{\bar{X}_k} \frac{N_k}{N_k^+}$$ where $\eta(P_m, X_k)\Big\vert_{\bar{X}_k}$ is the elasticity of P_m with respect to X_k given in Table 6, evaluated at the sample mean of X_k over N_k observations of the dummy; $\eta(P_m, X_k)\Big\vert_{\bar{X}_k^+}$ is the elasticity of P_m with respect to X_k after the correction for the positive observations of the dummy, evaluated at the sample mean of X_k over N_k^+ positive observations of X_k.

1.3 Examples to be produced

Five examples only from 20 allowed combinations. Presenting real examples from allowed multiplications of models requires combining regression outputs for ***Totals***, obtained from both LEVEL 1.6 and LEVEL 2.1 algorithms, and regression outputs for ***Splits***, obtained from both SHARE S-1/S-5 and PROBABILITY P-2/P-6 algorithms. As it is impossible to illustrate here all 20 representative combinations allowed by TABQDF, we choose 5 typical ones drawing, to the extent possible, from regression variants already documented at length in LEVEL, SHARE or PROBABILITY manuals: their databases and examples are listed in Table 8. In fact, the first 3 examples draw directly from the Canada 1976 database [framed in Table 8] for Generation-Distribution and Share variants but the last 2 examples will draw from the VIA RAIL 1987 database which is not listed in Table 8: these examples will consequently require a distinct treatment below.

Table 8. Available data bases and examples in LEVEL, PROBABILITY and SHARE manuals

Examples	1st	2nd	3rd	4th
LEVEL 1.6 data in AJD-107				
Bases	**Montreal** (Dec. 1956-Dec. 1971)		**Quebec** (Dec. 1956-Dec.1982)	
Model	Demtec transit demand model		DRAG-1 road safety model	
Equations	1 (Adults)	2 (Adults); 4 (Schoolchildren)	9 equations	
Tablex	AJD-106, p. 231	AJD-106, p. 240	AJD-106, p. 250	
LEVEL 2.1 data in AJD-110				
Bases	**Trade flows 1988** North America 1 proximity matrix	**Canada 1976** intercity passenger 1 proximity matrix	**Trade flows 1987** European Community 1 proximity matrix	**Canada 1976** intercity passenger, 2 proximity matrices
Model	Generation-Distribution models			
Equations	6 variants	6 variants (4-mode)	6 variants	6 variants (4-mode)
Tablex	AJD-109, p. 52	AJD-109, p. 55	AJD-118, Appendix 1	AJD-118, Appendix 1
PROBABILITY P-2/P-6 data in AJD-113				
Base	**Santiago 1983-1985**			
Model	Urban 9-mode choice			
Equations	9 variants			
Tablex	AJD-112, p. 125			
SHARE S-1/S-5 data in AJD-116				
Bases	**Canada 1976**		**Germany 1985**	
Model	Intercity passenger mode choice			
Equations	6 variants (4-mode)	3 variants (4-mode)	3 variants (3-mode)	
Tablex	AJD-115, p. 90	AJD-115, p. 96	AJD-115, p. 102	

As was the case in the appendix of the procedures report (Tran and Gaudry, 2008c), each example here effectively requires three Tables: the ***two source Tablexes*** of the models that are combined (we call these here the AB Tablex for the Generation-Distribution models and the D Tablex for the mode Split models) ***and the TABQDF Tablex itself***, (called an ABCDEF TABQDF). For each of the TABQDF tables produced with the 5 examples, the source Tablexes are found in the manuals or in appendices to this document: they are all referenced and accessible. The TABQDF Tables do not contain all the statistical information found in the source tables, but the reader can easily verify that features common to TABQDF and sources, such as the elasticities and *t*-statistics, duly match. The QDF algorithm checks for the consistency between the Generation-Distribution variants and the U terms automatically generated by the SHARE and PROBABILITY altorithms[3].

[3] These algorithms automatically create files for the U term and some related results: (C) and (D) elasticity terms, as well as conditional *t*-statistics of the regression coefficients.

A first group of 3 examples combining Level and Share models. The first group of examples combines *Total Generation-Distribution* and *Share Mode Split* model results:

- Example I combines 1 SHARE variant (1 S) and 1 LEVEL variant. (1 L_0) ;
- Example II combines 1 SHARE variant (1 S) and 7 LEVEL variants (7 L_2);
- Example III combines 6 SHARE variants (6 S) and 1 LEVEL variant (1 L_2);

Example I. Example I in the table below combines a simple logarithmic form (LOG) of the Generation-Distribution model found in Gaudry *et alii* (1994, 2002; reproduced in AJD-109, p. 55) re-estimated with the LEVEL-1.6 algorithm to document the use of this algorithm with a Mode choice variant estimated with 4 Box-Cox transformations (4BC-LOGIT from AJD-115, p. 96). As this case, found in Column 0, is the same as that of Column 1 in the next table explaining Example II, it naturally has the same Log Likelihood value.

ABCDEF TABQDF for Example I

			AB Tablex	
			0	
			LOG	
D Tablex	1	LIN-LOGIT		-537.74
	2	1BC-LOGIT		-494.56
	3	4BC-LOGIT	X	-479.51
			-1318.24	**Log. Lik.**

Source : Gaudry *et alii* (1994, 2002):
AB Tablex : Column 1, Table B.2, p. 50, re-estimated by LEVEL-1.6 algorithm.
D Tablex : Table B.1, p. 48-49.

Example II. The next example developed in the table below is identical to that found in Tran and Gaudry (2008c). It combines the SHARE S-1/S-5 variant used in Example I ─ estimated with 4 Box-Cox transformations (4BC-LOGIT), one on each variable of the mode choice model ─ to seven distinct LEVEL-2.1 variants: the first one is of logarithmic form (LOG) and the others add first order (AU1) or second order (AU2) spatial autocorrelation [and even the proximity term (PR) of a distributed lag as well]. In some cases, the form of the Generation-Distribution model is estimated with five Box-Cox transformations (BC5) ─ one on each variable of the model. Only Log likelihood values of the original models are supplied here, but the source papers and Tablexes provide details.

ABCDEF TABQDF for Example II

			AB Tablex							
			1	2	3	4	5	6	7	
			LOG	LOG + AU1	LOG + AU1 + PR	BC5 +AU1 + PR	LOG +AU1+AU2	LOG +AU1+AU2 + PR	BC5 +AU1+AU2 + PR	
D Tablex	1	LIN-LOGIT								-537.74
	2	1BC-LOGIT								-494.56
	3	4BC-LOGIT	**X**	**X**	**X**	**X**	**X**	**X**	**X**	-479.51
			-1318.24	-1294.58	-1294.48	-1262.98	-1309.49	-1301.11	-1272.27	**Log Lik**.

Source : Gaudry *et alii* (1994, 2002)
ABCDEF TABQDF: Table B.5, p. 53-55.
AB Tablex : Models from Columns 1,2,3 and 6 of Table B.2, p. 50 and Models from Columns 2,3 and 6 of Table B.3, p. 51.
D Tablex : Table B.1, p. 48-49.

Example III. The structure of Example II consisted in fixing the Mode choice model and modifying the Generation-Distribution model. We presently do the opposite for Example III in the table below.

In Column 8, we modify Case 7 of Example II in two ways: we reduce its total number of BCT from 5 to 4 (by imposing a common BCT to the variables REVE and POP previously obtaining distinct BCT), leaving everything else unchanged except for the replacement of the U term (of Line 3) by the U term generated from the Line 4 model.

This combination of changes reduces the Log Likelihood from -1272,27 (in Case 7) to -1385,25 (in Case 8). Cases 9-13 are then obtained by simple substitution of other U terms in accordance with the models defined in Lines 5-13. Different U terms naturally imply different Log likelihood values but these values are not nested.

ABCDEF TABQDF for Example III

			AB Tablex						
			8	**9**	**10**	**11**	**12**	**13**	
			BC4 +AU1+AU2 + PR	BC4 +AU1+AU2 + PR	BC4 +AU1+AU2 + PR	BC4 +AU1+AU2 + PR	BC4 +AU1+AU2 + PR	BC4 +AU1+AU2 + PR	
D Tablex	**4**	LIN-LOGIT	**X**						-527.45
	5	BC-LOGIT		**X**					-499.34
	6	S-DOGIT			**X**				-509.27
	7	G-DOGIT				**X**			-499.22
	8	LIN-IPT-L					**X**		-508.79
	9	BT-IPT-L						**X**	-490.46
			-1385.25	-1388.59	1387.56	1388.75	1388.13	1384.89	**Log Lik.**

Source :

D Tablex : Table 4, pages 15-18 of Gaudry (1989, 1990, 1993).

AB Tablex : new, obtained (i) in Column 8, from reducing the number of BCT of Case 7 in Example II by one (the Income and Population variables REVE and POP now share a BCT instead of obtaining one each) and by using the U term from the Line 4 model; (ii) in Columns 9-13, by using U terms obtained from from the models listed in lines 5-9.

A second group of 2 examples combining Level and Probability models. The second group of examples combines *Total Generation-Distribution* and *Probability Mode Split* model results:

- •Example IV combines 1 PROBABILITY variant (1 P) and 5 LEVEL variants (1 L_0);
- •Example V combines 1 PROBABILITY variant (1 P) and 3 LEVEL variants (1 L_2).

In these examples, we use the VIA RAIL 1987 database for the Quebec-Windsor corridor of Canada (see Chapter 4 below) supplied in the AJD-119 companion to this document (AJD-118). The discrete data base contains 4401 Business Purpose trips and 8536 trips for Other Purposes. We reproduce in Appendix 2 six key mode choice model variants (3 by trip purpose) developed with the full samples drawn from a study of High Speed Rail made for Transport Canada (Gaudry and Le Leyzour, 1994).

But, as the L-2.1 algorithm is restricted to 1000 observations (due to the size limit [1000 x 1000] imposed on the two allowed R matrices — defined in the frame on page 4 above), we need to construct reduced-size Mode choice and Generation-distribution models from the original data base. As our purpose is to demonstrate the numerical capabilities of TABQDF, not to develop refined proper usable models, the retained examples of simplistic models are constructed only from the Business trip sample data.

First, we make use of the available sample weights to derive aggregate flows from the discrete observations: this yields 3226 two-way averages $[(T_{ij} + T_{ji})/2]$ if both origins (emissions) and destinations (attractions) are required strictly positive (*i.e.* $T_i > 0$ and $T_j > 0$). For many of these symmetric O-D flows, one or more of the four modes may have no registered individual trip[4]. We then draw at random 1000 of the 3226 Total O-D flows and note that, for our purposes, the underlying subset of discrete observations is sufficient across the modes: Rail is available for 991 individuals, Air for 846, Bus for 797 and Road for 999.

Above mentioned data bases and relevant Tablex editions are listed in Table 9 where the data base subset used for the forthcoming examples is framed.

Table 9. Available VIA RAIL 1987 Quebec-Windsor data bases and examples in QDF manuals

QDF data in AJD-119						
			Example IV		Example V	
Bases	Business	Other	Business			
Model	4-mode choice		4-mode choice	Generation-Distrib.	4-mode choice	Generation-Distrib.
Equations	3 variants	3 variants	5 variants	5 variants	5 variants	3 variants
Tablexes in AJD-118	Appendix 2		Appendix 3, Section 8.1	Appendix 4, Section 9.1	Appendix 3, Section 8.2	Appendix 4, Section 9.2
	Col. 1-3	Col. 4-6				
Observations	4401	8536	1000			
Sample	Full		Reduced			

Second, simplified specifications of Mode choice and Generation-Distribution models are adopted: apart from intercepts, the former include only ***Frequency***, ***Cost***, ***Travel Time*** and ***Income*** in modal utility functions (allowing for up to 4 Box-Cox transformations); the latter include only ***trip ends*** T_i ***and*** T_j, as well as the ***U term*** constructed from a mode choice variant (allowing for up to 4 Box-Cox transformations, as the dependent variable is also transformable).

Example IV. Example IV in the table below combines the U terms from a Box-Cox Logit specification of the Mode Choice with 4 BCT to 5 progressively more flexible specifications of the Generation-Distribution component model estimated with the LEVEL-1.6 algorithm.

ABCDEF TABQDF for Example IV								
			AB Tablex from L-1.6 algorithm					
			14	**15**	**16**	**17**	**18**	
			LIN	LOG	BC1	BC2	BC4	
D Tablex	**10**	LOGIT LIN						-328.15
	11	LOGIT LOG						-329.11
	12	LOGIT BC1						-326.17
	13	LOGIT BC2						-325.38
	14	LOGIT BC4	**X**	**X**	**X**	**X**	**X**	-323.24
			-18797.91	-17486.28	17482.62	17477.05	17459.96	**Log Likelihood**
Source : this document AJD-118								
D Tablex : Appendix 3, Section 8.1.								
AB Tablex : Appendix 4, Section 9.1.								

Example V. The table below starts with the last combination found in Example IV (Line 14 and Column 18) and, using the LEVEL-2.1 algorithm, estimates progressively more complex versions of the Generation-Distribution model. At first, heteroskedasticity of a general form, based on the product of

[4] Had we selected only O-D pairs for which the 4 modes are used, there would be only 2275 aggregate flows.

trip end terms [T_i*T_j], yields very large gains in Log Likelihood (in Column 20) but the further addition of first order directed (spatial) autocorrelation based on flows originating or destinating less than 100 minutes by car from the origin or destination flow considered yields a non stationary result (as the first order autocorrelation parameter ρ_1 reaches -1, *i.e.* a corner solution) in Column 21. Although no statistical gain then occurs, the resulting model estimates pose no numerical problems for TABQDF computations. Another specification with a distributed lag (not reported on here) again yielded trivial statistical gain and posed no numerical problem for TABQDF computations.

ABCDEF TABQDF for Example V						
			AB Tablex from L-2.1 algorithm			
			19	**20**	**21**	
			BC4	BC4 +HG	BC4 +HG +AU1	
D Tablex	**15**	BC-LOGIT	X	X	X	-323.24
	16	S-DOGIT				-324.43
	17	G-DOGIT				-303.37
	18	LIN-IPT-L				-316.69
	19	BT-IPT-L				-308.62
			-17459.96	-17433.91	17433.38	**Log Likelihood**
Source : this document AJD-118						
D Tablex : Appendix 3, Section 8.2.						
AB Tablex : Appendix 4, Section 9.2.						

Table 10 summarises the combinations covered by these 5 TABQDF examples, which in fact include no estimates of serial autocorrelation based on a pure L_1 case defined in Table 2.

Table 10. The five examples to be presented in TABQDF tables

Model families		LEVEL 1.6		LEVEL 2.1	
		Autocorrelation		*Autocorrelation*	
		None	*Serial*	*None*	*Directed*
		Canada 1976			
SHARE S-1/S-5	***Full sample of O-D pairs with 4 modes***	**I (1 S • 1 L_0)**			**II (1 S • 7 L_2)** **III (6 S • 1 L_2)**
		Quebec-Windsor Corridor 1987			
PROBABILITY P-2/P-6	***Subset of 1000 OD pairs (Business)***	**IV (1 P • 5 L_0)**			**V (1 P • 3 L_2)**
	Column	**1**	**2**	**3**	**4**

2. QDF TABLE PROGRAM

2.1 Installation of QDF Table Program

The file *Tabqdf.zip*, which can be downloaded from the website of Agora Jules Dupuit currently found at www.e-ajd.net, contains all the folders used in this publication. To install the QDF Table Program with their examples of input/output files and associated database, unzip *Tabqdf.zip* to drive *d:*.

The *d:\Tabqdf* folder will be created with the following structure: in the root folder *d:\Tabqdf*, the Fortran file *Tabqdf.f* and specific files used in the compilation of the QDF program with Compaq Visual Fortran 6.6 are stored. Under this folder, there are four subfolders:

1. The *Databases* subfolder contains four folders:
    a. Via Rail 1987
    b. Intercity Passenger Flows Canada 1976
    c. Trade Flows (in tons) European Community 1987
    d. Trade Flows (in value) North America 1988
2. The *Documentation* subfolder contains all the publications cited in the text but not available from the website Agora Jules Dupuit.
3. The *Release* subfolder contains five compiled files: *Tabqdf.exe, Level1.exe, Level2.exe, Prob2007.exe* and *Share2007.exe*, for TABQDF, LEVEL L-1.6, LEVEL L-2.1, PROBABILITY P2/P6 and SHARE S1/S5 programs respectively.
4. The *TabqdfTests* subfolder includes Five folders *First Example, Second Example, Third Example, Fourth Example* and *Fifth Example* which contain the input (.in) / output (.txt) files and the data file associated with each example.

2.2 Program Input File

The input file for the QDF Table Program contains three sections:

SECTION I : QDF Table options	
First Card (List Options) **_$QDF ncol=…,nlevel=…,nshapro=…,nutil=…,…,…,nvs=…,nlist=…,nblxlev=…,nblxsha=…$** (Symbol _ indicates a blank. A value should be specified for each parameter after the equality sign)	
ncol	Number of columns or variants to be reported in QDF Table (max. 10).
nlevel	1 for LEVEL L-1.6 2 for LEVEL L-2.1
nshapro	1 for SHARE S1-S5 2 for PROBABILITY P2-P6
nutil(i), i=1,ncol	vector of *ncol* elements indicating if the Utility variable U is present in the i-th variant for LEVEL L-1.6 or L-2.1: nutil(i)=0 if U is not present in the i-th variant and 1 otherwise.
nvs	Language version of QDF Table: nvs=0 for French, nvs=1 for English.
nlist	Output option for TABLEX (nlist=0 for a normal run where only the tablex is produced, nlist=1 when only the list of blocks with the definitions of the independent variables X is produced).

nblxlev	Number of blocks of independent variables X for LEVEL L-1.6 or L-2.1. The number of blocks is unlimited for a total of 100 independent variables.
nblxsha	Number of blocks of independent variables X for SHARE S1-S5 or PROBABILITY P2-P6. The number of blocks is unlimited for a total of 100 independent variables.

SECTION II : List of Names and Definitions for the Blocks and the Independent Variables	
fname1	Name of the file containing the list of all the names and definitions for the blocks and the independent variables within each block for LEVEL L-1.6 or LEVEL L-2.1. A blank card shoud be added at the end of each block, including the last block.
fname2	Name of the file containing the list of all the names and definitions for the blocks and the independent variables within each block for SHARE S1-S5 or PROBABILITY P2-P6. A blank card shoud be added at the end of each block, including the last block.
SECTION III : Estimation Results	
fnames	Names of the elasticity files for the two groups of variants. The first group of variants corresponds to LEVEL L-1.6 or L-2.1, followed by the second group associated with SHARE S1-S5 or PROBABILITY P2-P6. Enter one file name per variant. The display order of the variants in the columns of QDF Table follows the entry order of the elasticity files specified in this section.

2.3 Running QDF Table program

Since the QDF Table program is a DOS-program, it can only be run in a DOS dialog box:

1. Select START/RUN menu and type the command **cmd** to open the DOS dialog box.
2. At the DOS prompt, go to the folder *d:\Tabqdf\\TabqdfTests\First Example* where the creation of a Tablex will be made for this example. If one is interested in creating a Tablex for the other examples, then go to the corresponding subfolder under the folder *TabqdfTests*. For each Example folder, there are two subfolders, namely *Level 1.6* or *Level 2.1*, and *Share* or *Probability* depending on the coupled models considered in the example. Each subfolder contains the input and output files for the estimation of the variants of the coupled models.
3. Before running the QDF Table program, all the variants of the coupled models should be estimated using the *Level 1.6*, *Level 2.1*, *Probability P2/P6 or Share S1/S5* programs which are included in subfolder *d:\Tabqdf\Release* to obtain the elasticities associated with the variants for each model. These elasticities will be used as inputs to the QDF Table program (See Section III of 2.2 Program Input File).
4. To run the QDF Table program, the input file (.in) for the program and the definition file (.txt) containing all the names and the definitions of the blocks and the independent variables in each block should be created in each Example folder. To run the program with the *First Example* for example, go to *d:\Tabqdf\\TabqdfTests\First Example* and type the command:

D:\Tabqdf\Release\Tabqdf.exe < input file name > output file name.

This command can also be written in a batch (.bat) file and double-click on it will run the QDF Table program with a specified input file. We will use the .bat files for the five examples to run the program.

The output file (.txt) containing the tablex for a set of estimated variants will be saved in the same folder as the input file.

2.4 First Example of Input File

TABQDF_Example_I.in

$QDF ncol=1,nlevel=1,nshapro=1,nutil=1,nvs=1,nlist=0,nblxlev=1,nblxsha=4$
D:\Tabqdf\TabqdfTests\First Example\Level 1.6\L1Listdef.txt
D:\Tabqdf\TabqdfTests\First Example\Share\S1S5Listdef.txt
D:\Tabqdf\TabqdfTests\First Example\Level 1.6\LOG1_ELA.txt
D:\Tabqdf\TabqdfTests\First Example\Share\ElaPxUx3.txt

2.5 First Example of Output File

TABQDF_Example_I.doc

```
&QDF
 NCOL    =           1,
 NLEVEL  =           1,
 NSHAPRO =           1,
 NUTIL   =           1, 9*0,
 NVS     =           1,
 NLIST   =           0,
 NBLXLEV =           1,
 NBLXSHA =           4
 /
LEVEL L-1.6
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE DEPENDENT VARIABLES
   The variable TOTFLOW   is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
   The variable POP       is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.

SHARE
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE ALTERNATIVES
   Alternative AIR       is the same in all variants.
   Alternative RAIL      is the same in all variants.
   Alternative BUS       is the same in all variants.
   Alternative CAR       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
  ALTERNATIVE  1: AIR
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable F1/D1     is the same in all variants.
   The variable D1/T1     is the same in all variants.
   The variable DIST1     is the same in all variants.
   The variable FREQU1    is the same in all variants.
  ALTERNATIVE  2: RAIL
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable F2/D2     is the same in all variants.
```

```
   The variable D2/T2     is the same in all variants.
   The variable DIST2     is the same in all variants.
   The variable FREQ2     is the same in all variants.
  ALTERNATIVE  3: BUS
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable F3/D3     is the same in all variants.
   The variable D3/T3     is the same in all variants.
   The variable DIST3     is the same in all variants.
   The variable FREQ3     is the same in all variants.
  ALTERNATIVE  4: CAR
   The variable NUITA     is the same in all variants.
   The variable F4/D4     is the same in all variants.
   The variable D4/T4     is the same in all variants.
   The variable DIST4     is the same in all variants.

===================================================================
LEVEL                                             FAMILY         1.6
                                                  VARIANT        LOG
                                                  NUMBER           1

SHARE                                             FAMILY       LOGIT
                                                  VARIANT  BC4-LOGIT
                                                  NUMBER           3
===================================================================
LEVEL

 POPULATION                          POP          EL(T,X)      1.389
                                                  (t)      (  24.92)
 REVENUE                             REVE         EL(T,X)      0.832
                                                  (t)      (   1.18)
 FRENCH LANGUAGE                     LANGFR       EL(T,X)      0.066
                                                  (t)      (   1.17)
 UTILITY-Share Variant 3             UTILITY      EL(T,U)      0.634
                                                  (t)      (  41.44)
-------------------------------------------------------------------
SHARE

ALTERNATIVE 1: AIR       (Mean share: 0.3649)
=============
 O-D WITHIN THE SAME PROVINCE        ODREG        EL(U)       -0.008
                                                  EL(Sm)      -0.029
                                                  (t)      (  -0.29)
                                                  EL(T)       -0.005
                                                  EL(Tm)      -0.034
                                                  D.R.        -0.583

 INCOME                              REVE         EL(U)        1.043
                                                  EL(Sm)       1.362
                                                  (t)      (   1.29)
                                                  EL(T)        1.493
                                                  EL(Tm)       2.855
                                                  D.R.         0.433

 LANGUAGE SIMILARITY O-D             LANGFR       EL(U)        0.274
                                                  EL(Sm)       0.396
                                                  (t)      (   2.53)
                                                  EL(T)        0.240
                                                  EL(Tm)       0.636
                                                  D.R.         0.036

 FARE(AIR) / DISTANCE(AIR)           F1/D1        EL(U)       -0.547
                                                  EL(Sm)      -0.952
                                                  (t)      (  -7.38)
                                                  EL(T)       -0.347
                                                  EL(Tm)      -1.298
                                                  D.R.        -0.268

 DISTANCE(AIR) / TIME(AIR)           D1/T1        EL(U)        0.670
                                                  EL(Sm)       1.164
```

		(t)	(6.24)
		EL(T)	0.424
		EL(Tm)	1.589
		D.R.	-0.268
DISTANCE(AIR)	DIST1	EL(U)	-0.702
		EL(Sm)	-1.221
		(t)	(-14.45)
		EL(T)	-0.445
		EL(Tm)	-1.665
		D.R.	-0.268
UTILITARY FREQUENCY (AIR)	FREQU1	EL(U)	0.147
		EL(Sm)	0.256
		(t)	(6.25)
		EL(T)	0.093
		EL(Tm)	0.349
		D.R.	-0.268

ALTERNATIVE 2: RAIL (Mean share: 0.0409)
=============

O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.008
		EL(Sm)	0.027
		(t)	(0.17)
		EL(T)	-0.005
		EL(Tm)	0.022
		D.R.	-6.832
INCOME	REVE	EL(U)	1.043
		EL(Sm)	1.925
		(t)	(1.57)
		EL(T)	1.493
		EL(Tm)	3.418
		D.R.	9.678
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.274
		EL(Sm)	-0.189
		(t)	(0.30)
		EL(T)	0.240
		EL(Tm)	0.051
		D.R.	113.610
FARE(RAIL) / DISTANCE(RAIL)	F2/D2	EL(U)	-0.017
		EL(Sm)	-0.316
		(t)	(-7.38)
		EL(T)	-0.011
		EL(Tm)	-0.327
		D.R.	-0.198
DISTANCE(RAIL) / TIME(RAIL)	D2/T2	EL(U)	0.075
		EL(Sm)	1.407
		(t)	(6.24)
		EL(T)	0.048
		EL(Tm)	1.455
		D.R.	-0.198
DISTANCE(RAIL)	DIST2	EL(U)	-0.094
		EL(Sm)	-1.760
		(t)	(-14.45)
		EL(T)	-0.060
		EL(Tm)	-1.819
		D.R.	-0.198
FREQUENCY (RAIL)	FREQ2	EL(U)	0.014
		EL(Sm)	0.253
		(t)	(6.25)
		EL(T)	0.009
		EL(Tm)	0.262
		D.R.	-0.198

```
ALTERNATIVE 3: BUS       (Mean share: 0.0261)
=============
 O-D WITHIN THE SAME PROVINCE        ODREG        EL(U)        -0.008
                                                  EL(Sm)        0.157
                                                  (t)      (    1.51)
                                                  EL(T)        -0.005
                                                  EL(Tm)        0.152
                                                  D.R.         -2.296

 INCOME                              REVE         EL(U)         1.043
                                                  EL(Sm)       -0.571
                                                  (t)      (    0.30)
                                                  EL(T)         1.493
                                                  EL(Tm)        0.922
                                                  D.R.         60.980

 LANGUAGE SIMILARITY O-D             LANGFR       EL(U)         0.274
                                                  EL(Sm)        0.593
                                                  (t)      (    3.62)
                                                  EL(T)         0.240
                                                  EL(Tm)        0.833
                                                  D.R.         10.036

 FARE(BUS) / DISTANCE(BUS)           F3/D3        EL(U)        -0.008
                                                  EL(Sm)       -0.256
                                                  (t)      (   -7.38)
                                                  EL(T)        -0.005
                                                  EL(Tm)       -0.261
                                                  D.R.         -0.282

 DISTANCE(BUS) / TIME(BUS)           D3/T3        EL(U)         0.043
                                                  EL(Sm)        1.435
                                                  (t)      (    6.24)
                                                  EL(T)         0.027
                                                  EL(Tm)        1.463
                                                  D.R.         -0.282

 DISTANCE(BUS)                       DIST3        EL(U)        -0.053
                                                  EL(Sm)       -1.773
                                                  (t)      (  -14.45)
                                                  EL(T)        -0.034
                                                  EL(Tm)       -1.807
                                                  D.R.         -0.282

 FREQUENCY (BUS)                     FREQ3        EL(U)         0.016
                                                  EL(Sm)        0.530
                                                  (t)      (    6.25)
                                                  EL(T)         0.010
                                                  EL(Tm)        0.540
                                                  D.R.         -0.282

ALTERNATIVE 4: CAR       (Mean share: 0.5681)
=============
 NUMBER OF NIGHTS                    NUITA        EL(U)        -0.360
                                     -----        EL(Sm)       -0.289
                                                  (t)      (   -8.43)
                                                  EL(T)        -0.228
                                                  EL(Tm)       -0.517
                                                  D.R.         -0.224

 FARE(CAR) / DISTANCE(CAR)           F4/D4        EL(U)        -0.175
                                                  EL(Sm)       -0.140
                                                  (t)      (   -7.38)
                                                  EL(T)        -0.111
                                                  EL(Tm)       -0.251
                                                  D.R.         -0.224

 DISTANCE(CAR) / TIME(CAR)           D4/T4        EL(U)         0.846
                                                  EL(Sm)        0.679
                                                  (t)      (    6.24)
                                                  EL(T)         0.536
```

```
                                                    EL(Tm)        1.216
                                                    D.R.         -0.224

 DISTANCE(CAR)                          DIST4       EL(U)        -1.033
                                                    EL(Sm)       -0.829
                                                    (t)        ( -14.45)
                                                    EL(T)        -0.655
                                                    EL(Tm)       -1.484
                                                    D.R.         -0.224
=====================================================================
```

2.6 Second Example of Input File

TABQDF_Example_II.in

$QDF ncol=7,nlevel=2,nshapro=1,nutil=1,1,1,1,1,1,1,nvs=1,nlist=0,nblxlev=1,nblxsha=4$
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\L2Listdef.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\S1S5Listdef.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\LOG1_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\LOG+AU12_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\LOG+AU1+P3_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\BC5+AU1+P4_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\LOG+A1+A25_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\LOG+A1A2P6_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Level 2.1\BC5+A1A2P7_ELA.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Second Example\Share\ElaPxUx3.txt

2.7 Second Example of Output File

TABQDF_Example_II.doc

```
&QDF
 NCOL    =           7,
 NLEVEL  =           2,
 NSHAPRO =           1,
 NUTIL   = 7*1, 3*0,
 NVS     =           1,
 NLIST   =           0,
 NBLXLEV =           1,
 NBLXSHA =           4
 /

LEVEL L-2.1
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE DEPENDENT VARIABLES
   The variable TOTFLOW   is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
   The variable POP       is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.

SHARE
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE ALTERNATIVES
   Alternative AIR       is the same in all variants.
   Alternative RAIL      is the same in all variants.
```

```
  Alternative BUS      is the same in all variants.
  Alternative CAR      is the same in all variants.
-CHECK ON THE INDEPENDENT VARIABLES
 ALTERNATIVE  1: AIR
  The variable ODREG     is the same in all variants.
  The variable REVE      is the same in all variants.
  The variable LANGFR    is the same in all variants.
  The variable F1/D1     is the same in all variants.
  The variable D1/T1     is the same in all variants.
  The variable DIST1     is the same in all variants.
  The variable FREQU1    is the same in all variants.
 ALTERNATIVE  2: RAIL
  The variable ODREG     is the same in all variants.
  The variable REVE      is the same in all variants.
  The variable LANGFR    is the same in all variants.
  The variable F2/D2     is the same in all variants.
  The variable D2/T2     is the same in all variants.
  The variable DIST2     is the same in all variants.
  The variable FREQ2     is the same in all variants.
 ALTERNATIVE  3: BUS
  The variable ODREG     is the same in all variants.
  The variable REVE      is the same in all variants.
  The variable LANGFR    is the same in all variants.
  The variable F3/D3     is the same in all variants.
  The variable D3/T3     is the same in all variants.
  The variable DIST3     is the same in all variants.
  The variable FREQ3     is the same in all variants.
 ALTERNATIVE  4: CAR
  The variable NUITA     is the same in all variants.
  The variable F4/D4     is the same in all variants.
  The variable D4/T4     is the same in all variants.
  The variable DIST4     is the same in all variants.
```

LEVEL		FAMILY	2.1	2.1	2.1	2.1	2.1	2.1	2.1
		VARIANT	LOG	LOG+AU1	LOG+AU1+P	BC5+AU1+P	LOG+A1+A2	LOG+A1A2P	BC5+A1A2P
		NUMBER	1	2	3	4	5	6	7
SHARE		FAMILY	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
		VARIANT	BC4-LOGIT	BC4-LOGIT	BC4-LOGIT	BC4-LOGIT	BC4-LOGIT	BC4-LOGIT	BC4-LOGIT
		NUMBER	3	3	3	3	3	3	3
LEVEL									
POPULATION	POP	EL(T,X)	1.389	1.419	1.425	1.474	1.414	1.430	1.430
		(t)	(25.45)	(28.04)	(28.36)	(37.37)	(25.61)	(29.06)	(30.01)
REVENUE	REVE	EL(T,X)	0.832	1.799	1.706	0.633	1.320	0.940	0.268
		(t)	(1.20)	(2.39)	(2.22)	(2.18)	(1.56)	(1.28)	(0.58)
FRENCH LANGUAGE	LANGFR	EL(T,X)	0.066	0.334	0.348	0.194	0.239	0.287	0.169
		(t)	(1.20)	(4.08)	(4.16)	(6.23)	(2.39)	(3.76)	(5.94)
UTILITY-Share Variant 3	UTILITY	EL(T,U)	0.634	0.571	0.569	0.486	0.596	0.540	0.493
		(t)	(42.33)	(25.07)	(25.27)	(24.51)	(21.25)	(18.76)	(22.83)
SHARE									
ALTERNATIVE 1: AIR (Mean share: 0.3649)									
O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
		EL(Sm)	-0.029	-0.029	-0.029	-0.029	-0.029	-0.029	-0.029
		(t)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)	(-0.29)
		EL(T)	-0.005	-0.005	-0.005	-0.004	-0.005	-0.004	-0.004
		EL(Tm)	-0.034	-0.033	-0.033	-0.033	-0.034	-0.033	-0.033
		D.R.	-0.583	-0.618	-0.619	-0.668	-0.604	-0.636	-0.664
INCOME	REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
		EL(Sm)	1.362	1.362	1.362	1.362	1.362	1.362	1.362
		(t)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)	(1.29)
		EL(T)	1.493	2.395	2.300	1.140	1.942	1.504	0.782
		EL(Tm)	2.855	3.757	3.662	2.502	3.304	2.866	2.144
		D.R.	0.433	0.747	0.721	0.249	0.611	0.438	0.000
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
		EL(Sm)	0.396	0.396	0.396	0.396	0.396	0.396	0.396
		(t)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)	(2.53)
		EL(T)	0.240	0.490	0.504	0.327	0.403	0.435	0.304
		EL(Tm)	0.636	0.886	0.899	0.723	0.798	0.831	0.700
		D.R.	0.036	0.517	0.535	0.241	0.382	0.435	0.191
FARE(AIR) / DISTANCE(AIR)	F1/D1	EL(U)	-0.547	-0.547	-0.547	-0.547	-0.547	-0.547	-0.547
		EL(Sm)	-0.952	-0.952	-0.952	-0.952	-0.952	-0.952	-0.952
		(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
		EL(T)	-0.347	-0.312	-0.311	-0.266	-0.326	-0.296	-0.270
		EL(Tm)	-1.298	-1.264	-1.263	-1.218	-1.278	-1.247	-1.221
		D.R.	-0.268	-0.323	-0.324	-0.401	-0.300	-0.350	-0.395
DISTANCE(AIR) / TIME(AIR)	D1/T1	EL(U)	0.670	0.670	0.670	0.670	0.670	0.670	0.670
		EL(Sm)	1.164	1.164	1.164	1.164	1.164	1.164	1.164
		(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
		EL(T)	0.424	0.382	0.381	0.326	0.399	0.362	0.330
		EL(Tm)	1.589	1.547	1.545	1.490	1.564	1.526	1.494
		D.R.	-0.268	-0.323	-0.324	-0.401	-0.300	-0.350	-0.395
DISTANCE(AIR)	DIST1	EL(U)	-0.702	-0.702	-0.702	-0.702	-0.702	-0.702	-0.702
		EL(Sm)	-1.221	-1.221	-1.221	-1.221	-1.221	-1.221	-1.221
		(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)

		EL(T)	-0.445	-0.401	-0.399	-0.341	-0.418	-0.379	-0.346
		EL(Tm)	-1.665	-1.621	-1.620	-1.562	-1.639	-1.600	-1.567
		D.R.	-0.268	-0.323	-0.324	-0.401	-0.300	-0.350	-0.395
UTILITARY FREQUENCY (AIR)	FREQU1	EL(U)	0.147	0.147	0.147	0.147	0.147	0.147	0.147
		EL(Sm)	0.256	0.256	0.256	0.256	0.256	0.256	0.256
		(t)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)
		EL(T)	0.093	0.084	0.084	0.072	0.088	0.079	0.073
		EL(Tm)	0.349	0.340	0.340	0.327	0.343	0.335	0.328
		D.R.	-0.268	-0.323	-0.324	-0.401	-0.300	-0.350	-0.395

ALTERNATIVE 2: RAIL (Mean share: 0.0409)
=============

O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
		EL(Sm)	0.027	0.027	0.027	0.027	0.027	0.027	0.027
		(t)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)
		EL(T)	-0.005	-0.005	-0.005	-0.004	-0.005	-0.004	-0.004
		EL(Tm)	0.022	0.022	0.022	0.023	0.022	0.022	0.023
		D.R.	-6.832	-6.133	-6.113	-5.241	-6.409	-5.805	-5.309
INCOME	REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
		EL(Sm)	1.925	1.925	1.925	1.925	1.925	1.925	1.925
		(t)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)	(1.57)
		EL(T)	1.493	2.395	2.300	1.140	1.942	1.504	0.782
		EL(Tm)	3.418	4.319	4.224	3.065	3.866	3.428	2.707
		D.R.	9.678	12.552	12.307	8.094	11.276	9.721	6.063
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
		EL(Sm)	-0.189	-0.189	-0.189	-0.189	-0.189	-0.189	-0.189
		(t)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)
		EL(T)	0.240	0.490	0.504	0.327	0.403	0.435	0.304
		EL(Tm)	0.051	0.301	0.315	0.138	0.214	0.246	0.115
		D.R.	113.610	38.784	38.128	56.865	45.082	42.231	63.656
FARE(RAIL) / DISTANCE(RAIL)	F2/D2	EL(U)	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017
		EL(Sm)	-0.316	-0.316	-0.316	-0.316	-0.316	-0.316	-0.316
		(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
		EL(T)	-0.011	-0.010	-0.010	-0.008	-0.010	-0.009	-0.008
		EL(Tm)	-0.327	-0.326	-0.326	-0.324	-0.326	-0.325	-0.324
		D.R.	-0.198	-0.275	-0.277	-0.379	-0.244	-0.312	-0.371
DISTANCE(RAIL) / TIME(RAIL)	D2/T2	EL(U)	0.075	0.075	0.075	0.075	0.075	0.075	0.075
		EL(Sm)	1.407	1.407	1.407	1.407	1.407	1.407	1.407
		(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)
		EL(T)	0.048	0.043	0.043	0.037	0.045	0.041	0.037
		EL(Tm)	1.455	1.450	1.450	1.444	1.452	1.448	1.445
		D.R.	-0.198	-0.275	-0.277	-0.379	-0.244	-0.312	-0.371
DISTANCE(RAIL)	DIST2	EL(U)	-0.094	-0.094	-0.094	-0.094	-0.094	-0.094	-0.094
		EL(Sm)	-1.760	-1.760	-1.760	-1.760	-1.760	-1.760	-1.760
		(t)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)	(-14.45)
		EL(T)	-0.060	-0.054	-0.054	-0.046	-0.056	-0.051	-0.046
		EL(Tm)	-1.819	-1.813	-1.813	-1.806	-1.816	-1.811	-1.806
		D.R.	-0.198	-0.275	-0.277	-0.379	-0.244	-0.312	-0.371
FREQUENCY (RAIL)	FREQ2	EL(U)	0.014	0.014	0.014	0.014	0.014	0.014	0.014
		EL(Sm)	0.253	0.253	0.253	0.253	0.253	0.253	0.253
		(t)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)	(6.25)
		EL(T)	0.009	0.008	0.008	0.007	0.008	0.007	0.007
		EL(Tm)	0.262	0.261	0.261	0.260	0.262	0.261	0.260
		D.R.	-0.198	-0.275	-0.277	-0.379	-0.244	-0.312	-0.371

ALTERNATIVE 3: BUS (Mean share: 0.0261)
=============

O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
		EL(Sm)	0.157	0.157	0.157	0.157	0.157	0.157	0.157
		(t)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)	(1.51)
		EL(T)	-0.005	-0.005	-0.005	-0.004	-0.005	-0.004	-0.004
		EL(Tm)	0.152	0.153	0.153	0.153	0.152	0.153	0.153
		D.R.	-2.296	-2.164	-2.160	-1.987	-2.217	-2.100	-2.001
INCOME	REVE	EL(U)	1.043	1.043	1.043	1.043	1.043	1.043	1.043
		EL(Sm)	-0.571	-0.571	-0.571	-0.571	-0.571	-0.571	-0.571
		(t)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)	(0.30)
		EL(T)	1.493	2.395	2.300	1.140	1.942	1.504	0.782
		EL(Tm)	0.922	1.823	1.728	0.569	1.370	0.932	0.211
		D.R.	60.980	49.253	49.913	75.681	53.216	60.711	140.922
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.274	0.274	0.274	0.274	0.274	0.274	0.274
		EL(Sm)	0.593	0.593	0.593	0.593	0.593	0.593	0.593
		(t)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)	(3.62)
		EL(T)	0.240	0.490	0.504	0.327	0.403	0.435	0.304
		EL(Tm)	0.833	1.083	1.097	0.920	0.996	1.028	0.897
		D.R.	10.036	16.320	16.578	12.611	14.476	15.194	11.969
FARE(BUS) / DISTANCE(BUS)	F3/D3	EL(U)	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
		EL(Sm)	-0.256	-0.256	-0.256	-0.256	-0.256	-0.256	-0.256
		(t)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)	(-7.38)
		EL(T)	-0.005	-0.004	-0.004	-0.004	-0.005	-0.004	-0.004
		EL(Tm)	-0.261	-0.261	-0.261	-0.260	-0.261	-0.260	-0.260
		D.R.	-0.282	-0.352	-0.354	-0.447	-0.324	-0.386	-0.439
DISTANCE(BUS) / TIME(BUS)	D3/T3	EL(U)	0.043	0.043	0.043	0.043	0.043	0.043	0.043
		EL(Sm)	1.435	1.435	1.435	1.435	1.435	1.435	1.435
		(t)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)	(6.24)

```
                                                   EL(T)        0.027       0.025       0.025       0.021       0.026       0.023       0.021
                                                   EL(Tm)       1.463       1.460       1.460       1.456       1.461       1.459       1.457
                                                   D.R.        -0.282      -0.352      -0.354      -0.447      -0.324      -0.386      -0.439

 DISTANCE(BUS)                    DIST3            EL(U)       -0.053      -0.053      -0.053      -0.053      -0.053      -0.053      -0.053
                                                   EL(Sm)      -1.773      -1.773      -1.773      -1.773      -1.773      -1.773      -1.773
                                                   (t)      ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)
                                                   EL(T)       -0.034      -0.031      -0.030      -0.026      -0.032      -0.029      -0.026
                                                   EL(Tm)      -1.807      -1.804      -1.804      -1.799      -1.805      -1.802      -1.800
                                                   D.R.        -0.282      -0.352      -0.354      -0.447      -0.324      -0.386      -0.439

 FREQUENCY (BUS)                  FREQ3            EL(U)        0.016       0.016       0.016       0.016       0.016       0.016       0.016
                                                   EL(Sm)       0.530       0.530       0.530       0.530       0.530       0.530       0.530
                                                   (t)      (   6.25)   (   6.25)   (   6.25)   (   6.25)   (   6.25)   (   6.25)   (   6.25)
                                                   EL(T)        0.010       0.009       0.009       0.008       0.010       0.009       0.008
                                                   EL(Tm)       0.540       0.539       0.539       0.538       0.539       0.539       0.538
                                                   D.R.        -0.282      -0.352      -0.354      -0.447      -0.324      -0.386      -0.439

ALTERNATIVE 4: CAR       (Mean share: 0.5681)
=============
 NUMBER OF NIGHTS                 NUITA            EL(U)       -0.360      -0.360      -0.360      -0.360      -0.360      -0.360      -0.360
                                  -----            EL(Sm)      -0.289      -0.289      -0.289      -0.289      -0.289      -0.289      -0.289
                                                   (t)      (  -8.43)   (  -8.43)   (  -8.43)   (  -8.43)   (  -8.43)   (  -8.43)   (  -8.43)
                                                   EL(T)       -0.228      -0.206      -0.205      -0.175      -0.215      -0.195      -0.177
                                                   EL(Tm)      -0.517      -0.495      -0.494      -0.464      -0.504      -0.484      -0.467
                                                   D.R.        -0.224      -0.268      -0.270      -0.336      -0.250      -0.292      -0.330

 FARE(CAR) / DISTANCE(CAR)        F4/D4            EL(U)       -0.175      -0.175      -0.175      -0.175      -0.175      -0.175      -0.175
                                                   EL(Sm)      -0.140      -0.140      -0.140      -0.140      -0.140      -0.140      -0.140
                                                   (t)      (  -7.38)   (  -7.38)   (  -7.38)   (  -7.38)   (  -7.38)   (  -7.38)   (  -7.38)
                                                   EL(T)       -0.111      -0.100      -0.099      -0.085      -0.104      -0.094      -0.086
                                                   EL(Tm)      -0.251      -0.240      -0.240      -0.225      -0.244      -0.235      -0.226
                                                   D.R.        -0.224      -0.268      -0.270      -0.336      -0.250      -0.292      -0.330

 DISTANCE(CAR) / TIME(CAR)        D4/T4            EL(U)        0.846       0.846       0.846       0.846       0.846       0.846       0.846
                                                   EL(Sm)       0.679       0.679       0.679       0.679       0.679       0.679       0.679
                                                   (t)      (   6.24)   (   6.24)   (   6.24)   (   6.24)   (   6.24)   (   6.24)   (   6.24)
                                                   EL(T)        0.536       0.483       0.482       0.412       0.504       0.457       0.417
                                                   EL(Tm)       1.216       1.163       1.161       1.091       1.184       1.137       1.097
                                                   D.R.        -0.224      -0.268      -0.270      -0.336      -0.250      -0.292      -0.330

 DISTANCE(CAR)                    DIST4            EL(U)       -1.033      -1.033      -1.033      -1.033      -1.033      -1.033      -1.033
                                                   EL(Sm)      -0.829      -0.829      -0.829      -0.829      -0.829      -0.829      -0.829
                                                   (t)      ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)   ( -14.45)
                                                   EL(T)       -0.655      -0.590      -0.588      -0.502      -0.616      -0.558      -0.509
                                                   EL(Tm)      -1.484      -1.419      -1.417      -1.332      -1.445      -1.388      -1.339
                                                   D.R.        -0.224      -0.268      -0.270      -0.336      -0.250      -0.292      -0.330

==================================================================================================================================================
```

2.8 Third Example of Input File

TABQDF_Example_III.in

$QDF ncol=6,nlevel=2,nshapro=1,nutil=1,1,1,1,1,1,nvs=1,nlist=0,nblxlev=1,nblxsha=4$
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\L2Listdef.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\S1S5Listdef.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P1_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P2_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P3_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P4_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P5_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Level 2.1\BC4+A1A2P6_ELA.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx1.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx2.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx4.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx5.txt
D:\Tabqdf\TabqdfTests\Third Example\Share\ElaPxUx6.txt

2.9 Third Example of Output File

TABQDF_Example_III.doc

```
&QDF
 NCOL    =           6,
 NLEVEL  =           2,
 NSHAPRO =           1,
 NUTIL   = 6*1, 4*0,
 NVS     =           1,
 NLIST   =           0,
 NBLXLEV =           1,
 NBLXSHA =           4
 /

LEVEL L-2.1
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE DEPENDENT VARIABLES
   The variable TOTFLOW   is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
   The variable POP       is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.

SHARE
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE ALTERNATIVES
   Alternative AIR       is the same in all variants.
   Alternative RAIL      is the same in all variants.
   Alternative BUS       is the same in all variants.
   Alternative CAR       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
  ALTERNATIVE  1: AIR
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable RETNLSC1  is the same in all variants.
   The variable RFANLSC1  is the same in all variants.
   The variable FREQU1    is the same in all variants.
  ALTERNATIVE  2: RAIL
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable RETNLSC2  is the same in all variants.
   The variable RFANLSC2  is the same in all variants.
   The variable FREQ2     is the same in all variants.
  ALTERNATIVE  3: BUS
   The variable ODREG     is the same in all variants.
   The variable REVE      is the same in all variants.
   The variable LANGFR    is the same in all variants.
   The variable RETNLSC3  is the same in all variants.
   The variable RFANLSC3  is the same in all variants.
   The variable FREQ3     is the same in all variants.
  ALTERNATIVE  4: CAR
   The variable NUITA     is the same in all variants.

=========================================================================================================================
LEVEL                                           FAMILY        2.1        2.1        2.1        2.1        2.1        2.1
                                                VARIANT  BC4+A1A2P  BC4+A1A2P  BC4+A1A2P  BC4+A1A2P  BC4+A1A2P  BC4+A1A2P
                                                NUMBER          1          2          3          4          5          6

SHARE                                           FAMILY      LOGIT      LOGIT    S-DOGIT    G-DOGIT  LIN-IPT-L   BT-IPT-L
                                                VARIANT  LIN-LOGIT   BC-LOGIT    S-DOGIT    G-DOGIT  LIN-IPT-L   BT-IPT-L
                                                NUMBER          1          2          3          4          5          6
=========================================================================================================================
LEVEL

 POPULATION                         POP         EL(T,X)     1.437      1.435      1.447      1.426      1.543      1.422
                                                (t)      (  13.52)  (  13.12)  (  13.24)  (  12.84)  (  13.17)  (  13.40)
 REVENUE                            REVE        EL(T,X)     1.114      1.334      1.412      1.384      2.586      0.102
                                                (t)      (   0.68)  (   0.83)  (   0.83)  (   0.90)  (   1.65)  (   0.06)
 FRENCH LANGUAGE                    LANGFR      EL(T,X)     0.639      0.560      0.560      0.510      0.723      0.545
                                                (t)      (   2.60)  (   2.28)  (   2.26)  (   2.20)  (   3.18)  (   2.51)
 UTILITY-Share Variant 1            UTIL1       EL(T,U)     0.086
                                                (t)      (   1.98)
 UTILITY-Share Variant 2            UTIL2       EL(T,U)                0.359
                                                (t)                 (   1.48)
 UTILITY-Share Variant 3            UTIL3       EL(T,U)                           0.324
                                                (t)                            (   1.36)
 UTILITY-Share Variant 4            UTIL4       EL(T,U)                                      0.208
                                                (t)                                       (   1.34)
 UTILITY-Share Variant 5            UTIL5       EL(T,U)                                                -0.220
                                                (t)                                                  (  -1.70)
 UTILITY-Share Variant 6            UTIL6       EL(T,U)                                                           1.220
                                                (t)                                                             (   3.01)
-------------------------------------------------------------------------------------------------------------------------
SHARE
```

ALTERNATIVE 1: AIR (Mean share: 0.3649)								
O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.028	-0.004	-0.006	0.095	-0.041	0.024
		EL(Sm)	-0.144	-0.125	-0.035	-0.145	-0.143	-0.019
		(t)	(-0.57)	(-0.41)	(-0.26)	(-0.28)	(-0.97)	(0.04)
		EL(T)	-0.002	-0.001	-0.002	0.020	0.009	0.029
		EL(Tm)	-0.146	-0.126	-0.037	-0.125	-0.135	0.011
		D.R.	-0.955	-0.972	-0.863	-1.429	-1.182	6.357
INCOME	REVE	EL(U)	0.655	0.650	0.862	0.217	1.448	1.296
		EL(Sm)	1.310	1.432	3.535	4.482	4.805	1.949
		(t)	(0.89)	(0.98)	(1.91)	(1.76)	(2.38)	(1.56)
		EL(T)	1.170	1.567	1.691	1.429	2.268	1.684
		EL(Tm)	2.480	2.998	5.225	5.911	7.073	3.633
		D.R.	0.293	0.432	-0.113	-0.337	-0.121	0.270
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.026	0.115	0.087	0.027	0.188	0.112
		EL(Sm)	0.018	0.227	0.401	0.829	0.682	0.117
		(t)	(0.13)	(1.65)	(1.46)	(2.02)	(1.72)	(0.55)
		EL(T)	0.641	0.601	0.589	0.516	0.682	0.682
		EL(Tm)	0.658	0.828	0.990	1.345	1.364	0.799
		D.R.	1.668	0.989	0.630	0.051	0.370	1.338
TIME(CAR) / TIME(AIR)	RETNLSC1	EL(U)	0.476	0.536	0.144	0.221	0.337	1.109
		EL(Sm)	1.782	1.410	0.894	1.146	1.434	1.736
		(t)	(7.93)	(7.05)	(3.22)	(2.80)	(2.25)	(5.65)
		EL(T)	0.041	0.192	0.046	0.046	-0.074	1.353
		EL(Tm)	1.823	1.602	0.940	1.192	1.360	3.090
		D.R.	-0.939	-0.671	-0.865	-0.895	-1.149	0.200
FARE(AIR) / FARE(CAR)	RFANLSC1	EL(U)	-0.179	-0.272	-0.471	-0.635	-0.572	-0.308
		EL(Sm)	-0.670	-0.716	-2.932	-3.291	-2.436	-0.482
		(t)	(-6.90)	(-2.55)	(-6.36)	(-4.91)	(-5.57)	(-2.51)
		EL(T)	-0.015	-0.098	-0.152	-0.132	0.126	-0.376
		EL(Tm)	-0.685	-0.814	-3.084	-3.423	-2.310	-0.858
		D.R.	-0.939	-0.671	-0.865	-0.895	-1.149	0.200
UTILITARY FREQUENCY (AIR)	FREQU1	EL(U)	0.066	0.083	0.049	0.093	0.060	0.039
		EL(Sm)	0.247	0.218	0.307	0.485	0.257	0.061
		(t)	(4.95)	(5.39)	(4.90)	(3.95)	(5.01)	(3.67)
		EL(T)	0.006	0.030	0.016	0.019	-0.013	0.048
		EL(Tm)	0.252	0.247	0.322	0.504	0.243	0.109
		D.R.	-0.939	-0.671	-0.865	-0.895	-1.149	0.200
ALTERNATIVE 2: RAIL (Mean share: 0.0409)								
O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.028	-0.004	-0.006	0.095	-0.041	0.024
		EL(Sm)	-0.338	0.287	-0.028	0.051	-0.069	0.222
		(t)	(-1.41)	(1.05)	(-0.27)	(0.55)	(-0.94)	(2.13)
		EL(T)	-0.002	-0.001	-0.002	0.020	0.009	0.029
		EL(Tm)	-0.341	0.285	-0.030	0.070	-0.060	0.252
		D.R.	-0.826	-1.111	0.481	5.807	-4.633	1.865
INCOME	REVE	EL(U)	0.655	0.650	0.862	0.217	1.448	1.296
		EL(Sm)	3.369	1.405	2.974	0.479	2.998	-0.102
		(t)	(1.95)	(1.07)	(1.86)	(0.18)	(2.24)	(0.59)
		EL(T)	1.170	1.567	1.691	1.429	2.268	1.684
		EL(Tm)	4.539	2.972	4.665	1.908	5.266	1.582
		D.R.	5.301	11.885	7.859	17.309	9.526	25.017
LANGUAGE SIMILARITY O-D	LANGFR	EL(U)	0.026	0.115	0.087	0.027	0.188	0.112
		EL(Sm)	-0.077	-0.048	-0.141	-0.272	-0.172	-0.127
		(t)	(-0.16)	(0.34)	(-0.17)	(-0.52)	(0.05)	(-0.05)
		EL(T)	0.641	0.601	0.589	0.516	0.682	0.682
		EL(Tm)	0.564	0.553	0.448	0.244	0.510	0.555
		D.R.	26.759	25.570	31.157	50.756	31.694	29.047
TIME(CAR) / TIME(RAIL)	RETNLSC2	EL(U)	0.023	0.093	0.012	0.013	0.019	0.089
		EL(Sm)	0.415	1.878	0.191	0.236	0.346	1.805
		(t)	(7.93)	(7.05)	(3.22)	(2.80)	(2.25)	(5.65)
		EL(T)	0.002	0.033	0.004	0.003	-0.004	0.109
		EL(Tm)	0.417	1.912	0.195	0.239	0.342	1.914
		D.R.	-0.886	-0.572	-0.505	-0.714	-1.291	0.387
FARE(RAIL) / FARE(CAR)	RFANLSC2	EL(U)	-0.015	-0.047	-0.072	-0.070	-0.057	-0.045
		EL(Sm)	-0.281	-0.949	-1.127	-1.220	-1.059	-0.903
		(t)	(-6.90)	(-2.55)	(-6.36)	(-4.91)	(-5.57)	(-2.51)
		EL(T)	-0.001	-0.017	-0.023	-0.014	0.012	-0.054
		EL(Tm)	-0.283	-0.966	-1.150	-1.235	-1.046	-0.957
		D.R.	-0.886	-0.572	-0.505	-0.714	-1.291	0.387
FREQUENCY (RAIL)	FREQ2	EL(U)	0.010	0.008	0.013	0.018	0.011	0.010
		EL(Sm)	0.183	0.169	0.208	0.317	0.197	0.203
		(t)	(4.95)	(5.39)	(4.90)	(3.95)	(5.01)	(3.67)
		EL(T)	0.001	0.003	0.004	0.004	-0.002	0.012
		EL(Tm)	0.184	0.172	0.212	0.321	0.194	0.215
		D.R.	-0.886	-0.572	-0.505	-0.714	-1.291	0.387
ALTERNATIVE 3: BUS (Mean share: 0.0261)								
O-D WITHIN THE SAME PROVINCE	ODREG	EL(U)	-0.028	-0.004	-0.006	0.095	-0.041	0.024
		EL(Sm)	0.056	0.530	0.062	0.739	0.039	0.350
		(t)	(-0.04)	(2.36)	(0.45)	(2.17)	(-0.02)	(3.67)
		EL(T)	-0.002	-0.001	-0.002	0.020	0.009	0.029
		EL(Tm)	0.054	0.529	0.061	0.759	0.048	0.379
		D.R.	-2.727	-1.094	-2.159	-0.010	6.139	1.974

```
 INCOME                            REVE       EL(U)         0.655      0.650      0.862      0.217      1.448      1.296
                                              EL(Sm)        0.403     -1.370     -0.171     -4.532     -0.383     -2.189
                                              (t)      (   0.61) (  -0.46) (   0.37) (  -1.68) (   0.62) (  -0.59)
                                              EL(T)         1.170      1.567      1.691      1.429      2.268      1.684
                                              EL(Tm)        1.573      0.196      1.519     -3.103      1.885     -0.506
                                              D.R.         27.463    304.095     41.585    -18.618     45.044   -128.422

 LANGUAGE SIMILARITY O-D           LANGFR     EL(U)         0.026      0.115      0.087      0.027      0.188      0.112
                                              EL(Sm)        0.620      0.496      0.578     -0.761      0.532      0.730
                                              (t)      (   2.46) (   3.81) (   2.52) (  -1.15) (   2.77) (   2.90)
                                              EL(T)         0.641      0.601      0.589      0.516      0.682      0.682
                                              EL(Tm)        1.261      1.097      1.167     -0.246      1.214      1.412
                                              D.R.         18.449     19.973     18.302    -81.305     20.485     17.480

 TIME(CAR) / TIME(BUS)             RETNLSC3   EL(U)         0.012      0.057      0.007      0.025      0.010      0.067
                                              EL(Sm)        0.397      1.915      0.183      0.196      0.328      2.361
                                              (t)      (   7.93) (   7.05) (   3.22) (   2.80) (   2.25) (   5.65)
                                              EL(T)         0.001      0.021      0.002      0.005     -0.002      0.082
                                              EL(Tm)        0.398      1.936      0.185      0.201      0.326      2.443
                                              D.R.         -0.899     -0.594     -0.559      0.004     -1.258      0.289

 FARE(BUS) / FARE(CAR)             RFANLSC3   EL(U)        -0.008     -0.029     -0.037     -0.126     -0.029     -0.032
                                              EL(Sm)       -0.257     -0.968     -1.034     -0.969     -0.959     -1.128
                                              (t)      (  -6.90) (  -2.55) (  -6.36) (  -4.91) (  -5.57) (  -2.51)
                                              EL(T)        -0.001     -0.010     -0.012     -0.026      0.006     -0.039
                                              EL(Tm)       -0.258     -0.979     -1.046     -0.995     -0.952     -1.168
                                              D.R.         -0.899     -0.594     -0.559      0.004     -1.258      0.289

 FREQUENCY (BUS)                   FREQ3      EL(U)         0.013      0.013      0.018      0.085      0.014      0.019
                                              EL(Sm)        0.433      0.430      0.494      0.653      0.462      0.657
                                              (t)      (   4.95) (   5.39) (   4.90) (   3.95) (   5.01) (   3.67)
                                              EL(T)         0.001      0.005      0.006      0.018     -0.003      0.023
                                              EL(Tm)        0.434      0.435      0.500      0.671      0.459      0.680
                                              D.R.         -0.899     -0.594     -0.559      0.004     -1.258      0.289

ALTERNATIVE 4: CAR       (Mean share: 0.5681)
=============
 NUMBER OF NIGHTS                  NUITA      EL(U)        -0.455     -0.371     -0.492     -0.403     -0.354     -0.486
                                   -----      EL(Sm)       -0.404     -0.432     -0.147     -0.109     -0.131     -0.422
                                              (t)      (  -7.43) (  -6.60) (  -4.46) (  -1.25) (  -4.48) (  -4.68)
                                              EL(T)        -0.039     -0.133     -0.159     -0.084      0.078     -0.593
                                              EL(Tm)       -0.443     -0.565     -0.306     -0.193     -0.054     -1.015
                                              D.R.         -0.845     -0.585     -0.084     -0.238     -3.551      0.029

========================================================================================================================
```

2.10 Fourth Example of Input File

TABQDF_Example_IV.in

```
$QDF ncol=5,nlevel=1,nshapro=2,nutil=1,1,1,1,1,nvs=1,nlist=0,nblxlev=1,nblxsha=3$
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\L1Listdef.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\P2P6Listdef_pour TABQDF.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\LIN6_ELA.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\LOG7_ELA.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\BC18_ELA.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\BC29_ELA.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Level 1.6\BC410_ELA.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fourth Example\Probability\ElaPxUx3.txt
```

2.11 Fourth Example of Output File

TABQDF_Example_IV.doc

```
&QDF
 NCOL    =           5,
 NLEVEL  =           1,
 NSHAPRO =           2,
 NUTIL   = 5*1, 5*0,
 NVS     =           1,
 NLIST   =           0,
 NBLXLEV =           1,
 NBLXSHA =           3
 /

LEVEL L-1.6
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE DEPENDENT VARIABLES
   The variable Tij       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
   The variable Ti        is the same in all variants.
   The variable Tj        is the same in all variants.

PROBABILITY
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE ALTERNATIVES
   Alternative RAIL      is the same in all variants.
   Alternative AIR       is the same in all variants.
   Alternative BUS       is the same in all variants.
   Alternative CAR       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
  ALTERNATIVE  1: RAIL
   The variable RAI_CTT   is the same in all variants.
   The variable RAI_TTT   is the same in all variants.
   The variable RAI_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  2: AIR
   The variable AIR_CTT   is the same in all variants.
   The variable AIR_TTT   is the same in all variants.
   The variable AIR_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  3: BUS
   The variable BUS_CTT   is the same in all variants.
   The variable BUS_TTT   is the same in all variants.
   The variable BUS_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  4: CAR
   The variable CAR_C     is the same in all variants.
   The variable CAR_RT    is the same in all variants.

==============================================================================================================
LEVEL                                            FAMILY        1.6        1.6        1.6        1.6        1.6
                                                 VARIANT       LIN        LOG        BC1        BC2        BC4
                                                 NUMBER          6          7          8          9         10

PROBABILITY                                      FAMILY      LOGIT      LOGIT      LOGIT      LOGIT      LOGIT
                                                 VARIANT   bus_bc4    bus_bc4    bus_bc4    bus_bc4    bus_bc4
                                                 NUMBER          3          3          3          3          3
==============================================================================================================
LEVEL

 TRIPS FROM ORIG_i TO ALL DEST.       Ti         EL(T,X)     0.170      0.533      0.517      0.481      0.363
                                                 (t)      (  1.49)  (  8.05)  (  8.64)  (  8.52)  (  8.71)
 TRIPS FROM ALL ORIG. TO DEST_j       Tj         EL(T,X)     0.321      0.651      0.641      0.588      0.821
                                                 (t)      (  3.43)  ( 11.00)  ( 12.00)  ( 11.76)  ( 10.26)
 UTILITY from VARIANT bus_bc4         Util_BC4   EL(T,U)     0.255      0.584      0.596      0.533      0.528
                                                 (t)      ( 13.94)  ( 36.04)  ( 35.57)  ( 35.90)  ( 37.59)
--------------------------------------------------------------------------------------------------------------
PROBABILITY

ALTERNATIVE 1: RAIL      (Mean prob.: 0.1660)
=============
 TOTAL COST, RAIL(RUN + ACCESS COST) RAI_CTT     EL(U)      -0.194     -0.194     -0.194     -0.194     -0.194
                                                 EL(Pm)     -3.273     -3.273     -3.273     -3.273     -3.273
                                                 (t)      ( -5.12)  ( -5.12)  ( -5.12)  ( -5.12)  ( -5.12)
                                                 EL(T)      -0.050     -0.113     -0.116     -0.104     -0.103
                                                 EL(Tm)     -3.322     -3.386     -3.388     -3.376     -3.375
                                                 D.R.       -0.910     -0.798     -0.794     -0.815     -0.817

 TOTAL TRAVEL TIME, RAIL             RAI_TTT     EL(U)      -0.161     -0.161     -0.161     -0.161     -0.161
                                                 EL(Pm)     -2.714     -2.714     -2.714     -2.714     -2.714
                                                 (t)      ( -3.45)  ( -3.45)  ( -3.45)  ( -3.45)  ( -3.45)
                                                 EL(T)      -0.041     -0.094     -0.096     -0.086     -0.085
                                                 EL(Tm)     -2.756     -2.808     -2.810     -2.800     -2.799
                                                 D.R.       -0.910     -0.798     -0.794     -0.815     -0.817

 FREQUENCY, RAIL                     RAI_FR      EL(U)       0.072      0.072      0.072      0.072      0.072
                                                 EL(Pm)      1.206      1.206      1.206      1.206      1.206
                                                 (t)      (  7.59)  (  7.59)  (  7.59)  (  7.59)  (  7.59)
                                                 EL(T)       0.018      0.042      0.043      0.038      0.038
                                                 EL(Tm)      1.224      1.247      1.248      1.244      1.243
                                                 D.R.       -0.910     -0.798     -0.794     -0.815     -0.817

 INCOME                              INCOME      EL(U)       0.053      0.053      0.053      0.053      0.053
                                                 EL(Pm)      0.140      0.140      0.140      0.140      0.140
```

```
                                                (t)      (   1.11)  (   1.11)  (   1.11)  (   1.11)  (   1.11)
                                                EL(T)        0.014      0.031      0.032      0.028      0.028
                                                EL(Tm)       0.153      0.171      0.171      0.168      0.168
                                                D.R.        -0.466      0.096      0.115      0.017      0.009

ALTERNATIVE 2: AIR       (Mean prob.: 0.3590)
=============
 TOTAL COST, AIR (RUN + ACCESS COST) AIR_CTT    EL(U)       -0.901     -0.901     -0.901     -0.901     -0.901
                                                EL(Pm)      -3.688     -3.688     -3.688     -3.688     -3.688
                                                (t)      (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)
                                                EL(T)       -0.230     -0.526     -0.537     -0.480     -0.476
                                                EL(Tm)      -3.917     -4.214     -4.225     -4.168     -4.163
                                                D.R.        -0.837     -0.652     -0.646     -0.679     -0.682

 TOTAL TRAVEL TIME, AIR              AIR_TTT    EL(U)       -0.221     -0.221     -0.221     -0.221     -0.221
                                                EL(Pm)      -0.904     -0.904     -0.904     -0.904     -0.904
                                                (t)      (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)
                                                EL(T)       -0.056     -0.129     -0.132     -0.118     -0.117
                                                EL(Tm)      -0.960     -1.033     -1.035     -1.021     -1.020
                                                D.R.        -0.837     -0.652     -0.646     -0.679     -0.682

 FREQUENCY, AIR                      AIR_FR     EL(U)        0.343      0.343      0.343      0.343      0.343
                                                EL(Pm)       1.405      1.405      1.405      1.405      1.405
                                                (t)      (   7.59)  (   7.59)  (   7.59)  (   7.59)  (   7.59)
                                                EL(T)        0.088      0.200      0.205      0.183      0.181
                                                EL(Tm)       1.492      1.605      1.609      1.588      1.586
                                                D.R.        -0.837     -0.652     -0.646     -0.679     -0.682

 INCOME                              INCOME     EL(U)        0.053      0.053      0.053      0.053      0.053
                                                EL(Pm)       0.140      0.140      0.140      0.140      0.140
                                                (t)      (   1.11)  (   1.11)  (   1.11)  (   1.11)  (   1.11)
                                                EL(T)        0.014      0.031      0.032      0.028      0.028
                                                EL(Tm)       0.153      0.171      0.171      0.168      0.168
                                                D.R.        -0.753     -0.493     -0.484     -0.530     -0.533

ALTERNATIVE 3: BUS       (Mean prob.: 0.0040)
=============
 TOTAL COST, BUS (RUN + ACCESS COST) BUS_CTT    EL(U)       -0.067     -0.067     -0.067     -0.067     -0.067
                                                EL(Pm)      -2.796     -2.796     -2.796     -2.796     -2.796
                                                (t)      (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)
                                                EL(T)       -0.017     -0.039     -0.040     -0.036     -0.036
                                                EL(Tm)      -2.814     -2.836     -2.837     -2.832     -2.832
                                                D.R.         0.529      2.471      2.544      2.173      2.143

 TOTAL TRAVEL TIME, BUS              BUS_TTT    EL(U)       -0.072     -0.072     -0.072     -0.072     -0.072
                                                EL(Pm)      -2.971     -2.971     -2.971     -2.971     -2.971
                                                (t)      (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)
                                                EL(T)       -0.018     -0.042     -0.043     -0.038     -0.038
                                                EL(Tm)      -2.989     -3.013     -3.013     -3.009     -3.009
                                                D.R.         0.529      2.471      2.544      2.173      2.143

 FREQUENCY, BUS                      BUS_FR     EL(U)        0.036      0.036      0.036      0.036      0.036
                                                EL(Pm)       1.491      1.491      1.491      1.491      1.491
                                                (t)      (   7.59)  (   7.59)  (   7.59)  (   7.59)  (   7.59)
                                                EL(T)        0.009      0.021      0.021      0.019      0.019
                                                EL(Tm)       1.500      1.512      1.512      1.510      1.510
                                                D.R.         0.529      2.471      2.544      2.173      2.143

 INCOME                              INCOME     EL(U)        0.053      0.053      0.053      0.053      0.053
                                                EL(Pm)       0.140      0.140      0.140      0.140      0.140
                                                (t)      (   1.11)  (   1.11)  (   1.11)  (   1.11)  (   1.11)
                                                EL(T)        0.014      0.031      0.032      0.028      0.028
                                                EL(Tm)       0.153      0.171      0.171      0.168      0.168
                                                D.R.        21.148     44.487     45.286     41.205     40.869

ALTERNATIVE 4: CAR       (Mean prob.: 0.4710)
=============
 COST, CAR (RUN COST)                CAR_C      EL(U)       -2.602     -2.602     -2.602     -2.602     -2.602
                                                EL(Pm)      -0.992     -0.992     -0.992     -0.992     -0.992
                                                (t)      (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)  (  -5.12)
                                                EL(T)       -0.664     -1.519     -1.552     -1.387     -1.374
                                                EL(Tm)      -1.655     -2.511     -2.543     -2.379     -2.365
                                                D.R.        -0.149      0.285      0.295      0.238      0.233

 RUN TIME, CAR                       CAR_RT     EL(U)       -1.314     -1.314     -1.314     -1.314     -1.314
                                                EL(Pm)      -0.501     -0.501     -0.501     -0.501     -0.501
                                                (t)      (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)  (  -3.45)
                                                EL(T)       -0.335     -0.767     -0.784     -0.701     -0.694
                                                EL(Tm)      -0.836     -1.268     -1.284     -1.201     -1.195
                                                D.R.        -0.149      0.285      0.295      0.238      0.233

===============================================================================================================
```

2.12 Fifth Example of Input File

TABQDF_Example_V.in

$QDF ncol=3,nlevel=2,nshapro=2,nutil=1,1,1,nvs=1,nlist=0,nblxlev=1,nblxsha=3$
D:\Tabqdf\TabqdfTests\Fifth Example\Level 2.1\L2Listdef.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Probability\P2P6Listdef_pour TABQDF.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Level 2.1\BC410_ELA.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Level 2.1\BC4HG17_ELA.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Level 2.1\BC4HGAUPR24_ELA.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Probability\ElaPxUx3.txt
D:\Tabqdf\TabqdfTests\Fifth Example\Probability\ElaPxUx3.txt

2.13 Fifth Example of Output File

TABQDF_Example_V.doc

```
&QDF
 NCOL    =           3,
 NLEVEL  =           2,
 NSHAPRO =           2,
 NUTIL   = 3*1, 7*0,
 NVS     =           1,
 NLIST   =           0,
 NBLXLEV =           1,
 NBLXSHA =           3
 /

LEVEL L-2.1
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE DEPENDENT VARIABLES
   The variable Tij       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
   The variable Ti        is the same in all variants.
   The variable Tj        is the same in all variants.

PROBABILITY
 -CHECK ON THE NUMBER OF OBSERVATIONS
   The number of observations is the same in all variants.
 -CHECK ON THE ALTERNATIVES
   Alternative RAIL      is the same in all variants.
   Alternative AIR       is the same in all variants.
   Alternative BUS       is the same in all variants.
   Alternative CAR       is the same in all variants.
 -CHECK ON THE INDEPENDENT VARIABLES
  ALTERNATIVE  1: RAIL
   The variable RAI_CTT   is the same in all variants.
   The variable RAI_TTT   is the same in all variants.
   The variable RAI_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  2: AIR
   The variable AIR_CTT   is the same in all variants.
   The variable AIR_TTT   is the same in all variants.
   The variable AIR_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  3: BUS
   The variable BUS_CTT   is the same in all variants.
   The variable BUS_TTT   is the same in all variants.
   The variable BUS_FR    is the same in all variants.
   The variable INCOME    is the same in all variants.
  ALTERNATIVE  4: CAR
   The variable CAR_C     is the same in all variants.
   The variable CAR_RT    is the same in all variants.

=========================================================================================
LEVEL                                             FAMILY         2.1        2.1        2.1
                                                  VARIANT        BC4      BC4HG  BC4HGAUPR
                                                  NUMBER          10         17         24

PROBABILITY                                       FAMILY       LOGIT      LOGIT      LOGIT
                                                  VARIANT    bus_bc4    bus_bc4    bus_bc4
                                                  NUMBER           3          3          3
=========================================================================================
LEVEL

 TRIPS FROM ORIG_i TO ALL DEST.       Ti          EL(T,X)      0.363      0.398      0.398
                                                  (t)      (   8.71)  (   7.74)  (   7.73)
 TRIPS FROM ALL ORIG. TO DEST_j       Tj          EL(T,X)      0.821      0.806      0.807
                                                  (t)      (  10.26)  (  12.71)  (  12.88)
 UTILITY from VARIANT bus_bc4         Util_BC4    EL(T,U)      0.528      0.508      0.500
                                                  (t)      (  37.59)  (  40.84)  (  45.44)
-----------------------------------------------------------------------------------------
```

PROBABILITY

ALTERNATIVE 1: RAIL (Mean prob.: 0.1660)
=============

TOTAL COST, RAIL(RUN + ACCESS COST)	RAI_CTT	EL(U)	-0.194	-0.194	-0.194
		EL(Pm)	-3.273	-3.273	-3.273
		(t)	(-5.12)	(-5.12)	(-5.12)
		EL(T)	-0.103	-0.099	-0.097
		EL(Tm)	-3.375	-3.371	-3.370
		D.R.	-0.817	-0.824	-0.826
TOTAL TRAVEL TIME, RAIL	RAI_TTT	EL(U)	-0.161	-0.161	-0.161
		EL(Pm)	-2.714	-2.714	-2.714
		(t)	(-3.45)	(-3.45)	(-3.45)
		EL(T)	-0.085	-0.082	-0.081
		EL(Tm)	-2.799	-2.796	-2.795
		D.R.	-0.817	-0.824	-0.826
FREQUENCY, RAIL	RAI_FR	EL(U)	0.072	0.072	0.072
		EL(Pm)	1.206	1.206	1.206
		(t)	(7.59)	(7.59)	(7.59)
		EL(T)	0.038	0.036	0.036
		EL(Tm)	1.243	1.242	1.241
		D.R.	-0.817	-0.824	-0.826
INCOME	INCOME	EL(U)	0.053	0.053	0.053
		EL(Pm)	0.140	0.140	0.140
		(t)	(1.11)	(1.11)	(1.11)
		EL(T)	0.028	0.027	0.027
		EL(Tm)	0.168	0.167	0.166
		D.R.	0.009	-0.024	-0.036

ALTERNATIVE 2: AIR (Mean prob.: 0.3590)
=============

TOTAL COST, AIR (RUN + ACCESS COST)	AIR_CTT	EL(U)	-0.901	-0.901	-0.901
		EL(Pm)	-3.688	-3.688	-3.688
		(t)	(-5.12)	(-5.12)	(-5.12)
		EL(T)	-0.476	-0.457	-0.451
		EL(Tm)	-4.163	-4.145	-4.138
		D.R.	-0.682	-0.693	-0.697
TOTAL TRAVEL TIME, AIR	AIR_TTT	EL(U)	-0.221	-0.221	-0.221
		EL(Pm)	-0.904	-0.904	-0.904
		(t)	(-3.45)	(-3.45)	(-3.45)
		EL(T)	-0.117	-0.112	-0.110
		EL(Tm)	-1.020	-1.016	-1.014
		D.R.	-0.682	-0.693	-0.697
FREQUENCY, AIR	AIR_FR	EL(U)	0.343	0.343	0.343
		EL(Pm)	1.405	1.405	1.405
		(t)	(7.59)	(7.59)	(7.59)
		EL(T)	0.181	0.174	0.172
		EL(Tm)	1.586	1.579	1.576
		D.R.	-0.682	-0.693	-0.697
INCOME	INCOME	EL(U)	0.053	0.053	0.053
		EL(Pm)	0.140	0.140	0.140
		(t)	(1.11)	(1.11)	(1.11)
		EL(T)	0.028	0.027	0.027
		EL(Tm)	0.168	0.167	0.166
		D.R.	-0.533	-0.549	-0.554

ALTERNATIVE 3: BUS (Mean prob.: 0.0040)
=============

TOTAL COST, BUS (RUN + ACCESS COST)	BUS_CTT	EL(U)	-0.067	-0.067	-0.067
		EL(Pm)	-2.796	-2.796	-2.796
		(t)	(-5.12)	(-5.12)	(-5.12)
		EL(T)	-0.036	-0.034	-0.034
		EL(Tm)	-2.832	-2.831	-2.830
		D.R.	2.143	2.023	1.980
TOTAL TRAVEL TIME, BUS	BUS_TTT	EL(U)	-0.072	-0.072	-0.072
		EL(Pm)	-2.971	-2.971	-2.971
		(t)	(-3.45)	(-3.45)	(-3.45)
		EL(T)	-0.038	-0.036	-0.036
		EL(Tm)	-3.009	-3.007	-3.007
		D.R.	2.143	2.023	1.980
FREQUENCY, BUS	BUS_FR	EL(U)	0.036	0.036	0.036
		EL(Pm)	1.491	1.491	1.491
		(t)	(7.59)	(7.59)	(7.59)
		EL(T)	0.019	0.018	0.018
		EL(Tm)	1.510	1.509	1.509
		D.R.	2.143	2.023	1.980
INCOME	INCOME	EL(U)	0.053	0.053	0.053
		EL(Pm)	0.140	0.140	0.140
		(t)	(1.11)	(1.11)	(1.11)
		EL(T)	0.028	0.027	0.027
		EL(Tm)	0.168	0.167	0.166
		D.R.	40.869	39.516	39.017

```
ALTERNATIVE 4: CAR       (Mean prob.: 0.4710)
=============
 COST, CAR (RUN COST)               CAR_C       EL(U)        -2.602      -2.602      -2.602
                                                EL(Pm)       -0.992      -0.992      -0.992
                                                (t)      (  -5.12)  (  -5.12)  (  -5.12)
                                                EL(T)        -1.374      -1.321      -1.301
                                                EL(Tm)       -2.365      -2.312      -2.293
                                                D.R.          0.233       0.213       0.205

 RUN TIME, CAR                      CAR_RT      EL(U)        -1.314      -1.314      -1.314
                                                EL(Pm)       -0.501      -0.501      -0.501
                                                (t)      (  -3.45)  (  -3.45)  (  -3.45)
                                                EL(T)        -0.694      -0.667      -0.657
                                                EL(Tm)       -1.195      -1.168      -1.158
                                                D.R.          0.233       0.213       0.205

==========================================================================================
```

3. PROGRAM LISTING

The program consists of the main module **Tabqdf** and one subroutine **underline** . It was compiled with Compaq Visual Fortran 6.6.

Program listing

Tabqdf.f

```
        program Tabqdf
c Compiled with Visual Compaq Fortran 6.6 (February 2 2012)
    implicit double precision (a-h,o-z)

    parameter (maxcol=10,maxlev=100,maxalt=20,maxsha=100)
    dimension elatotal(maxcol),elamodal(maxcol),divrate(maxcol)

    dimension elastxlev(maxlev,maxcol),studxlev(maxlev,maxcol)
    dimension nxlevel(maxcol),nutil(maxcol),nobslev(maxcol)
        dimension numvlev(maxlev),numvsha(maxsha)

        dimension numxsha(maxalt,maxcol),nobssha(maxcol)
        dimension nxshare(maxalt,maxcol),sharemean(maxalt)
        dimension elapx(maxalt,maxsha,maxcol),elaux(maxalt,maxsha,maxcol)

        integer levnumber(maxcol),shanumber(maxcol)

        character*1 under(10)
        character*8 nambl,namx,namvlev(maxlev),namvsha(maxsha)
    character*8 namalt,namalt1,namalter(maxalt)
    character*10 namylev(maxcol),namxlev(maxlev,maxcol)
        character*10 levfamily(maxcol),levvariant(maxcol)
        character*10 namU(maxcol),elastU(maxcol,maxcol)
        character*10 studU(maxcol,maxcol)
    character*10 namxsha(maxalt,maxsha,maxcol),shavariant(maxcol)
        character*10 shafamily(maxcol),nomy(maxalt,maxcol)
    character*10 studxsha(maxalt,maxsha,maxcol)

    character*20 idraw1,idraw2
        character*35 def,defvlev(maxlev),defvsha(maxsha)
    character*80 ifmt,DefFile,LevElastFile,ShaElastFile

    namelist/QDF/ncol,nlevel,nshapro,nutil,nvs,nlist,nblxlev,nblxsha

        data studU/100*' '/
        data elastU/100*' '/
```

```
* 8 INPUT PARAMETERS in namelist QDF:
* ncol       : number of columns (variants) listed in the table (max.10).
* nlevel     : 1 for LEVEL 1.6
*              2 for LEVEL 2.1
* nshapro    : 1 for SHARE S1-S5
*              2 for PROBABILITY P2-P6
* nutil(i)   : vector of ncol elements indicating if the Utility variable
* (i=1,ncol)   U is present in the i-th variant in LEVEL L-1.6 or L-2.1,
*              nutil(i)=0 if U is not present and 1 otherwise.
* nvs        : 0 (French version of the table)
*            : 1 (English version of the table)
* nlist      : 0 No list for the definitions of the blocks and the independent
*                variables X.
*            : 1 List the definitions of the blocks and the independent
*                variables X.
* nblxlev    : Number of blocks of independent variables X for LEVEL.
* nblxsha    : Number of blocks of independent variables X for SHARE
*              or PROBABILITY.

      read QDF
         print QDF
* LEVEL: Read DefFile containing the list of definitions
* of the blocks and the variables X on UNIT=1.
      read 5,DefFile
5        format(a80)
      open(unit=1,file=DefFile)
* nvarlev is the total number of the variables X in DefFile for LEVEL.
         if(nlist.eq.1)then
              print*,' '
              if(nvs.eq.0)then
                   print*,'NIVEAU: Liste des Définitions des Blocs et des Var
     +iables'
              else
                   print*,'LEVEL: List of Definitions of the Blocks and the V
     +ariables'
              endif
         endif
      nvarlev=0
      do 1000 i=1,nblxlev
      if(nvs.eq.0)read(1,1001)num,nambl,def
      if(nvs.eq.1)read(1,1002)num,nambl,def
1001     format(i2,a8,a35)
1002     format(i2,a8,35x,a35)
      if(nlist.eq.1)print 1003,i,nambl,def
1003     format(i5,2x,a8,2x,a35)
1005     continue
```

```
* numvlev() : category (0,1,2,3) of a variable, only a variable
*           of category 2 (quasi-dummy) or category 3 (dummy)
*           is underlined.
* namvlev() : name of a variable
* defvlev() : definition of a variable
      if(nvs.eq.0)read(1,1001)num,namx,def
      if(nvs.eq.1)read(1,1002)num,namx,def
      if(namx.eq.' ')go to 1000
      nvarlev=nvarlev+1
      numvlev(nvarlev)=num
      namvlev(nvarlev)=namx
            defvlev(nvarlev)=def
* Print the list of definitions of the blocks and the variables.
      if(nlist.eq.1)print 1003,num,namvlev(nvarlev),def
      go to 1005
1000    continue
            close(1)

* SHARE/PROBABILITY: Read DefFile containing the list of definitions
* of the blocks and the variables X on UNIT=1.
      read 5,DefFile
      open(unit=1,file=DefFile)
* nvarsha is the total number of the variables X
* in DefFile for SHARE/PROBABILITY.
            if(nlist.eq.1)then
                        print*,' '
                        if(nvs.eq.0)then
                                    print*,'PART: Liste des Définitions des Blocs et des Varia
     +bles'
                        else
                                    print*,'SHARE: List of Definitions of the Blocks and the V
     +ariables'
                        endif
            endif
      nvarsha=0
      do 1100 i=1,nblxsha
      if(nvs.eq.0)read(1,1001)num,nambl,def
      if(nvs.eq.1)read(1,1002)num,nambl,def
      if(nlist.eq.1)print 1003,i,nambl,def
1105    continue
* numvsha() : category (0,1,2,3) of a variable, only a variable
*           of category 2 (quasi-dummy) or category 3 (dummy)
*           is underlined.
* namvsha() : name of a variable
* defvsha() : definition of a variable
      if(nvs.eq.0)read(1,1001)num,namx,def
```

```
      if(nvs.eq.1)read(1,1002)num,namx,def
      if(namx.eq.' ')go to 1100
      nvarsha=nvarsha+1
      numvsha(nvarsha)=num
      namvsha(nvarsha)=namx
        defvsha(nvarsha)=def
* Print the list of definitions of the blocks and the variables.
      if(nlist.eq.1)print 1003,num,namvsha(nvarsha),def
      go to 1105
1100    continue
        close(1)

* For "ncol" variants
* LEVEL: Read file name for the Elasticities
*        Read name and number of a Variant.
* maxU is the number of the Utility variables which are different across the
* 'ncol' variants.
        maxU=0
        do 1 j=1,ncol
        levfamily(j)='1.6'
        if(nlevel.eq.2)levfamily(j)='2.1'
        read 5,LevElastFile
      open(unit=1,file=LevElastFile)
        read(1,2)levvariant(j),levnumber(j),namylev(j),nobslev(j)
2       format(a9,i4/a10,i5)
* For "nxlevel" independent variables (excluding the contant)
* READ the name of an independent variable X in LEVEL,
*      the conditional t of the associated beta
*      and the elasticity of T (sample) with respect to X,
        ix=1
100   read(1,4,end=110)namxlev(ix,j),studxlev(ix,j),elastxlev(ix,j)
4     format(a10,2d15.7)
        nxlevel(j)=ix
* nxlev1 is the total number of independent variables X other than
* the Utility variable U in a variant.
        if(nutil(j).eq.1)nxlev1=ix-1
        ix=ix+1
      go to 100
110     close(1)
* Check if U is present in j-th variant.
        if(nutil(j).eq.1)then
                maxU=maxU+1
                nx=nxlevel(j)
                if(maxU.eq.1)then
* First U found
                        namU(maxU)=namxlev(nx,j)
```

```
                    encode(10,40,studU(maxU,j))studxlev(nx,j)
40                  format(1h(,f7.2,1h))
                    encode(10,41,elastU(maxU,j))elastxlev(nx,j)
41                  format(f10.3)
               else
                    k=maxU-1
                    do 42 i=1,k
                    if(namxlev(nx,j).eq.namU(i))then
* Case where the current U is the same as one of the previous U's
                         maxU=maxU-1
                         encode(10,40,studU(i,j))studxlev(nx,j)
                         encode(10,41,elastU(i,j))elastxlev(nx,j)
                         go to 1
                    endif
42                  continue
* Case where the current U is different from all the previous U's
                    namU(maxU)=namxlev(nx,j)
                    encode(10,40,studU(maxU,j))studxlev(nx,j)
                    encode(10,41,elastU(maxU,j))elastxlev(nx,j)
               endif
          endif
1         continue

          if(nvs.eq.0)print 135,levfamily(1)
135       format(/'NIVEAU L-',a10)
          if(nvs.eq.1)print 136,levfamily(1)
136       format(/'LEVEL L-',a10)

* Check if the number of observations is the same across the variants.
          if(nvs.eq.0)print*,"-VÉRIFICATION SUR LES NOMBRES D'OBSERVATIONS"
          if(nvs.eq.1)print*,'-CHECK ON THE NUMBER OF OBSERVATIONS'
          ier=0
          do j=2,ncol
               if(nobslev(1).ne.nobslev(j))ier=1
          enddo
          if(ier.eq.0)then
               if(nvs.eq.0)then
                    print *,"  Le nombre d'observations ",
     +'est le même dans toutes les variantes.'
               else
                    print *,'  The number of observations ',
     +'is the same in all variants.'
               endif
          else
               if(nvs.eq.0)then
                    print*," Le nombre d'observations ",
```

```
     +"n'est pas le même dans toutes les variantes:"
                              print 4510,(levnumber(i),nobslev(i),i=1,ncol)
4510                          format(5x,"Variante  Nombre d'observations"/(5x,i8,2x,i5))
                      else
                              print*,' The number of observations ',
     +'is not the same in all variants:'
                              print 4511,(levnumber(i),nobslev(i),i=1,ncol)
4511                          format(7x,'Variant  Number of observations'/(6x,i8,2x,i5))
                      endif
              endif
* Check if the dependent variable Y is the same across the variants.
              if(nvs.eq.0)print*,'-VÉRIFICATION SUR LES VARIABLES DÉPENDANTES'
              if(nvs.eq.1)print*,'-CHECK ON THE DEPENDENT VARIABLES'
              nydiff=0
              do 138 j=2,ncol
                      if(namylev(1).ne.namylev(j))then
                              ier=1
                              nydiff=1
                      endif
138           continue
              if(nydiff.eq.0)then
                      if(nvs.eq.0)then
                              print *,'  La variable ',namylev(1),
     +'est la même dans toutes les variantes.'
                      else
                              print *,'  The variable ',namylev(1),
     +'is the same in all variants.'
                      endif
                      go to 139
              else
                      if(nvs.eq.0)then
                              print 451,(levnumber(i),namylev(i),i=1,ncol)
                      else
                              print 45,(levnumber(i),namylev(i),i=1,ncol)
                      endif
              endif
139           continue
* Check if each independent variable X is the same across the variants.
              if(nvs.eq.0)print*,'-VÉRIFICATION SUR LES VARIABLES INDÉPENDANTES'
              if(nvs.eq.1)print*,'-CHECK ON THE INDEPENDENT VARIABLES'
              do 48 k=1,nxlev1
              nxdiff=0
              do 44 j=2,ncol
                      if(namxlev(k,1).ne.namxlev(k,j))then
                              ier=1
                              nxdiff=1
```

```
                endif
44      continue
        if(nxdiff.eq.0)then
                if(nvs.eq.0)then
                        print*,'  La variable ',namxlev(k,1),'est la même dans',
   +' toutes les variantes.'
                else
                        print*,'  The variable ',namxlev(k,1),'is the same in',
   +' all variants.'
                endif
                go to 48
        else
                if(nvs.eq.0)then
                        print*,'  La variable ',namxlev(k,1),"n'est pas la même",
   +" dans toutes les variantes:"
                        print 451,(levnumber(i),namxlev(k,i),i=1,ncol)
451                     format(5x,'Variante  Variable'/(5x,i8,2x,a10))
                else
                        print*,'  The variable ',namxlev(k,1),"is not the same",
   +" in all variants:"
                        print 45,(levnumber(i),namxlev(k,i),i=1,ncol)
45                      format(7x,'Variant  Variable'/(6x,i8,2x,a10))
                endif
        endif
48      continue

* SHARE or PROBABILITY
*    Read file name for the elasticities
*    Read model family, name and number of a variant, number of alternatives and observations.
*    Read number of X for each alternative, excluding the constant.
*    Read the mean of SHARE or PROBABILITY for each alternative.
        do 10 j=1,ncol
        read 5,ShaElastFile
      open(unit=1,file=ShaElastFile)
        read(1,6)shafamily(j),shavariant(j),shanumber(j),nalt,nobssha(j)
6       format(2a10,2i3,i5)
        read(1,7)(nxshare(i,j),i=1,nalt)
7       format(12x,10i4)
* Read Alternative names
        read(1,8)(nomy(i,j),i=1,nalt)
8       format(12x,10a10)
* Read Mean Share, used to compute Diversion Rate (D.R.)
        read(1,9)(sharemean(i),i=1,nalt)
9       format(10x,10d15.7)
* Read the name of an alternative,
*     the name of an independent variable in SHARE (excluding the constant).
```

```
*     the elasticity of SHARE with respect to an independent variable X,
*     the conditional t of the associated beta
*     and the elasticity of UTILITY with respect to an independent variable X.
          do 200 i=1,nalt
          do 201 ix=1,nxshare(i,j)
          read(1,11)namalter(i),namxsha(i,ix,j),elapx(i,ix,j),studx,
     +elaux(i,ix,j)
11        format(a8,4x,a8,3f15.7)
          encode(10,40,studxsha(i,ix,j))studx
201       continue
200       continue
      close(1)
10        continue

          print*,' '
          if(nshapro.eq.1)then
                if(nvs.eq.0)then
                      print 342
342                   format('PART')
                else
                      print 344
344                   format('SHARE')
                endif
          endif
          if(nshapro.eq.2)then
                if(nvs.eq.0)then
                      print 343
343                   format('PROBABILITÉ')
                else
                      print 345
345                   format('PROBABILITY')
                endif
          endif
* Check on the number of observations across the variants.
          if(nvs.eq.0)print*,"-VÉRIFICATION SUR LES NOMBRES D'OBSERVATIONS"
          if(nvs.eq.1)print*,'-CHECK ON THE NUMBER OF OBSERVATIONS'
          nobsdiff=0
          do j=2,ncol
                if(nobssha(1).ne.nobssha(j))then
                      ier=1
                      nobsdiff=1
                endif
          enddo
          if(nobsdiff.eq.0)then
                if(nvs.eq.0)then
                      print *,"  Le nombre d'observations ",
```

```
     +'est le même dans toutes les variantes.'
                  else
                        print *,'  The number of observations ',
     +'is the same in all variants.'
                  endif
            else
                  if(nvs.eq.0)then
                        print*,'' Le nombre d'observations '',
     +''n'est pas le même dans toutes les variantes:''
                        print 4510,(shanumber(i),nobssha(i),i=1,ncol)
                  else
                        print*,' The number of observations ',
     +'is not the same in all variants:'
                        print 4511,(shanumber(i),nobssha(i),i=1,ncol)
                  endif
            endif

* Check if each alternative is the same across the variants.
            if(nvs.eq.0)print*,'-VÉRIFICATION SUR LES ALTERNATIVES'
            if(nvs.eq.1)print*,'-CHECK ON THE ALTERNATIVES'
            do i=1,nalt
            naltdiff=0
            do j=2,ncol
                  if(nomy(i,1).ne.nomy(i,j))then
                        ier=1
                        naltdiff=1
                  endif
            enddo
                  if(naltdiff.eq.1)then
                        if(nvs.eq.0)then
                              print*,'  Alternative ',nomy(i,1),''n'est pas la même'',
     +'' dans toutes les variantes:''
                              print 452,(shanumber(j),nomy(i,j),j=1,ncol)
452                           format(5x,'Variante  Alternative'/(5x,i8,2x,a10))
                        else
                              print*,'  Alternative ',nomy(i,1),''is not the same'',
     +'' in all variants:''
                              print 453,(shanumber(j),nomy(i,j),j=1,ncol)
453                           format(6x,'Variant  Alternative'/(5x,i8,2x,a10))
                        endif
                  else
                        if(nvs.eq.0)then
                              print*,'  Alternative ',nomy(i,1),'est la même',
     +                  ' dans toutes les variantes.'
                        else
                              print*,'  Alternative ',nomy(i,1),'is the same',
```

```
     +                        ' in all variants.'
                              endif
                    endif
          enddo
* Check if each independent variable is the same across the variants.
          if(nvs.eq.0)print*,'-VÉRIFICATION SUR LES VARIABLES INDÉPENDANTES'
          if(nvs.eq.1)print*,'-CHECK ON THE INDEPENDENT VARIABLES'
          do 60 m=1,nalt
          print 65,m,namalter(m)
65        format(2x,'ALTERNATIVE ',i2,': ',a8)
          nv=nxshare(m,1)
          do 61 k=1,nv
          nxdiff=0
          do 63 j=2,ncol
                    if(namxsha(m,k,1).ne.namxsha(m,k,j))then
                              ier=1
                              nxdiff=1
                    endif
63        continue
          if(nxdiff.eq.0)then
                    if(nvs.eq.0)then
                              print*,'  La variable ',namxsha(m,k,1),'est la même dans
     +toutes les variantes.'
                    else
                              print*,'  The variable ',namxsha(m,k,1),'is the same in a
     +ll variants.'
                    endif
                    go to 61
          else
                    if(nvs.eq.0)then
                              print*,'  La variable ',namxsha(m,k,1),"n'est pas la même",
     +" dans toutes les variantes:"
                              print 451,(shanumber(j),namxsha(m,k,j),j=1,ncol)
                    else
                              print*,'  The variable ',namxsha(m,k,1),"is not the same",
     +" in all variants:"
                              print 45,(shanumber(j),namxsha(m,k,j),j=1,ncol)
                    endif
          endif
62        continue
61        continue
60        continue
          if(ier.eq.1)then
                    print*,' '
                    if(nvs.eq.0)then
                              print*,'ARRÊT DU PROGRAMME ...'
```

```
                else
                        print*,'PROGRAM STOP ...'
                endif
                stop
        endif

        print*,' '
***BEGINNING of TABLE
    nlen=(ncol+1)*11 + 45
    encode(20,112,idraw1)nlen
    encode(20,222,idraw2)nlen
112     format(1h(,i3,6h(1h-)))
222     format(1h(,i3,6h(1h=)))
* PRINT column headings for LEVEL and SHARE
    print idraw2
        if(nvs.eq.0)then
                print 301,(levfamily(i),i=1,ncol)
301             format('NIVEAU',t50,'FAMILLE',t64,10a11)
                print 302,(adjustr(levvariant(i)),i=1,ncol)
302             format(t50'VARIANTE',a10,9a11)
                print 303,(levnumber(i),i=1,ncol)
303             format(t50'NUMÉRO',1x,10i11)
        else
                print 101,(levfamily(i),i=1,ncol)
101             format('LEVEL',t50,'FAMILY',t64,10a11)
                print 102,(adjustr(levvariant(i)),i=1,ncol)
102             format(t50'VARIANT',10a11)
                print 103,(levnumber(i),i=1,ncol)
103             format(t50'NUMBER',1x,10i11)
        endif

        if(nshapro.eq.1)then
                if(nvs.eq.0)then
                        print 304,(adjustr(shafamily(i)),i=1,ncol)
304                     format(/'PART',t50,'FAMILLE',t57,10a11)
                else
                        print 104,(adjustr(shafamily(i)),i=1,ncol)
104                     format(/'SHARE',t50,'FAMILY',t57,10a11)
                endif
        else
                if(nvs.eq.0)then
                        print 305,(adjustr(shafamily(i)),i=1,ncol)
305                     format(/'PROBABILITÉ',t50,'FAMILLE',t57,10a11)
                else
                        print 105,(adjustr(shafamily(i)),i=1,ncol)
105                     format(/'PROBABILITY',t50,'FAMILY',t57,10a11)
```

```
            endif
        endif
        if(nvs.eq.0)then
            print 302,(adjustr(shavariant(i)),i=1,ncol)
            print 303,(shanumber(i),i=1,ncol)
        else
            print 102,(adjustr(shavariant(i)),i=1,ncol)
            print 103,(shanumber(i),i=1,ncol)
        endif

      print idraw2
* LEVEL: PRINT elasticities and t-statistics
        if(nvs.eq.0)then
            print 245
245         format('NIVEAU'/)
        else
            print 246
246         format('LEVEL'/)
        endif
        encode(40,250,ifmt)ncol
250     format(14h(2x,'(t)',t10,,i2,17h(2x'(',f7.2,')')))
* Print the definition of the variable X
      do 15 i=1,nxlev1
        do 31 k=1,nvarlev
        if(namxlev(i,1).eq.namvlev(k))then
            print 16,defvlev(k),namxlev(i,1),(elastxlev(i,j),j=1,ncol)
16          format(1x,a35,1x,a10,2x,'EL(T,X)',t57,10f11.3)
* Underline a quasi-dummy (category 2) or a dummy (category 3).
* numvlev() is the category (0,1,2,3) of a variable X for LEVEL.
            call underline(numvlev(m),namxlev(i,1),under)
            print 50,under
50          format(1h ,36x,10a1,$)
            print ifmt,(studxlev(i,j),j=1,ncol)
        endif
31      continue
15      continue

        if(maxU.gt.0)then
            do 18 i=1,maxU
            do 32 k=1,nvarlev
                if(namU(i).eq.namvlev(k))then
                    print 17,defvlev(k),namU(i),(elastU(i,j),j=1,ncol)
17                  format(1x,a35,1x,a10,2x,'EL(T,U)',t57,10a11)
                    print 19,(studU(i,j),j=1,ncol)
19                  format(49x,'(t)',t58,10a11)
                endif
```

```
32              continue
18              continue
        endif

    print idraw1
* SHARE or PROBABILITY: PRINT elasticities and t-statistics
    if(nshapro.eq.1) then
                if(nvs.eq.0)then
                        print 142
142                     format('PART'/)
                else
                        print 143
143                     format('SHARE'/)
                endif
        else
                if(nvs.eq.0)then
                        print 144
144                     format('PROBABILITÉ'/)
                else
                        print 145
145                     format('PROBABILITY'/)
                endif
        endif
    do 20 j=1,nalt
        if(j.lt.10)then
                if(nshapro.eq.1)then
                        if(nvs.eq.0)then
                                print 220,j,namalter(j),sharemean(j)
220                     format('ALTERNATIVE'i2': 'a8,2x,'(Part moyenne:'f7.4,1h)
   +                    /13(1h=))
                        else
                                print 120,j,namalter(j),sharemean(j)
120                             format('ALTERNATIVE'i2': 'a8,2x,'(Mean share:'f7.4,1h)
   +                    /13(1h=))
                        endif
                else
                        if(nvs.eq.0)then
                                print 230,j,namalter(j),sharemean(j)
230                     format('ALTERNATIVE'i2': 'a8,2x,'(Prob. moyenne:'f7.4,1h)
   +                    /13(1h=))
                        else
                                print 130,j,namalter(j),sharemean(j)
130                             format('ALTERNATIVE'i2': 'a8,2x,'(Mean prob.:'f7.4,1h)
   +                    /13(1h=))
                        endif
                endif
```

```
         else
                  if(nshapro.eq.1)then
                           if(nvs.eq.0)then
                                    print 221,j,namalter(j),sharemean(j)
221                                 format('ALTERNATIVE'i3': 'a8,2x,'(Part moyenne:'f7.4,1h)
    +                      /14(1h=))
                           else
                                    print 121,j,namalter(j),sharemean(j)
121                                 format('ALTERNATIVE'i3': 'a8,2x,'(Mean share:'f7.4,1h)
    +                      /14(1h=))
                           endif
                  else
                           if(nvs.eq.0)then
                                    print 231,j,namalter(j),sharemean(j)
231                                 format('ALTERNATIVE'i3': 'a8,2x,'(Prob. moyenne:'f7.4,1h)
    +                      /14(1h=))
                           else
                                    print 131,j,namalter(j),sharemean(j)
131                                 format('ALTERNATIVE'i3': 'a8,2x,'(Mean prob.:'f7.4,1h)
    +                      /14(1h=))
                           endif
                  endif
         endif
    nv=nxshare(j,1)
    do 21 k=1,nv
* If a SHARE/PROB variable appears also in LEVEL,
* then numxlev is the corresponding index of X in LEVEL.
    numxlev=0
    do 23 kk=1,nxlev1
23   if(namxsha(j,k,1).eq.namxlev(kk,1))numxlev=kk
         do 33 m=1,nvarsha
         if(namxsha(j,k,1).eq.namvsha(m))then
                  print 25,defvsha(m),namxsha(j,k,1),(elaux(j,k,i),i=1,ncol)
25                format(1x,a35,1x,a10,2x,'EL(U)',t57,10f11.3)
* Underline a quasi-dummy (category 2) or a dummy (category 3)
* numvsha() is the category (0,1,2,3) of a variable X for SHARE/PROB.
                  call underline(numvsha(m),namxsha(j,k,1),under)
                  print 50,under
                  if(nshapro.eq.1)print 26,(elapx(j,k,i),i=1,ncol)
26                format(2x'EL(Sm)',1x,10f11.3)
                  if(nshapro.eq.2)print 27,(elapx(j,k,i),i=1,ncol)
27                format(2x'EL(Pm)',1x,10f11.3)
         endif
33       continue
    print 19,(studxsha(j,k,i),i=1,ncol)
```

```
* Compute ELATOTAL (Total elasticity)
*       ELAMODAL (Modal Elasticity)
*       DIVRATE  (Diversion rate).
      do 22 i=1,ncol
      elatotal(i)=elastxlev(nxlevel(1),i)*elaux(j,k,i)
* If a SHARE variable appears also in LEVEL (numxlev > 0),
* add ELASTXLEV to ELATOTAL.
      if(numxlev.gt.0)elatotal(i)=elatotal(i)+elastxlev(numxlev,i)
      elamodal(i)=elatotal(i)+elapx(j,k,i)
22    divrate(i)=(elatotal(i)/(sharemean(j)*elamodal(i))) - 1
      print 28,(elatotal(i),i=1,ncol)
28    format(49x,'EL(T)',t57,10f11.3)
      print 29,(elamodal(i),i=1,ncol)
29    format(49x,'EL(Tm)',t57,10f11.3)
      print 30,(divrate(i),i=1,ncol)
30    format(49x,'D.R.',t57,10f11.3)
      print *,' '
21          continue

20          continue
      print idraw2
      stop
            end

            subroutine underline(catx,namx,under)
* Input : catx  = 2 for a quasi-dummy
*               = 3 for a real dummy
*       namx  = name of an independent variable X
* Output: under = underscore whose length is the same as the length of namx
*                without trailing blank characters
            integer catx
      character*10 namx
      character*1 under(10)
            do 1 i=1,10
1           under(i)=' '
            nonblank=len_trim(namx)
      if(catx.eq.2)then
                  do 2 i=1,nonblank
2                 under(i)='-'
            elseif(catx.eq.3)then
               do 3 i=1,nonblank
3                 under(i)='='
            endif
      return
            end
```

4. DATABASES

4.1 Via Rail 1987

In Examples IV and V considered in Table 9 of section 1.3, the complete database produced by the Canadian passenger railway company Via Rail in 1987 for the Quebec-Windsor corridor contains 4401 O-D pairs for business purpose and 8536 O-D pairs for other purposes, for four transportation modes: Rail, Air, Bus and Car, (Gaudry and Le Leyzour, 1994; AJD-120). The 4401 O-D pairs are used to create randomly a reduced sample of 1000 O-D pairs for the estimation of the Probability, Level L-1.6 and L-2.1 variants in examples IV and V. This reduced set is necessary due to the inversion of the big matrices involved in L-2.1 algorithm in example V.

Four files can be found in ***d:\Tabqdf\Databases\Via Rail***:

1. Complete database: ***Business4401.txt*** and ***NonBusiness8536.txt***.
2. Reduced database: ***Business1000.txt***.
3. Observation numbers for the 1000 O-D pairs in the reduced database: ***Numobs1000.txt***.

The reduced sample is used in the Probability variants of Examples IV and V, and also used to create the dependent and independent variables in L-1.6 and L-2.1 variants, namely:

1. The dependent variable Tij and the independent variables Ti and Tj used in L-1.6 variants of Example IV. The associated file is stored in *d:\Tabqdf\TabqdfTests\Four Example\Level 1.6* under the name *Tij-3indep.in*. In the same folder, the definitions of the variables are given in the file *L1Listdef.txt;*
2. The dependent variable Tij, the independent variables Ti and Tj, and the heteroskedasticity variable $TiTj$ used in L-2.1 variants of Example V. The associated file is stored in *d:\Tabqdf \TabqdfTests\Fifth Example\Level 2.1* under the name *TijL2.in*. In the same folder, the definitions of the variables are given in the file *L2Listdef.txt;*
3. The contiguity matrix *DIST_OD* used in L-2.1 variants of Example V is stored in the file *Rdist100.in* in folder *d:\Tabqdf \TabqdfTests\Fifth Example\Level 2.1*. This matrix is based on the distance between Montreal and neighbouring cities, Toronto and neighbouring cities, or Ottawa and neighbouring cities under 100 minutes of car runtime (variable *CAR_RT* in the reduced database): a value 1 is assigned to an O-D pair in the matrix if car_rt < 100 and a value 0 otherwise.

   Note that the proportions of business trips originating or destinating in these cities are:

	as destination	as origin
Montreal	10%	10%
Ottawa	9%	12%
Toronto	15%	16%

4.2 Intercity passenger flows, Canada 1976

For Canada, the flow indicator for year 1976 is the ***passenger flow*** for 120 Canadian city pairs.

The database can be found in folder ***d:\Tabqdf\Databases\Canada 1976*** which includes four files:

1. ***Canada1976.in*** contains all the variables for Level 2.1.
2. ***R320km.in*** s the contiguity matrix *DIST_OD* based on the distance of 320 km between an origin and a destination. A value of 1 is assigned to an O-D pair in the matrix if the distance between an origin and a destination is less than or equal to 300 km, and a value of 0 otherwise.
3. ***R1.in*** is the contiguity matrix *DIST_O* based on the distance of 320 km from an origin to a destination. A value of 1 is assigned to an O-D pair in the matrix if the distance between an origin and a destination is less than or equal to 320 km, and a value of 0 otherwise.
4. ***R2.in*** is the contiguity matrix *DIST_D* based on the distance of 320 km from a destination to an origin. A value of 1 is assigned to an O-D pair in the matrix if the distance between an origin and a destination is less than or equal to 320 km, and a value of 0 otherwise.

4.3 Trade flows in tons, European Community 1987

For the European Community, the flow indicator for year 1987 is the ***tonnage shipped*** for Europe (because value of shipment data were not available on a regional basis, but only on a national basis), as in Blum and Mende (1996). Flows are observed among the 11 German provinces, as well as between them and 10 other European countries (but not among European countries): there are 316 observations.

The database can be found in folder ***d:\Tabqdf\Databases\European Community 1987*** which includes three files:

5. ***Europe.in*** contains all the variables for Level 2.1.
6. ***R300km.in*** is the contiguity matrix *DIST_OD* based on the distance of 300 km between an origin and a destination. A value of 1 is assigned to an O-D pair in the matrix if the distance between an origin and a destination is less than or equal to 300 km, and a value of 0 otherwise.
7. ***R450km.in*** is the contiguity matrix *DIST_OD* based on the distance of 450 km between an origin and a destination. A value of 1 is assigned to an O-D pair in the matrix if the distance between an origin and a destination is less than or equal to 450 km, and a value of 0 otherwise.

4.4 Trade flows in value, North America 1988

For North America, the flow indicator for year 1988 is the ***value of shipments*** , as in McCallum (1995). Flows are observed among the 10 Canadian provinces, as well as between them and 30 American states (but not among American states): there are 683 observations.

The database can be found in folder ***d:\Tabqdf\Databases\North America 1988*** which includes two files:

8. ***America.in*** contains all the variables for Level 2.1.
9. ***R400miles.in*** is the contiguity matrix *DIST_OD* based on the distance of 400 miles between an origin and a destination. A value of 1 is assigned to an element of the matrix if the distance between an origin and a destination is less than or equal to 400 miles, and a value of 0 otherwise.

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6. Appendix 1. Other generation-distribution models constructed from supplied data bases

6.1 Trade flows, European Community, 1987 (from Gaudry, 2004)

EUROPEAN COMMUNITY 2

TRADE FLOW MODELS BETWEEN 10 EUROPEAN COUNTRIES AND 11 GERMAN PROVINCES,
AND AMONG THE LATTER (1987 SHIPMENTS IN TONS)

I.	TYPE =	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2	LEVEL-2
ELASTICITY E(y) (EP)	VARIANT =	LOG	LOG+ AU	LOG+ AU+PR	BC1+ AU+PR	BC2+ AU+PR	BC2GH1+ AU+PR
(COND. T-STATISTIC)	VERSION =	1	1	1	1	1	1
LEVEL VARIABLES							
GROSS DOMESTIC PRODUCT AT ORIGIN	mecu87_o	.709 (14.65) LAM 1	.812 (12.49) LAM 1	.834 (12.14) LAM 1	.867 (12.49) LAM 1	.972 (12.75) LAM 1	.983 (13.24) LAM 1
GROSS DOMESTIC PRODUCT AT DESTINATION	mecu87_d	.731 (15.15) LAM 1	.829 (12.92) LAM 1	.855 (12.49) LAM 1	.903 (13.10) LAM 1	1.016 (13.37) LAM 1	1.038 (13.56) LAM 1
DISTANCE	dist	-1.298 (-15.54) LAM 1	-1.342 (-12.39) LAM 1	-1.406 (-13.50) LAM 1	-1.734 (-17.61) LAM 1	-2.345 (-16.05) LAM 1	-2.474 (-17.44) LAM 1
INTERPROVINCIAL TRADE DUMMY	gdummy ======	0.990 (6.38)	1.082 (3.47)	1.024 (3.19)	0.799 (2.97)	0.762 (2.72)	0.769 (3.10)
HETEROSKEDASTICITY STRUCTURE							
DELTA COEFFICIENTS							
PRODUCT OF GDP AT ORIGIN AND DESTINATION	mecuprod						-.087 (-4.33) LAM
II. PARAMETERS							
HETEROSKEDASTICITY STRUCTURE							
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]							
LAMBDA(Z)	mecuprod						.000 FIXED
BOX-COX TRANSFORMATIONS: UNCOND: [T-STATISTIC=0] / [T-STATISTIC=1]							
LAMBDA(Y)	goods					.079 [4.07] [-47.33]	.074 [3.21] [-40.17]
LAMBDA(Y) - GROUP 1	LAM 1	.000 FIXED	.000 FIXED	.000 FIXED	.078 [4.04] [-47.85]		
LAMBDA(X) - GROUP 1	LAM 1	.000 FIXED	.000 FIXED	.000 FIXED	.078 [4.04] [-47.85]	.270 [4.40] [-11.89]	.028 [4.86] [-12.47]
SPATIAL AUTOCORRELATION: COND: [T-STATISTIC=0] / [T-STATISTIC=1]							
O and D: DIST (300 km)							
RHO (DIST_OD)			.425 [4.77]	.543 [5.26]	.760 [11.35]	.870 [23.19]	.876 [27.28]
PI (DIST_OD)			1.000 FIXED	.435 [1.08] [-1.40]	.186 [1.53] [-6.73]	.142 [2.07] [-12.51]	.125 [2.25] [-15.78]

III.GENERAL STATISTICS						
LOG-LIKELIHOOD	-2356.63	-2341.23	-2339.99	-2329.95	-2327.14	-2318.49
SAMPLE : - NUMBER OF OBSERVATIONS	316	316	316	316	316	316
- FIRST OBSERVATION	1	1	1	1	1	1
- LAST OBSERVATION	316	316	316	316	316	316
NUMBER OF ESTIMATED PARAMETERS :						
- FIXED PART :						
. BETAS	5	5	5	5	5	5
. BOX-COX	0	0	0	1	2	2
. ASSOCIATED DUMMIES	0	0	0	0	0	0
- SPATIAL AUTOCORRELATION :						
. RHO	0	1	1	1	1	1
. PI	0	0	1	1	1	1
- HETEROSKEDASTICITY :						
. DELTAS	0	0	0	0	0	1
. BOX-COX	0	0	0	0	0	0
. ASSOCIATED DUMMIES	0	0	0	0	0	0

6.2 Intercity passenger flows, Canada 1976 (with two orders of autocorrelation, from Gaudry *et al*., 1994, 2002)

PART I. Presented statistics							
Model type		L-2.1	L-2.1	L-2.1	L-2.1	L-2.1	L-2.1
Name of the variant		LOG	LOG+AU	LOG+AU+PR	BC1+AU+PR	BC2+AU+PR	BC5+AU+PR
Version number of the variant		1	2	3	4	5	6
Dependent variable in the variant		TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW
IND = INDEPENDENT VARIABLES							
POPULATION	POP						
Elasticity of E(Y)		1.389	1.414	1.430	1.457	1.499	1.498
Conditional t-statistic for BETA		(25.45)	(25.61)	(29.06)	(31.40)	(37.05)	(30.01)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1	LAM 1
REVENUE	REVE						
Elasticity of E(Y)		0.832	1.320	0.940	1.350	0.209	0.308
Conditional t-statistic for BETA		(1.20)	(1.56)	(1.28)	(1.95)	(0.31)	(0.58)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1	LAM 2
FRENCH LANGUAGE	LANGFR						
Elasticity of E(Y)		0.066	0.239	0.287	0.312	0.251	0.195
Conditional t-statistic for BETA		(1.20)	(2.39)	(3.76)	(4.24)	(4.22)	(5.94)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1	LAM 3
UTILITY	UTILITY						
Elasticity of E(Y)		0.634	0.596	0.540	0.511	0.655	0.674
Conditional t-statistic for BETA		(42.33)	(21.25)	(18.76)	(19.10)	(22.20)	(22.83)
		LAM 1	LAM 1	LAM 1	LAM 1	LAM 1	LAM 4
REGRESSION CONSTANT	CONSTANT						
Elasticity of E(Y)		-10.784	-16.193	-13.857	-14.640	-20.263	2.213
Conditional t-statistic for BETA		(-1.96)	(-2.30)	(-2.31)	(-2.92)	(-2.98)	(4.43)

PART II. Parameters						
t-statistic unconditional (=0) [=1]						
Model type	L-2.1	L-2.1	L-2.1	L-2.1	L-2.1	L-2.1
Name of the variant	LOG	LOG+AU	LOG+AU+PR	BC1+AU+PR	BC2+AU+PR	BC5+AU+PR
Version number of the variant	1	2	3	4	5	6
Dependent variable in the variant	TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW	TOTFLOW
Box-Cox Transformations						
LAMBDA(Y)	0.000	0.000	0.000	0.030	0.058	0.066
	Fixed	Fixed	Fixed	(1.57)	(3.21)	(2.07)
				[-51.44]	[-51.72]	[-29.05]
LAMBDA(X) - Group 1	0.000	0.000	0.000	0.030	-0.055	0.182
	Fixed	Fixed	Fixed	(1.57)	(-2.54)	(2.49)
				[-51.44]	[-48.52]	[-11.22]
LAMBDA(X) - Group 2						6.552
						(0.14)
						[0.12]
LAMBDA(X) - Group 3						-0.251
						(-0.42)
						[-2.12]
LAMBDA(X) - Group 4						-0.067
						(-2.75)
						[-44.08]

```
Directed Autocorrelation
------------------------
  O: DIST < 320 km                  DIST_O

     RHO                                                   0.425     0.591     0.627     0.517     0.574
                                                       (   3.38) (   6.15) (   7.25) (   5.01) (   5.72)

     PI                                                    1.000     0.000     0.014     0.000     0.000
                                                           Fixed (   0.00) (   0.23) (   0.00) (   0.00)
                                                                 [ -22.21] [ -16.22] [ -22.57] [  -7.19]

  D: DIST < 320 km                  DIST_D

     RHO                                                   0.316     0.213     0.217     0.238     0.250
                                                       (   1.88) (   1.31) (   1.64) (   1.40) (   1.84)

     PI                                                    1.000     1.000     1.000     1.000     1.000
                                                           Fixed (   0.64) (   0.69) (   0.67) (   0.88)
                                                                 [   0.00] [   0.00] [   0.00] [   0.00]
==========================================================================================================
PART III. General statistics
  Model type                                     L-2.1     L-2.1     L-2.1     L-2.1     L-2.1     L-2.1
  Name of the variant                              LOG    LOG+AU LOG+AU+PR BC1+AU+PR BC2+AU+PR BC5+AU+PR
  Version number of the variant                      1         2         3         4         5         6
  Dependent variable in the variant            TOTFLOW   TOTFLOW   TOTFLOW   TOTFLOW   TOTFLOW   TOTFLOW

==========================================================================================================

Log-likelihood                                -1318.23  -1306.12  -1301.11  -1300.22  -1287.21  -1272.27

Pearson R2                                       0.885     0.920     0.946     0.958     0.970     0.995
Pearson R2 adjusted for D.F.                     0.881     0.916     0.942     0.955     0.967     0.995

Pseudo-(L)-R2                                    0.944     0.977     0.983     0.982     0.975     0.983
Pseudo-(L)-R2 adjusted for D.F.                  0.942     0.976     0.982     0.980     0.973     0.981

Average probability (Y=limit observation)        0.000     0.000     0.000     0.000     0.000     0.000

Sample     - Number of observations                120       120       120       120       120       120

Number of estimated parameters
           - Beta                                    5         5         5         5         5         5
           - Lambda(y) and Lambda(x)                 0         0         0         1         2         5
           - Heteroskedasticity: Delta               0         0         0         0         0         0
                                 Lambda(z)           0         0         0         0         0         0
           - Autocorrélation:    Rho                 0         2         2         2         2         2
                                 Pi                  0         0         2         2         2         2
           - Total                                   5         7         9        10        11        14

Mean of observed Y                            0.15D+06  0.15D+06  0.15D+06  0.15D+06  0.15D+06  0.15D+06

Mean of estimated E(Y)                        0.19D+06  0.17D+06  0.17D+06  0.16D+06  0.15D+06  0.15D+06

==========================================================================================================
```

7. Appendix 2. Key full sample mode choice models for the Quebec-Windsor Corridor of Canada, Business and Other trip purposes (from Gaudry and Le Leyzour, 1994, and Gaudry, 2011)

The first 3 columns (Business trips) are virtually identical to those found in the above mentioned papers, notably to those found in Table 3 of the most recent (the sample differs by one observation). The last 3 columns (Other trips) are identical.

PART I. Presented statistics							
Model type		LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
Name of the variant		BENBUS1	BENBUS1	BENBUS1	BENNBUS	BENNBUS	BENNBUS
Version number of the variant		8	18	46	43	48	105
Dependent variable in the variant		CHOICE	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE
ALTERNATIVE 1 : RAIL							
N NETWORK VARIABLES							
S)TOTAL COST, RAIL(RUN + ACCESS COS	RAI_CTT						
BETA coefficient		-0.0222	-0.8105	-2.9866	0.0185	-4.9475	-0.9471
Elast. of P(RAIL) - w.aggregate		-0.9028	-2.1613	-1.2094	0.3536	-1.0042	-0.7869
Change of P(RAIL) - w.aggregate		-0.0555	-0.1383	-0.0757	0.0223	-0.0612	-0.0599
T-statistic conditional on LAMBDA(X)		(-3.66)	(-7.23)	(-2.85)	(2.14)	(-6.91)	(-4.18)
		L 1 (GE)	L 4 (GE)	L 4 (GE)	L 1 (GE)	L 4 (GE)	L 2 (GE)
TOTAL TRAVEL TIME, RAIL	RAI_TTT						
BETA coefficient		-0.0136	-0.0001	-0.0708	-0.0072	0.0354	0.0000
Elast. of P(RAIL) - w.aggregate		-2.9894	-1.6558	-1.9302	-1.4093	0.4163	-0.1543
Change of P(RAIL) - w.aggregate		-0.1825	-0.1052	-0.1237	-0.0857	0.0278	-0.0226
T-statistic conditional on LAMBDA(X)		(-10.19)	(-5.91)	(-4.32)	(-6.65)	(1.44)	(-1.51)
		L 1 (GE)	L 3 (GE)	L 3 (GE)	L 1 (GE)	L 3 (GE)	L 3 (GE)
FREQUENCY, RAIL	RAI_FR						
BETA coefficient		0.0796	0.0138	0.0275	0.0574	0.0606	0.1056
Elast. of P(RAIL) - w.aggregate		0.3434	0.1282	0.1907	0.2718	0.3091	0.3779
Change of P(RAIL) - w.aggregate		0.0221	0.0080	0.0120	0.0156	0.0199	0.0278
T-statistic conditional on LAMBDA(X)		(12.02)	(11.67)	(11.88)	(8.35)	(10.03)	(11.25)
		L 1 (GE)	L 1 (GE)	L 1 (GE)	L 1 (GE)	L 1 (GE)	L 5 (GE)
RUN TIME, CAR	CAR_RT						
BETA coefficient				0.0000			0.0002
Elast. of P(RAIL) - w.aggregate				1.4467			0.8169
Change of P(RAIL) - w.aggregate				0.0978			0.0838
T-statistic conditional on LAMBDA(X)				(4.59)			(8.38)
				L 5 (GE)			L 4 (GE)
S SOCIOECONOMIC VARIABLES							
INCOME	INCOME						
BETA coefficient		0.0000	-0.1148	-0.0911	0.0000	-0.0179	-0.0028
Elast. of P(RAIL) - w.aggregate		-0.8051	-0.7062	-0.7488	-0.3530	-0.3606	-0.1283
Change of P(RAIL) - w.aggregate		-0.0437	-0.0412	-0.0441	-0.0156	-0.0175	-0.0041
T-statistic conditional on LAMBDA(X)		(-2.61)	(-2.99)	(-3.00)	(-3.86)	(-5.09)	(-2.48)
		(SP)	L 2 (SP)	L 2 (SP)	L 1 (SP)	L 2 (SP)	L 1 (SP)
ET ET CETERA							
REGRESSION CONSTANT	CONSTANT						
BETA coefficient		-3.4603	-1.1284	-1.2173	-3.2599	-1.8570	-3.4338
Elast. of P(RAIL) - w.aggregate		-2.4517	-0.0664	0.3004	-2.5674	-1.5525	-2.6533
Change of P(RAIL) - w.aggregate		-0.1402	-0.0030	0.0289	-0.1295	-0.0931	-0.1779
T-statistic conditional on LAMBDA(X)		(-10.17)	(-1.03)	(-1.18)	(-16.49)	(-5.35)	(-10.74)
		(SP)	(SP)	(SP)	(SP)	(SP)	(SP)
ALTERNATIVE 2 : AIR							
N NETWORK VARIABLES							
S)TOTAL COST, AIR (RUN + ACCESS COS	AIR_CTT						
BETA coefficient		-0.0222	-0.8105	-2.9866	0.0185	-4.9475	-0.9471
Elast. of P(AIR) - w.aggregate		-1.2290	-1.1239	-0.3245	1.4481	-0.4537	-0.7381
Change of P(AIR) - w.aggregate		-0.5360	-0.4626	-0.1282	0.1680	-0.0415	-0.0766
T-statistic conditional on LAMBDA(X)		(-3.66)	(-7.23)	(-2.85)	(2.14)	(-6.91)	(-4.18)
		L 1 (GE)	L 4 (GE)	L 4 (GE)	L 1 (GE)	L 4 (GE)	L 2 (GE)
TOTAL TRAVEL TIME, AIR	AIR_TTT						
BETA coefficient		-0.0136	-0.0001	-0.0708	-0.0072	0.0354	0.0000
Elast. of P(AIR) - w.aggregate		-0.7519	-0.2698	-0.5196	-1.0051	0.3384	-0.0074
Change of P(AIR) - w.aggregate		-0.3209	-0.1184	-0.2163	-0.1087	0.0325	-0.0010
T-statistic conditional on LAMBDA(X)		(-10.19)	(-5.91)	(-4.32)	(-6.65)	(1.44)	(-1.51)
		L 1 (GE)	L 3 (GE)	L 3 (GE)	L 1 (GE)	L 3 (GE)	L 3 (GE)
FREQUENCY, AIR	AIR_FR						
BETA coefficient		0.0796	0.0138	0.0275	0.0574	0.0606	0.1056
Elast. of P(AIR) - w.aggregate		0.7458	0.7854	0.7998	1.5333	2.1475	1.8924
Change of P(AIR) - w.aggregate		0.3955	0.4597	0.4514	0.2092	0.2598	0.2372
T-statistic conditional on LAMBDA(X)		(12.02)	(11.67)	(11.88)	(8.35)	(10.03)	(11.25)
		L 1 (GE)	L 1 (GE)	L 1 (GE)	L 1 (GE)	L 1 (GE)	L 5 (GE)
RUN TIME, CAR	CAR_RT						
BETA coefficient				0.0000			0.0002
Elast. of P(AIR) - w.aggregate				1.0341			1.4831
Change of P(AIR) - w.aggregate				0.5291			0.1982
T-statistic conditional on LAMBDA(X)				(4.59)			(8.38)
				L 5 (GE)			L 4 (GE)

```
  ------------------------------------
  S          SOCIOECONOMIC VARIABLES
  ------------------------------------
    INCOME                            INCOME
    BETA coefficient                              0.0000     0.2088     0.1664     0.0000     0.0227    -0.0012
    Elast. of P(AIR     ) - w.aggregate           0.5420     0.4401     0.4477     1.2382     0.8209     0.0319
    Change of P(AIR     ) - w.aggregate           0.2292     0.1803     0.1845     0.1546     0.0856     0.0045
    T-statistic conditional on LAMBDA(X)       (   4.99)  (   5.00)  (   5.05)  (   4.01)  (   2.72)  (  -0.57)
                                                  (SP)    L 2  (SP)  L 2  (SP)  L 1  (SP)  L 2  (SP)  L 1  (SP)
  ----------------------
  ET         ET CETERA
  ----------------------
    REGRESSION CONSTANT               CONSTANT
    BETA coefficient                             -4.9408    -8.0472    -9.8209    -8.4258    -6.7295    -4.7762
    Elast. of P(AIR     ) - w.aggregate          -1.5830    -2.6748    -3.2123    -6.6413    -5.5119    -3.4503
    Change of P(AIR     ) - w.aggregate          -0.6282    -1.0720    -1.2839    -0.7042    -0.5202    -0.3404
    T-statistic conditional on LAMBDA(X)       (  -7.19)  (  -5.86)  (  -7.50)  ( -11.24)  (  -7.27)  (  -6.86)
                                                  (SP)       (SP)       (SP)       (SP)       (SP)       (SP)
------------------------------------------------------------------------------------------------------------------------
---------------------
ALTERNATIVE  3 :  BUS
---------------------
  ------------------------------
  N          NETWORK VARIABLES
  ------------------------------
   S)TOTAL COST, BUS (RUN + ACCESS COS  BUS_CTT
    BETA coefficient                             -0.0222    -0.8105    -2.9866     0.0185    -4.9475    -0.9471
    Elast. of P(BUS     ) - w.aggregate          -0.4598    -1.7555    -1.3819     0.2777    -0.9118    -0.6952
    Change of P(BUS     ) - w.aggregate          -0.0381    -0.1363    -0.1022     0.0481    -0.1712    -0.1294
    T-statistic conditional on LAMBDA(X)       (  -3.66)  (  -7.23)  (  -2.85)  (   2.14)  (  -6.91)  (  -4.18)
                                               L 1  (GE)  L 4  (GE)  L 4  (GE)  L 1  (GE)  L 4  (GE)  L 2  (GE)
    TOTAL TRAVEL TIME, BUS            BUS_TTT
    BETA coefficient                             -0.0136    -0.0001    -0.0708    -0.0072     0.0354     0.0000
    Elast. of P(BUS     ) - w.aggregate          -3.1446    -1.7196    -1.8989    -1.3370     0.3679    -0.1030
    Change of P(BUS     ) - w.aggregate          -0.2606    -0.1467    -0.1478    -0.2295     0.0721    -0.0222
    T-statistic conditional on LAMBDA(X)       ( -10.19)  (  -5.91)  (  -4.32)  (  -6.65)  (   1.44)  (  -1.51)
                                               L 1  (GE)  L 3  (GE)  L 3  (GE)  L 1  (GE)  L 3  (GE)  L 3  (GE)
    FREQUENCY, BUS                    BUS_FR
    BETA coefficient                              0.0796     0.0138     0.0275     0.0574     0.0606     0.1056
    Elast. of P(BUS     ) - w.aggregate           1.3705     1.1397     1.2877     0.6303     0.7784     0.8297
    Change of P(BUS     ) - w.aggregate           0.1209     0.1000     0.1097     0.1249     0.1740     0.1703
    T-statistic conditional on LAMBDA(X)       (  12.02)  (  11.67)  (  11.88)  (   8.35)  (  10.03)  (  11.25)
                                               L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 5  (GE)
    RUN TIME, CAR                     CAR_RT
    BETA coefficient                                                    0.0000                           0.0002
    Elast. of P(BUS     ) - w.aggregate                                 1.5082                           0.6145
    Change of P(BUS     ) - w.aggregate                                 0.1348                           0.1222
    T-statistic conditional on LAMBDA(X)                              (   4.59)                        (   8.38)
                                                                      L 5  (GE)                        L 4  (GE)
  ------------------------------------
  S          SOCIOECONOMIC VARIABLES
  ------------------------------------
    INCOME                            INCOME
    BETA coefficient                             -0.0001    -0.3364    -0.2684     0.0000    -0.0439    -0.0120
    Elast. of P(BUS     ) - w.aggregate          -1.9311    -1.6553    -1.6767    -0.9334    -0.9075    -0.7894
    Change of P(BUS     ) - w.aggregate          -0.1032    -0.1076    -0.1062    -0.1170    -0.1486    -0.1259
    T-statistic conditional on LAMBDA(X)       (  -5.80)  (  -6.89)  (  -6.93)  ( -13.13)  ( -15.52)  ( -12.31)
                                                  (SP)    L 2  (SP)  L 2  (SP)  L 1  (SP)  L 2  (SP)  L 1  (SP)
  ----------------------
  ET         ET CETERA
  ----------------------
    REGRESSION CONSTANT               CONSTANT
    BETA coefficient                             -3.9796     1.9263     1.8537    -1.6915     0.5048    -1.7337
    Elast. of P(BUS     ) - w.aggregate          -2.9303     2.4469     2.4637    -1.0841     0.5928    -0.9860
    Change of P(BUS     ) - w.aggregate          -0.2076     0.1817     0.1775    -0.1666     0.1314    -0.1480
    T-statistic conditional on LAMBDA(X)       (  -7.01)  (   1.47)  (   1.46)  (  -8.61)  (   1.68)  (  -6.12)
                                                  (SP)       (SP)       (SP)       (SP)       (SP)       (SP)
------------------------------------------------------------------------------------------------------------------------
---------------------
ALTERNATIVE  4 :  CAR
---------------------
  ------------------------------
  N          NETWORK VARIABLES
  ------------------------------
    COST, CAR (RUN COST)              CAR_C
    BETA coefficient                             -0.0222    -0.8105    -2.9866     0.0185    -4.9475    -0.9471
    Elast. of P(CAR     ) - w.aggregate          -0.0880    -0.1563    -0.0745     0.0171    -0.0865    -0.0542
    Change of P(CAR     ) - w.aggregate          -0.0500    -0.1020    -0.0531     0.0133    -0.0691    -0.0425
    T-statistic conditional on LAMBDA(X)       (  -3.66)  (  -7.23)  (  -2.85)  (   2.14)  (  -6.91)  (  -4.18)
                                               L 1  (GE)  L 4  (GE)  L 4  (GE)  L 1  (GE)  L 4  (GE)  L 2  (GE)
    RUN TIME, CAR                     CAR_RT
    BETA coefficient                             -0.0136    -0.0001    -0.0708    -0.0072     0.0354     0.0000
    Elast. of P(CAR     ) - w.aggregate          -0.1944    -0.1053    -0.1711    -0.0776     0.0237    -0.0674
    Change of P(CAR     ) - w.aggregate          -0.1124    -0.0541    -0.0925    -0.0608     0.0183    -0.0484
    T-statistic conditional on LAMBDA(X)       ( -10.19)  (  -5.91)  (  -4.32)  (  -6.65)  (   1.44)  (  -1.51)
                                               L 1  (GE)  L 3  (GE)  L 3  (GE)  L 1  (GE)  L 3  (GE)  L 3  (GE)
```

PART II. Parameters
T-statistic unconditional (=0) [=1]

Model type	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
Name of the variant	BENBUS1	BENBUS1	BENBUS1	BENNBUS	BENNBUS	BENNBUS
Version number of the variant	8	18	46	43	48	105
Dependent variable in the variant	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE
BOX-COX TRANSFORMATIONS						
LAMBDA(X) 1	1.0000	1.5309	1.3403	1.0000	1.0700	0.4419
	FIXED	(3.52)	(3.15)	FIXED	(0.44)	(0.00)
		[1.22]	[0.80]		[0.03]	[0.00]
LAMBDA(X) 2		0.1620	0.1850		0.3330	-0.0149
		(0.51)	(0.58)		(0.00)	(0.00)
		[-2.63]	[-2.55]		[0.00]	[-0.03]
LAMBDA(X) 3		1.8020	0.6116		0.4766	4.8121
		(2.83)	(1.66)		(0.02)	(0.48)
		[1.26]	[-1.05]		[-0.02]	[0.38]
LAMBDA(X) 4		0.2805	-0.2198		-0.4875	1.6089
		(1.28)	(-0.38)		(-0.01)	(0.02)
		[-3.28]	[-2.13]		[-0.03]	[0.01]
LAMBDA(X) 5			4.9437			0.8783
			(4.09)			(0.01)
			[3.26]			[0.00]

PART III. General statistics

Model type	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
Name of the variant	BENBUS1	BENBUS1	BENBUS1	BENNBUS	BENNBUS	BENNBUS
Version number of the variant	8	18	46	43	48	105
Dependent variable in the variant	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE
LOG-LIKELIHOOD:						
- FINAL VALUE (LF)	-1068.84	-1058.37	-1049.87	-2179.83	-2123.74	-2100.86
- INITIAL VALUE	-1923.77	-1923.77	-1923.77	-2722.43	-2721.59	-2722.43
- WITH CONSTANTS ONLY (LC)	-1923.77	-1923.77	-1923.77	-2722.43	-2721.59	-2722.43
- RATIO TEST [-2(LC-LF)]	1709.85	1730.79	1747.80	1085.21	1195.72	1243.14
- DEGREES OF FREEDOM	9	13	15	10	14	16
RHO-SQUARED	0.4444	0.4498	0.4543	0.1993	0.2197	0.2283
RHO-SQUARED BAR - AKAIKE	0.4382	0.4415	0.4449	0.1945	0.2134	0.2213
- HOROWITZ	0.4413	0.4457	0.4496	0.1969	0.2165	0.2248
- HENSHER AND JOHNSON	0.4438	0.4491	0.4534	0.1988	0.2190	0.2276
PERCENT RIGHT	75.69	76.48	76.51	57.46	57.28	58.00
SAMPLE - NUMBER OF ALTERNATIVES	4	4	4	4	4	4
- NUMBER OF OBSERVATIONS	4401	4401	4401	8536	8536	8536
- AVAILABLE OBSERVATIONS:						
.ALTERNATIVE 1 : RAIL	4290	4290	4290	8229	8229	8229
.ALTERNATIVE 2 : AIR	3624	3624	3624	5902	5902	5902
.ALTERNATIVE 3 : BUS	3264	3264	3264	5668	5668	5668
.ALTERNATIVE 4 : CAR	4397	4397	4397	8452	8452	8452
TOTAL NUMBER OF FIXED OR ESTIMATED PARAMETERS:						
- BETA .Estimated	9	9	10	10	10	11
.CONSTANTS	3	3	3	3	3	3
- LAMBDA(X)						
.Fixed (always in G-DOGIT)	1	0	0	1	0	0
.Estimated	0	4	5	0	4	5
- EXTRA PARAMETERS						
.Fixed	0	0	0	0	0	0
.Estimated	0	0	0	0	0	0
- TOTAL						
.Fixed	1	0	0	1	0	0
.Estimated	12	16	18	13	17	19

8. Appendix 3. Simplified reduced sample Quebec-Windsor Corridor mode choice models (business trips)

8.1 Linear and Box-Cox Logit variants

```
=============================================================================================================
PART I. Presented statistics
  Model type                                                    LOGIT      LOGIT      LOGIT      LOGIT      LOGIT
  Name of the variant                                          bus_lin    bus_log    bus_bc1    bus_bc2    bus_bc4
  Version number of the variant                                  4          5          1          2          3
  Dependent variable in the variant                             CHOICE     CHOICE     CHOICE     CHOICE     CHOICE
=============================================================================================================
----------------------
ALTERNATIVE  1 :  RAIL
----------------------
  -----------------------------
  N          NETWORK VARIABLES
  -----------------------------
    TOTAL COST, RAIL(RUN + ACCESS COST)    RAI_CTT
      BETA coefficient                                          -0.0383    -3.5300    -0.3011    -0.2873    -1.2269
      Elast. of P(RAIL     ) - at mean(X)                       -1.9436    -3.2840    -2.2302    -2.2407    -3.2726
      T-statistic conditional on LAMBDA(X)                    (  -3.34)  (  -3.36)  (  -2.84)  (  -2.88)  (  -5.12)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, RAIL                RAI_TTT
      BETA coefficient                                          -0.0137    -1.8319    -0.1776    -0.1667    -0.0005
      Elast. of P(RAIL     ) - at mean(X)                       -3.8492    -1.7042    -3.1881    -3.2241    -2.7144
      T-statistic conditional on LAMBDA(X)                    (  -5.79)  (  -1.74)  (  -3.76)  (  -3.83)  (  -3.45)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)
    FREQUENCY, RAIL                        RAI_FR
      BETA coefficient                                           0.0968     1.5329     0.3833     0.3667     0.9766
      Elast. of P(RAIL     ) - at mean(X)                        0.4075     1.4261     0.7819     0.7622     1.2055
      T-statistic conditional on LAMBDA(X)                    (   7.60)  (   7.55)  (   7.46)  (   7.42)  (   7.59)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 3  (GE)
  -----------------------------------
  S          SOCIOECONOMIC VARIABLES
  -----------------------------------
    INCOME                                 INCOME
      BETA coefficient                                           0.0000     0.1752     0.0003   115.3837    51.9802
      Elast. of P(RAIL     ) - at mean(X)                        0.0141     0.1152     0.0549     0.1405     0.1397
      T-statistic conditional on LAMBDA(X)                    (   0.05)  (   0.68)  (   0.23)  (   1.18)  (   1.11)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)  L 4  (GE)
  ---------------------
  ET         ET CETERA
  ---------------------
    REGRESSION CONSTANT                    CONSTANT
      BETA coefficient                                          -2.4862    -6.3588    -3.0032  -199.0354  -104.8983
      Elast. of P(RAIL     ) - at mean(X)                       -2.0999    -4.3537    -2.2322  -145.7892   -76.2022
      T-statistic conditional on LAMBDA(X)                    (  -5.81)  (  -2.27)  (  -4.36)  (  -1.20)  (  -1.16)
                                                                 (SP)       (SP)       (SP)       (SP)       (SP)
-------------------------------------------------------------------------------------------------------------
---------------------
ALTERNATIVE  2 :  AIR
---------------------
  -----------------------------
  N          NETWORK VARIABLES
  -----------------------------
    TOTAL COST, AIR (RUN + ACCESS COST)    AIR_CTT
      BETA coefficient                                          -0.0383    -3.5300    -0.3011    -0.2873    -1.2269
      Elast. of P(AIR      ) - at mean(X)                       -5.0992    -2.6641    -3.3397    -3.3990    -3.6877
      T-statistic conditional on LAMBDA(X)                    (  -3.34)  (  -3.36)  (  -2.84)  (  -2.88)  (  -5.12)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, AIR                 AIR_TTT
      BETA coefficient                                          -0.0137    -1.8319    -0.1776    -0.1667    -0.0005
      Elast. of P(AIR      ) - at mean(X)                       -1.8639    -1.3825    -1.9891    -1.9925    -0.9037
      T-statistic conditional on LAMBDA(X)                    (  -5.79)  (  -1.74)  (  -3.76)  (  -3.83)  (  -3.45)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)
    FREQUENCY, AIR                         AIR_FR
      BETA coefficient                                           0.0968     1.5329     0.3833     0.3667     0.9766
      Elast. of P(AIR      ) - at mean(X)                        2.0751     1.1569     1.6511     1.6446     1.4047
      T-statistic conditional on LAMBDA(X)                    (   7.60)  (   7.55)  (   7.46)  (   7.42)  (   7.59)
                                                              L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 3  (GE)
```

```
  -----------------------------------
  S          SOCIOECONOMIC VARIABLES
  -----------------------------------
    INCOME                             INCOME
      BETA coefficient                              0.0000     0.1752     0.0003   115.3837    51.9802
      Elast. of P(AIR     ) - at mean(X)            0.0141     0.1152     0.0549     0.1405     0.1397
      T-statistic conditional on LAMBDA(X)        (  0.05)  (  0.68)  (  0.23)  (  1.18)  (  1.11)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)  L 4  (GE)
  ---------------------
  ET         ET CETERA
  ---------------------
    REGRESSION CONSTANT                CONSTANT
      BETA coefficient                             -1.2628    -5.1082    -2.4883  -198.4701  -103.1104
      Elast. of P(AIR     ) - at mean(X)           -0.8765    -3.1031    -1.7173  -145.2239   -74.4143
      T-statistic conditional on LAMBDA(X)        ( -1.07)  ( -1.65)  ( -1.77)  ( -1.20)  ( -1.14)
                                                    (SP)       (SP)       (SP)       (SP)       (SP)
----------------------------------------------------------------------------------------------------------
---------------------
ALTERNATIVE  3 :  BUS
---------------------
  -----------------------------
  N          NETWORK VARIABLES
  -----------------------------
    TOTAL COST, BUS (RUN + ACCESS COST)    BUS_CTT
      BETA coefficient                             -0.0383    -3.5300    -0.3011    -0.2873    -1.2269
      Elast. of P(BUS     ) - at mean(X)           -0.9598    -3.4336    -1.5774    -1.5698    -2.7964
      T-statistic conditional on LAMBDA(X)        ( -3.34)  ( -3.36)  ( -2.84)  ( -2.88)  ( -5.12)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, BUS             BUS_TTT
      BETA coefficient                             -0.0137    -1.8319    -0.1776    -0.1667    -0.0005
      Elast. of P(BUS     ) - at mean(X)           -4.0953    -1.7819    -3.3563    -3.3963    -2.9707
      T-statistic conditional on LAMBDA(X)        ( -5.79)  ( -1.74)  ( -3.76)  ( -3.83)  ( -3.45)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)
    FREQUENCY, BUS                     BUS_FR
      BETA coefficient                              0.0968     1.5329     0.3833     0.3667     0.9766
      Elast. of P(BUS     ) - at mean(X)            1.1220     1.4911     1.3473     1.3307     1.4910
      T-statistic conditional on LAMBDA(X)        (  7.60)  (  7.55)  (  7.46)  (  7.42)  (  7.59)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 3  (GE)
  -----------------------------------
  S          SOCIOECONOMIC VARIABLES
  -----------------------------------
    INCOME                             INCOME
      BETA coefficient                              0.0000     0.1752     0.0003   115.3837    51.9802
      Elast. of P(BUS     ) - at mean(X)            0.0141     0.1152     0.0549     0.1405     0.1397
      T-statistic conditional on LAMBDA(X)        (  0.05)  (  0.68)  (  0.23)  (  1.18)  (  1.11)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)  L 4  (GE)
  ---------------------
  ET         ET CETERA
  ---------------------
    REGRESSION CONSTANT                CONSTANT
      BETA coefficient                             -4.8949   -11.3210    -6.2281  -202.2235  -109.3414
      Elast. of P(BUS     ) - at mean(X)           -4.5086    -9.3159    -5.4571  -148.9773   -80.6453
      T-statistic conditional on LAMBDA(X)        ( -7.37)  ( -3.72)  ( -5.86)  ( -1.22)  ( -1.21)
                                                    (SP)       (SP)       (SP)       (SP)       (SP)
----------------------------------------------------------------------------------------------------------
---------------------
ALTERNATIVE  4 :  CAR
---------------------
  -----------------------------
  N          NETWORK VARIABLES
  -----------------------------
    COST, CAR (RUN COST)               CAR_C
      BETA coefficient                             -0.0383    -3.5300    -0.3011    -0.2873    -1.2269
      Elast. of P(CAR     ) - at mean(X)           -0.4795    -1.2082    -0.6685    -0.6828    -0.9916
      T-statistic conditional on LAMBDA(X)        ( -3.34)  ( -3.36)  ( -2.84)  ( -2.88)  ( -5.12)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    RUN TIME, CAR                      CAR_RT
      BETA coefficient                             -0.0137    -1.8319    -0.1776    -0.1667    -0.0005
      Elast. of P(CAR     ) - at mean(X)           -0.6113    -0.6270    -0.7606    -0.7774    -0.5008
      T-statistic conditional on LAMBDA(X)        ( -5.79)  ( -1.74)  ( -3.76)  ( -3.83)  ( -3.45)
                                                  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 2  (GE)
```

```
=========================================================================================================
PART II. Parameters
         T-statistic unconditional (=0) [=1]
  Model type                                                  LOGIT      LOGIT      LOGIT      LOGIT      LOGIT
  Name of the variant                                        bus_lin    bus_log    bus_bc1    bus_bc2    bus_bc4
  Version number of the variant                                 4          5          1          2          3
  Dependent variable in the variant                           CHOICE     CHOICE     CHOICE     CHOICE     CHOICE
=========================================================================================================
  ----------------------
  BOX-COX TRANSFORMATIONS
  ----------------------

    LAMBDA(X)  1                                               1.0000     0.0000     0.5182     0.5314     0.2616
                                                                FIXED      FIXED  (    2.31) (    0.26) (    0.14)
                                                                                  [   -2.15] [   -0.23] [   -0.38]

    LAMBDA(X)  2                                                                               -0.5870     1.5194
                                                                                             (    0.00) (    0.03)
                                                                                             [    0.00] [    0.01]

    LAMBDA(X)  3                                                                                           0.1809
                                                                                                        (    0.00)
                                                                                                        [   -0.02]

    LAMBDA(X)  4                                                                                          -0.5133
                                                                                                        (    0.00)
                                                                                                        [    0.00]
=========================================================================================================
PART III. General statistics
  Model type                                                  LOGIT      LOGIT      LOGIT      LOGIT      LOGIT
  Name of the variant                                        bus_lin    bus_log    bus_bc1    bus_bc2    bus_bc4
  Version number of the variant                                 4          5          1          2          3
  Dependent variable in the variant                           CHOICE     CHOICE     CHOICE     CHOICE     CHOICE
=========================================================================================================
  LOG-LIKELIHOOD:
                - FINAL VALUE (LF)                            -328.15    -329.11    -326.17    -325.38    -323.24
                - INITIAL VALUE                               -523.78    -523.78    -523.78    -523.78    -523.78
                - WITH CONSTANTS ONLY (LC)                    -523.78    -523.78    -523.78    -523.78    -523.78
                - RATIO TEST [-2(LC-LF)]                       391.26     389.34     395.23     396.79     401.08
                - DEGREES OF FREEDOM                                4          4          5          6          8

  RHO-SQUARED                                                  0.3735     0.3717     0.3773     0.3788     0.3829
  RHO-SQUARED BAR - AKAIKE                                     0.3601     0.3583     0.3620     0.3616     0.3619
                - HOROWITZ                                     0.3668     0.3650     0.3696     0.3702     0.3724
                - HENSHER AND JOHNSON                          0.3718     0.3700     0.3754     0.3766     0.3803

  PERCENT RIGHT                                                 75.50      73.50      74.60      75.20      75.50

  SAMPLE - NUMBER OF ALTERNATIVES                                   4          4          4          4          4
         - NUMBER OF OBSERVATIONS                                1000       1000       1000       1000       1000
         - AVAILABLE OBSERVATIONS:
           .ALTERNATIVE  1 : RAIL                                 991        991        991        991        991
           .ALTERNATIVE  2 : AIR                                  846        846        846        846        846
           .ALTERNATIVE  3 : BUS                                  797        797        797        797        797
           .ALTERNATIVE  4 : CAR                                  999        999        999        999        999

  TOTAL NUMBER OF ESTIMATED PARAMETERS:                             7          7          8          9         11
         - BETA .VARIABLES                                          4          4          4          4          4
                .CONSTANTS                                          3          3          3          3          3
         - LAMBDA(X)                                                0          0          1          2          4
         - EXTRA PARAMETERS                                         0          0          0          0          0
=========================================================================================================
```

8.2 Logit, Dogit and Inverse Power Transformation-Logit variants

```
================================================================================================================
PART I. Presented statistics
  Model type                                                 LOGIT     S-DOGIT    G-DOGIT    LIN-IPTL   BT-IPTL
  Name of the variant                                       bus_bc4    bus_bc3    bus_bc4    bus_bc4    bus_bc4
  Version number of the variant                                3          6          7          8          9
  Dependent variable in the variant                          CHOICE     CHOICE     CHOICE     CHOICE     CHOICE
================================================================================================================
---------------------
ALTERNATIVE  1 :  RAIL
---------------------
  ----------------------------
  N       =   NETWORK VARIABLES
  ----------------------------
    TOTAL COST, RAIL(RUN + ACCESS COST)    RAI_CTT
      BETA coefficient                                    -1.2269    -0.1368    -0.1432    -0.0286    -0.1285
      Elast. of P(RAIL    ) - at mean(X)                  -3.2726    -1.5417    -3.1391    -2.1846    -3.2943
      T-statistic conditional on LAMBDA(X)               (  -5.12)  (  -1.69)  (  -0.03)  (  -1.78)  (  -1.60)
                                                         L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, RAIL                RAI_TTT
      BETA coefficient                                    -0.0005    -0.0864    -0.0344    -0.3017    -0.1575
      Elast. of P(RAIL    ) - at mean(X)                  -2.7144    -3.2082    -4.1594    -1.4168    -2.1962
      T-statistic conditional on LAMBDA(X)               (  -3.45)  (  -3.21)  (   0.00)  (  -0.87)  (  -1.22)
                                                         L 2  (GE)  L 1  (GE)  L 2  (GE)  L 2  (GE)  L 2  (GE)
    FREQUENCY, RAIL                        RAI_FR
      BETA coefficient                                     0.9766     0.6816     0.2339     0.1060     0.0621
      Elast. of P(RAIL    ) - at mean(X)                   1.2055     0.7856     0.4257     0.5546     0.2794
      T-statistic conditional on LAMBDA(X)               (   7.59)  (   3.73)  (   0.06)  (   3.19)  (   1.49)
                                                         L 3  (GE)  L 2  (GE)  L 3  (GE)  L 3  (GE)  L 3  (GE)
  ----------------------------------
  S       =   SOCIOECONOMIC VARIABLES
  ----------------------------------
    INCOME                                 INCOME
      BETA coefficient                                    51.9802    82.1814     0.0000    -0.0014     0.0000
      Elast. of P(RAIL    ) - at mean(X)                   0.1397     0.1495    -0.1815    -0.2904    -0.3055
      T-statistic conditional on LAMBDA(X)               (   1.11)  (   1.91)  (   0.00)  (  -0.85)  (  -1.11)
                                                         L 4  (GE)  L 3  (GE)  L 4  (GE)  L 4  (GE)  L 4  (GE)
  ---------------------
  ET      =   ET CETERA
  ---------------------
    REGRESSION CONSTANT                    CONSTANT
      BETA coefficient                                  -104.8983  -162.6390    -0.1633    -2.3527    -1.4566
      Elast. of P(RAIL    ) - at mean(X)                 -76.2022   -80.6748     0.1461    -1.9707    -0.7511
      T-statistic conditional on LAMBDA(X)               (  -1.16)  (  -1.95)  (  -0.09)  (  -3.25)  (  -0.93)
                                                            (SP)       (SP)       (SP)       (SP)       (SP)
----------------------------------------------------------------------------------------------------------------
--------------------
ALTERNATIVE  2 :  AIR
--------------------
  ----------------------------
  N       =   NETWORK VARIABLES
  ----------------------------
    TOTAL COST, AIR (RUN + ACCESS COST)    AIR_CTT
      BETA coefficient                                    -1.2269    -0.1368    -0.1432    -0.0286    -0.1285
      Elast. of P(AIR     ) - at mean(X)                  -3.6877    -3.7365   -13.7590    -3.7108    -5.2644
      T-statistic conditional on LAMBDA(X)               (  -5.12)  (  -1.69)  (  -0.03)  (  -1.78)  (  -1.60)
                                                         L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, AIR                 AIR_TTT
      BETA coefficient                                    -0.0005    -0.0864    -0.0344    -0.3017    -0.1575
      Elast. of P(AIR     ) - at mean(X)                  -0.9037    -2.3928    -3.3650    -0.6095    -0.8233
      T-statistic conditional on LAMBDA(X)               (  -3.45)  (  -3.21)  (   0.00)  (  -0.87)  (  -1.22)
                                                         L 2  (GE)  L 1  (GE)  L 2  (GE)  L 2  (GE)  L 2  (GE)
    FREQUENCY, AIR                         AIR_FR
      BETA coefficient                                     0.9766     0.6816     0.2339     0.1060     0.0621
      Elast. of P(AIR     ) - at mean(X)                   1.4047     1.5982     3.6215     2.1993     2.1420
      T-statistic conditional on LAMBDA(X)               (   7.59)  (   3.73)  (   0.06)  (   3.19)  (   1.49)
                                                         L 3  (GE)  L 2  (GE)  L 3  (GE)  L 3  (GE)  L 3  (GE)
  ----------------------------------
  S       =   SOCIOECONOMIC VARIABLES
  ----------------------------------
    INCOME                                 INCOME
      BETA coefficient                                    51.9802    82.1814     0.0000    -0.0014     0.0000
      Elast. of P(AIR     ) - at mean(X)                   0.1397     0.2282    -0.2117    -0.1395    -0.1658
      T-statistic conditional on LAMBDA(X)               (   1.11)  (   1.91)  (   0.00)  (  -0.85)  (  -1.11)
                                                         L 4  (GE)  L 3  (GE)  L 4  (GE)  L 4  (GE)  L 4  (GE)
```

```
  --------------------
  ET      =   ET CETERA
  --------------------
    REGRESSION CONSTANT                     CONSTANT
      BETA coefficient                                  -103.1104  -162.2811     7.0737    -1.6096     1.8203
      Elast. of P(AIR     ) - at mean(X)                 -74.4143  -122.8363     5.2950    -0.5508     0.4918
      T-statistic conditional on LAMBDA(X)              (  -1.14)  (  -1.94)  (   3.84)  (  -0.59)  (   0.29)
                                                           (SP)       (SP)       (SP)       (SP)       (SP)
-------------------------------------------------------------------------------------------------------------
--------------------
ALTERNATIVE  3 :  BUS
--------------------
  ----------------------------
  N       =   NETWORK VARIABLES
  ----------------------------
    TOTAL COST, BUS (RUN + ACCESS COST)    BUS_CTT
      BETA coefficient                                    -1.2269    -0.1368    -0.1432    -0.0286    -0.1285
      Elast. of P(BUS     ) - at mean(X)                  -2.7964    -1.2825    -3.1424    -1.0463   -15.0643
      T-statistic conditional on LAMBDA(X)              (  -5.12)  (  -1.69)  (  -0.03)  (  -1.78)  (  -1.60)
                                                        L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    TOTAL TRAVEL TIME, BUS                  BUS_TTT
      BETA coefficient                                    -0.0005    -0.0864    -0.0344    -0.3017    -0.1575
      Elast. of P(BUS     ) - at mean(X)                  -2.9707    -4.5582    -8.9710    -1.5487   -21.4458
      T-statistic conditional on LAMBDA(X)              (  -3.45)  (  -3.21)  (   0.00)  (  -0.87)  (  -1.22)
                                                        L 2  (GE)  L 1  (GE)  L 2  (GE)  L 2  (GE)  L 2  (GE)
    FREQUENCY, BUS                          BUS_FR
      BETA coefficient                                     0.9766     0.6816     0.2339     0.1060     0.0621
      Elast. of P(BUS     ) - at mean(X)                   1.4910     1.5068     2.3759     1.9196    12.0573
      T-statistic conditional on LAMBDA(X)              (   7.59)  (   3.73)  (   0.06)  (   3.19)  (   1.49)
                                                        L 3  (GE)  L 2  (GE)  L 3  (GE)  L 3  (GE)  L 3  (GE)
  ----------------------------------
  S       =   SOCIOECONOMIC VARIABLES
  ----------------------------------
    INCOME                                  INCOME
      BETA coefficient                                    51.9802    82.1814     0.0000    -0.0014     0.0000
      Elast. of P(BUS     ) - at mean(X)                   0.1397     0.2332    -0.2117    -0.3016    -3.1372
      T-statistic conditional on LAMBDA(X)              (   1.11)  (   1.91)  (   0.00)  (  -0.85)  (  -1.11)
                                                        L 4  (GE)  L 3  (GE)  L 4  (GE)  L 4  (GE)  L 4  (GE)
  --------------------
  ET      =   ET CETERA
  --------------------
    REGRESSION CONSTANT                     CONSTANT
      BETA coefficient                                  -109.3414  -165.4200    -6.1438    -6.0983    -1.2884
      Elast. of P(BUS     ) - at mean(X)                 -80.6453  -128.6407    -7.9043    -5.7923    -5.7931
      T-statistic conditional on LAMBDA(X)              (  -1.21)  (  -1.98)  (  -2.79)  (  -4.81)  (  -0.17)
                                                           (SP)       (SP)       (SP)       (SP)       (SP)
-------------------------------------------------------------------------------------------------------------
--------------------
ALTERNATIVE  4 :  CAR
--------------------
  ----------------------------
  N       =   NETWORK VARIABLES
  ----------------------------
    COST, CAR (RUN COST)                    CAR_C
      BETA coefficient                                    -1.2269    -0.1368    -0.1432    -0.0286    -0.1285
      Elast. of P(CAR     ) - at mean(X)                  -0.9916    -0.4736    -1.2691    -0.5221    -0.6056
      T-statistic conditional on LAMBDA(X)              (  -5.12)  (  -1.69)  (  -0.03)  (  -1.78)  (  -1.60)
                                                        L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)  L 1  (GE)
    RUN TIME, CAR                           CAR_RT
      BETA coefficient                                    -0.0005    -0.0864    -0.0344    -0.3017    -0.1575
      Elast. of P(CAR     ) - at mean(X)                  -0.5008    -0.7248    -1.0825    -0.2662    -0.2932
      T-statistic conditional on LAMBDA(X)              (  -3.45)  (  -3.21)  (   0.00)  (  -0.87)  (  -1.22)
                                                        L 2  (GE)  L 1  (GE)  L 2  (GE)  L 2  (GE)  L 2  (GE)
=============================================================================================================
PART II. Parameters
       T-statistic unconditional (=0) [=1]
  Model type                                              LOGIT     S-DOGIT    G-DOGIT   LIN-IPTL   BT-IPTL
  Name of the variant                                    bus_bc4    bus_bc3    bus_bc4    bus_bc4    bus_bc4
  Version number of the variant                             3          6          7          8          9
  Dependent variable in the variant                       CHOICE     CHOICE     CHOICE     CHOICE     CHOICE
=============================================================================================================
  ----------------------
  BOX-COX TRANSFORMATIONS
  ----------------------

    LAMBDA(X)  1                                           0.2616     0.6974     1.0000     1.1151     1.0083
                                                        (    0.14) (    1.46)     FIXED  (    0.91) (    1.89)
                                                        [   -0.38] [   -0.63]            [    0.09] [    0.02]
```

```
  LAMBDA(X)  2                                                   1.5194     0.3299     1.0000     0.2883     0.5977
                                                             (    0.03) (    0.66)     FIXED  (    0.08) (    0.22)
                                                             [    0.01] [   -1.34]            [   -0.20] [   -0.15]

  LAMBDA(X)  3                                                   0.1809    -0.5145     1.0000     1.1776     1.5268
                                                             (    0.00) (   -0.50)     FIXED  (    1.46) (    2.22)
                                                             [   -0.02] [   -1.47]            [    0.22] [    0.77]

  LAMBDA(X)  4                                                  -0.5133                1.0000     0.5088     1.4154
                                                             (    0.00)                FIXED  (    0.16) (    0.42)
                                                             [    0.00]                       [   -0.16] [    0.12]
----------------
EXTRA PARAMETERS
----------------

  STANDARD DOGIT
  --------------

    THETA  1                                                                0.0124
                                                                        (    0.78)

    THETA  2                                                                0.0000
                                                                        (    0.00)

    THETA  3                                                                0.0000
                                                                        (    0.00)

    THETA  4                                                                0.0258
                                                                        (    0.24)

  GENERALIZED DOGIT
  -----------------

    THETA  1  1                                                                        0.0000
                                                                                        FIXED

    THETA  1  2                                                                        0.0548
                                                                                   (    0.01)

    THETA  1  3                                                                        0.0325
                                                                                   (    0.02)

    THETA  1  4                                                                        0.0084
                                                                                   (    0.00)

    THETA  2  1                                                                        0.0204
                                                                                   (    0.01)

    THETA  2  2                                                                        0.0000
                                                                                        FIXED

    THETA  2  3                                                                        0.0000
                                                                                   (    0.00)

    THETA  2  4                                                                        0.0000
                                                                                   (    0.00)

    THETA  3  1                                                                        0.0000
                                                                                   (    0.00)

    THETA  3  2                                                                        0.0000
                                                                                   (    0.00)

    THETA  3  3                                                                        0.0000
                                                                                        FIXED

    THETA  3  4                                                                        0.0000
                                                                                   (    0.00)

    THETA  4  1                                                                        5.4455
                                                                                   (   11.93)

    THETA  4  2                                                                        0.1547
                                                                                   (    0.04)

    THETA  4  3                                                                        6.2693
                                                                                   (   10.45)

    THETA  4  4                                                                        0.0000
                                                                                        FIXED
```

```
LIN-IPTL
--------

  PHI  1                                                                                         0.0002
                                                                                              (    0.00)
                                                                                              [    0.00]

  MU   1                                                                                         1.0000
                                                                                              ( 7693.86)
                                                                                              [   -0.03]

  PHI  2                                                                                         1.0000
                                                                                              (    0.00)
                                                                                              [    0.00]

  MU   2                                                                                         0.9999
                                                                                              (  293.23)
                                                                                              [   -0.03]

  PHI  3                                                                                         1.0000
                                                                                              (    0.00)
                                                                                              [    0.00]

  MU   3                                                                                         1.0000
                                                                                              (0.11D+05)
                                                                                              [    0.00]

  PHI  4                                                                                         0.0000
                                                                                              (    0.00)
                                                                                              [   -0.01]

  MU   4                                                                                         1.0000
                                                                                              (0.12D+05)
                                                                                              [    0.00]

BT-IPTL
-------

  PHI  1                                                                                                    0.0306
                                                                                                         (    0.63)
                                                                                                         [  -20.08]

  MU   1                                                                                                    0.0000
                                                                                                         (    0.04)
                                                                                                         [-.13D+07]

  PHI  2                                                                                                    0.0579
                                                                                                         (    0.63)
                                                                                                         [  -10.19]

  MU   2                                                                                                    0.0000
                                                                                                         (    0.03)
                                                                                                         [-.26D+08]

  PHI  3                                                                                                   -0.1354
                                                                                                         (   -0.16)
                                                                                                         [   -1.36]

  MU   3                                                                                                    0.0000
                                                                                                         (    0.00)
                                                                                                         [ -268.69]

  PHI  4                                                                                                    0.0596
                                                                                                         (    0.59)
                                                                                                         [   -9.25]

  MU   4                                                                                                    0.0000
                                                                                                         (    0.00)
                                                                                                         [-.86D+13]
```

PART III. General statistics

Model type	LOGIT	S-DOGIT	G-DOGIT	LIN-IPTL	BT-IPTL
Name of the variant	bus_bc4	bus_bc3	bus_bc4	bus_bc4	bus_bc4
Version number of the variant	3	6	7	8	9
Dependent variable in the variant	CHOICE	CHOICE	CHOICE	CHOICE	CHOICE
LOG-LIKELIHOOD:					
- FINAL VALUE (LF)	-323.24	-324.43	-303.37	-316.69	-308.62
- INITIAL VALUE	-523.78	-523.78	-523.78	-523.78	-523.78
- WITH CONSTANTS ONLY (LC)	-523.78	-523.78	-523.78	-523.78	-523.78
- RATIO TEST [-2(LC-LF)]	401.08	398.69	440.82	414.17	430.31
- DEGREES OF FREEDOM	8	11	20	16	16
RHO-SQUARED	0.3829	0.3806	0.4208	0.3954	0.4108
RHO-SQUARED BAR - AKAIKE	0.3619	0.3539	0.3769	0.3591	0.3745
- HOROWITZ	0.3724	0.3672	0.3988	0.3772	0.3926
- HENSHER AND JOHNSON	0.3803	0.3773	0.4157	0.3910	0.4065
PERCENT RIGHT	75.50	75.60	75.80	75.80	75.90
SAMPLE - NUMBER OF ALTERNATIVES	4	4	4	4	4
- NUMBER OF OBSERVATIONS	1000	1000	1000	1000	1000
- AVAILABLE OBSERVATIONS:					
.ALTERNATIVE 1 : RAIL	991	991	991	991	991
.ALTERNATIVE 2 : AIR	846	846	846	846	846
.ALTERNATIVE 3 : BUS	797	797	797	797	797
.ALTERNATIVE 4 : CAR	999	999	999	999	999
TOTAL NUMBER OF ESTIMATED PARAMETERS:	11	14	23	19	19
- BETA .VARIABLES	4	4	4	4	4
.CONSTANTS	3	3	3	3	3
- LAMBDA(X)	4	3	4	4	4
- EXTRA PARAMETERS	0	4	12	8	8

9. Appendix 4. Simplified reduced sample Generation-Distibution models, Quebec-Windsor Corridor (Business trips)

9.1 Models estimated with L-1.6

```
===========================================================================================
 PART I. Presented statistics
   Model type                                       L-1.6     L-1.6     L-1.6     L-1.6     L-1.6
   Name of the variant                                LIN       LOG       BC1       BC2       BC4
   Version number of the variant                      6         7         8         9        10
   Dependent variable in the variant                  Tij       Tij       Tij       Tij       Tij

 ===========================================================================================


 -----------------------------
 IND     INDEPENDENT VARIABLES
 -----------------------------
   TRIPS FROM ORIG_i TO ALL DEST.       Ti
     BETA coefficient                            0.24D+02  0.53D+00  0.61D+00  0.15D+01  0.18D+03
     Elasticity of Y (sample value)                 0.170     0.533     0.517     0.481     0.363
     Conditional t-statistic for BETA            (  1.49)  (  8.05)  (  8.64)  (  8.52)  (  8.71)
                                                    FL1       FL1       EL1       EL1       EL1
   TRIPS FROM ALL ORIG. TO DEST_j       Tj
     BETA coefficient                            0.45D+02  0.65D+00  0.76D+00  0.18D+01  0.27D-07
     Elasticity of Y (sample value)                 0.321     0.651     0.641     0.588     0.821
     Conditional t-statistic for BETA            (  3.43)  ( 11.00)  ( 12.00)  ( 11.76)  ( 10.26)
                                                    FL1       FL1       EL1       EL1       EL2
   UTILITY from VARIANT bus_bc4         Util_BC4
     BETA coefficient                            0.89D+10  0.58D+00  0.14D+01  0.16D+01  0.14D+01
     Elasticity of Y (sample value)                 0.255     0.584     0.596     0.533     0.528
     Conditional t-statistic for BETA            ( 13.94)  ( 36.04)  ( 35.57)  ( 35.90)  ( 37.59)
                                                    FL1       FL1       EL1       EL1       EL3
   REGRESSION CONSTANT                  constant
     BETA coefficient                            0.89D+10  0.71D+01  0.12D+02  0.38D+01 -0.39D+03
     Conditional t-statistic for BETA            ( 13.98)  (  6.67)  (  7.97)  (  1.30)  ( -7.88)


 ===========================================================================================
 PART II. Parameters
          t-statistic unconditional (=0)  [=1]
   Model type                                       L-1.6     L-1.6     L-1.6     L-1.6     L-1.6
   Name of the variant                                LIN       LOG       BC1       BC2       BC4
   Version number of the variant                      6         7         8         9        10
   Dependent variable in the variant                  Tij       Tij       Tij       Tij       Tij

 ===========================================================================================

 Box-Cox transformations (estimated)
 -----------------------------------
   Lambda(y)                            ELY                               0.035     0.065     0.066
                                                                       (  2.84)  (  4.11)  (  4.23)
                                                                       [-79.03]  [-59.28]  [-59.90]

   Lambda(x) - group 1                  EL1                               0.035    -0.002    -0.419
                                                                       (  2.84)  ( -0.11)  ( -1.42)
                                                                       [-79.03]  [-59.04]  [ -4.82]

   Lambda(x) - group 2                  EL2                                                   1.513
                                                                                           (  4.47)
                                                                                           [  1.52]

   Lambda(x) - group 3                  EL3                                                  -0.016
                                                                                           ( -1.02)
                                                                                           [-65.06]

 Box-Cox transformations (fixed)
 -------------------------------
   Lambda(y)                            FLY         1.000     0.000

   Lambda(x) - group 1                  FL1         1.000     0.000
```

PART III. General statistics

Model type	L-1.6	L-1.6	L-1.6	L-1.6	L-1.6
Name of the variant	LIN	LOG	BC1	BC2	BC4
Version number of the variant	6	7	8	9	10
Dependent variable in the variant	Tij	Tij	Tij	Tij	Tij
Log-likelihood	-18797.91	-17486.28	-17482.62	-17477.05	-17459.96
Pearson R2	0.144	0.243	0.248	0.276	0.296
Pearson R2 adjusted for D.F.	0.141	0.241	0.245	0.273	0.291
Pseudo-(L)-R2	0.165	0.578	0.574	0.575	0.589
Pseudo-(L)-R2 adjusted for D.F.	0.163	0.577	0.572	0.573	0.586
Average probability (Y=limit observation)	0.251	0.000	0.000	0.000	0.000
Sample - Number of observations	1000	1000	1000	1000	1000
Estimation - Number of observations	1000	1000	1000	1000	1000
- First observation	1	1	1	1	1
- Last observation	1000	1000	1000	1000	1000
Number of estimated parameters					
- Beta	4	4	4	4	4
- Lambda(y) and Lambda(x)	0	0	1	2	4
- Heteroskedasticity: Delta	0	0	0	0	0
Lambda(z)	0	0	0	0	0
- Autocorrelation: Rho	0	0	0	0	0
- Total	4	4	5	6	8
Mean of observed Y	0.26D+08	0.26D+08	0.26D+08	0.26D+08	0.26D+08
Standard error of observed Y	0.39D+08	0.39D+08	0.39D+08	0.39D+08	0.39D+08
Skewness of observed Y	0.34D+01	0.34D+01	0.34D+01	0.34D+01	0.34D+01
Mean of Y, estimated at the mean	0.31D+08	0.59D+08	0.53D+08	0.51D+08	0.46D+08
Standard error of Y, estimated at the mean	0.28D+08	0.94D+08	0.71D+08	0.60D+08	0.53D+08
Skewness of Y, estimated at the mean	0.80D+00	0.83D+01	0.55D+01	0.42D+01	0.41D+01
Mean of estimated E(Y)	0.31D+08	0.34D+08	0.31D+08	0.28D+08	0.28D+08
Standard error of estimated E(Y)	0.14D+08	0.46D+08	0.41D+08	0.32D+08	0.32D+08
Skewness of estimated E(Y)	0.45D+01	0.31D+01	0.30D+01	0.23D+01	0.23D+01
M.R.S. between Measures M1, M2 and M3					
M1 / M2	0.31D+01	0.63D+00	0.79D+00	0.92D+00	0.94D+00
M2 / M1	0.33D+00	0.16D+01	0.13D+01	0.11D+01	0.11D+01
M1 / M3	-0.31D+08	-0.67D+22	-0.13D+09	-0.10D+09	-0.93D+08
M3 / M1	-0.32D-07	-0.15D-21	-0.80D-08	-0.10D-07	-0.11D-07
M2 / M3	-0.10D+08	-0.11D+23	-0.16D+09	-0.11D+09	-0.99D+08
M3 / M2	-0.99D-07	-0.94D-22	-0.63D-08	-0.92D-08	-0.10D-07
Elasticity of substitution between M1, M2 and M3					
M1 / M2	0.29D+01	0.10D+01	0.11D+01	0.11D+01	0.11D+01
M2 / M1	0.35D+00	0.10D+01	0.95D+00	0.91D+00	0.91D+00
M1 / M3	-0.81D+00	-0.94D+15	-0.13D+02	-0.84D+01	-0.84D+01
M3 / M1	-0.12D+01	-0.11D-14	-0.77D-01	-0.12D+00	-0.12D+00
M2 / M3	-0.28D+00	-0.94D+15	-0.12D+02	-0.77D+01	-0.77D+01
M3 / M2	-0.35D+01	-0.11D-14	-0.81D-01	-0.13D+00	-0.13D+00

9.2 Models estimated with L-2.1

```
================================================================================
 PART I. Presented statistics
   Model type                                            L-2.1     L-2.1     L-2.1
   Name of the variant                                     BC4     BC4HG BC4HGAUPR
   Version number of the variant                            10        17        24
   Dependent variable in the variant                       Tij       Tij       Tij
 ================================================================================
 -----------------------------
 IND   = INDEPENDENT VARIABLES
 -----------------------------
   TRIPS FROM ORIG_i TO ALL DEST.       Ti
     BETA coefficient                                0.18D+03  0.80D+02  0.81D+02
     Elasticity of Y (sample value)                     0.363     0.398     0.398
     Conditional t-statistic for BETA               (   8.71) (   7.74) (   7.73)
                                                       LAM 1     LAM 1     LAM 1

   TRIPS FROM ALL ORIG. TO DEST_j       Tj
     BETA coefficient                                0.27D-07  0.76D-08  0.83D-08
     Elasticity of Y (sample value)                     0.821     0.806     0.807
     Conditional t-statistic for BETA               (  10.26) (  12.71) (  12.88)
                                                       LAM 2     LAM 2     LAM 2

   UTILITY from VARIANT bus_bc4         Util_BC4
     BETA coefficient                                0.14D+01  0.15D+01  0.15D+01
     Elasticity of Y (sample value)                     0.528     0.508     0.500
     Conditional t-statistic for BETA               (  37.59) (  40.84) (  45.44)
                                                       LAM 3     LAM 3     LAM 3

   REGRESSION CONSTANT                  CONSTANT
     BETA coefficient                               -0.39D+03 -0.19D+03 -0.20D+03
     Elasticity of Y (sample value)                  -125.090   -52.443   -51.857
     Conditional t-statistic for BETA               (  -7.88) (  -6.27) (  -6.28)


 Heteroskedasticity
 ------------------
   PRODUCT Ti*Tj                        TiTj
     DELTA coefficient                                        -0.12D-19 -0.96D-21
     Elasticity of Y (sample value)                               0.034     0.030
     Conditional t-statistic for DELTA                        (  -6.99) (  -6.91)
```

```
==============================================================================
 PART II. Parameters
          t-statistic unconditional (=0) [=1]
   Model type                                           L-2.1     L-2.1     L-2.1
   Name of the variant                                    BC4     BC4HG BC4HGAUPR
   Version number of the variant                           10        17        24
   Dependent variable in the variant                      Tij       Tij       Tij

 ==============================================================================
 Box-Cox Transformations
 -----------------------
   LAMBDA(Y)                                            0.066     0.076     0.078
                                                   (   4.23) (   4.32) (   4.20)
                                                   [ -59.90] [ -52.29] [ -49.92]

   LAMBDA(X) - Group 1                                 -0.419    -0.330    -0.329
                                                   (  -1.42) (  -0.93) (  -0.93)
                                                   [  -4.82] [  -3.76] [  -3.77]

   LAMBDA(X) - Group 2                                  1.513     1.631     1.625
                                                   (   4.47) (   5.77) (   5.85)
                                                   [   1.52] [   2.23] [   2.25]

   LAMBDA(X) - Group 3                                 -0.016    -0.032    -0.035
                                                   (  -1.02) (  -2.35) (  -2.89)
                                                   [ -65.06] [ -76.49] [ -86.18]
 Heteroskedasticity
 ------------------
   PRODUCT Ti*Tj                        TiTj

      LAMBDA(Z)                                                   1.861     1.964
                                                             (   3.78) (   3.84)
                                                             [   1.75] [   1.88]
 Directed Autocorrelation
 ------------------------
   CAR_RT < 100 min                     DIST_OD

      RHO                                                                  -1.000
                                                                       (  -0.17)

      PI                                                                    1.000
                                                                            Fixed
```

```
==============================================================================
PART III. General statistics
  Model type                                        L-2.1     L-2.1     L-2.1
  Name of the variant                                 BC4     BC4HG BC4HGAUPR
  Version number of the variant                        10        17        24
  Dependent variable in the variant                   Tij       Tij       Tij
==============================================================================
Log-likelihood                                  -17459.96 -17433.91 -17433.38

Degrees of freedom                                      7         9        10

Pearson R2                                          0.296     0.313     0.318
Pearson R2 adjusted for Degrees of freedom          0.291     0.306     0.311

Pseudo-(L)-R2                                       0.589     0.884     0.955
Pseudo-(L)-R2 adjusted for Degrees of freedom       0.586     0.883     0.955

Average probability (Y=limit observation)           0.000     0.000     0.000

Sample     - Number of observations                  1000      1000      1000

Total number of fixed or estimated parameters:
           - Beta .Estimated                            3         3         3
                  .Constant                             1         1         1
           - Lambda(y)
                  .Fixed                                0         0         0
                  .Estimated .Constrained               0         0         0
                             .Distinct                  1         1         1
           - Lambda(x)
                  .Fixed                                0         0         0
                  .Estimated                            3         3         3
           - Heteroskedasticity
              Delta
                  .Fixed                                0         0         0
                  .Estimated                            0         1         1
              Lambda(z)
                  .Fixed                                0         0         0
                  .Estimated                            0         1         1
           - Autocorrelation
              Rho .Estimated                            0         0         1
              Pi  .Fixed                                0         0         1
                  .Estimated                            0         0         0
           - Total
                  .Fixed                                0         0         1
                  .Estimated                            8        10        11

Mean of observed Y                                0.26D+08  0.26D+08  0.26D+08

Mean of estimated E(Y)                            0.28D+08  0.28D+08  0.27D+08

==============================================================================
```

10. Appendix 5. Characteristics of TRIO algorithms implemented in TRIO Version 2 for Windows and self-standing for Version 6

10.1 Maximum dimensions

In this table, values for the first column are drawn from Appendix E of Gaudry *et al*., 1993, which contains more information on the maximum dimensions of TRIO, Version 2.0.

Algorithms for each Version of TRIO	Three implemented in Version 2.0 for Windows	Five self-standing for future Version 6				
		LEVEL L-1.6	LEVEL L-2.1	SHARE S1/S5	PROB. P2/P6	QDF
DATABASE						
Number of variables	unlimited	100	100	300	300	Depend on the coupled models: see the four algorithms on the left.
Number of variants	100	999	999	999	999	
Number of groups	100	100	100	100	100	
ESTIMATION per variant						
Number of observations	unlimited	10000	1000	unlimited	unlimited	Maximum dimensions depend on the coupled models: see the four algorithms on the left.
Number of regression coefficients LEVEL 1 LEVEL L-2.1 SHARE S1/S5 PROBABILITY	 (L-1.4) 200[5] n.a. 60[6] (P-2) 60[7]	 100 n.a. n.a. n.a.	 n.a 100 n.a. n.a.	 n.a. n.a. 100 n.a.	 n.a. n.a. n.a. 80	
Number of Box-Cox parameters estimated or fixed	included in 60 below	10	10	10	10	
Number of estimated parameters other than regression coefficients	60	90	90	90	90	
Number of periods for L-1 forecast	100[8]	60	n.a.	n.a.	n.a.	
Number of variables for heteroskedasticity	5	10	10	n.a.	n.a.	
Number of autocorrelation orders	24	24	2	n.a.	n.a.	
Lag for autocorrelation function	168	96	n.a.	n.a.	n.a.	
Lag for partial autocorrelation function	84	36	n.a	n.a.	n.a.	
Number of alternatives in SHARE or PROBABILITY	20[9]	n.a.	n.a.	10	20	

[5] The number is 201 if the regression constant is counted.
[6] Excluding the regression constants of alternatives.
[7] P-2 algorithm because P-3 to P-6 are not implemented in Version 2.0. Excludes the regression constants of alternatives.
[8] Pertains to L-1.4.
[9] Except for the GENERALIZED DOGIT in SHARE, where it is 5.

10.2 Number of variants displayed in a TABLEX

PRINTING FORMAT	**Three algorithms implemented in TRIO V.2 Tablex tables produced for algorithms L-1.4, P-2 and S-1/S-5**		
	Number of variants displayed on first page/on other pages[10]		
	80 characters	128 characters	160 characters
Normal	3/6	7/9	9/12
Compressed	4/8	10/14	13/17
	Five algorithms for Version 6 Tablex tables produced by algorithms L-1.6, L-2.1, P-2/P-6, S-1/S-5 and QDF		
	Number of variants displayed on each page		
Normal	10 (max. 160 characters per line)		

10.3 Page width, character size and lines per printed page

How character size and interline space on a printed page change with page width

Three algorithms implemented in TRIO Version 2.0				
Page width: Number of characters	Maximum number of lines per printed page		Font type is “courier”	
	US letter	A4	Character size	Interline space
80	65	70	6	10 pts
128	70	75	6	10 pts
160	116	123	5	6 pts
Five algorithms for Version 6				
The page width has a maximum of 160 characters depending on the number of variants displayed. The number of lines per printed page will depend on the character size and interline space. The font type is “courier new”. In WORD, the user chooses the proper character size so that a whole line can fit to the page width.				

10.4 Format for numerical values in TABLEX

Exponential and fixed point formats used, with right-justified printed values

Three algorithms implemented in TRIO Version 2.0			
PRINTING FORMAT	Format used for		
	betas, derivatives	elasticities	t-statistics
Normal	E12.5	F12.3	(F10.2)
Compressed	E8.2	F8.3	(F6.2)
Five algorithms for Version 6			
	Format used for		
	betas, derivatives	elasticities	t-statistics
Normal	D9.2	F9.3	(F6.2)

[10] If all variant scan be printed on the same line, the definitions of variables are printed on the first and subsequent pages ; if not (depending on the fomat chosen), the definitions will be printed only on the first page.