TESTING THE LINEAR INVERSE POWER TRANSFORMATION LOGIT MODE CHOICE MODEL

BY

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RÉSUMÉ

Nous étudions les propriétés empiriques et numériques d'un membre de la classe IPT, le modèle probabiliste IPT-Logit Linéaire. Ce modèle rend possible le traitement de trois problèmes pratiques de modélisation du choix du mode de transport: la présence de courbes de réaction asymétriques, de captivité aux alternatives et de compatibilité à l'axiome IIA de la théorie des choix, axiome qui exclut d'emblée la complémentarité entre alternatives. Notre analyse formule pour la première fois des fonctions d'utilité représentative complètes et modélise l'interaction entre ces fonctions élargies, l'asymétrie et la captivité des fonctions de réaction. Des tests statistiques et empiriques sur un cas réel à deux modes de transport urbain à Winnipeg montrent que le modèle IPT-Logit Linéaire permet des gains importants sur son cas particulier connu, le Logit Linéaire, et comporte trois dimensions supplémentaires toutes utiles.

Mots-clés: CHOIX PROBABILISTE, CAPTIVITÉ, RÉACTION ASYMÉTRIQUE, SPÉCIFICITÉ MODALE, LOGIT, TRANSFORMATION PUISSANCE INVERSE, IPT, WINNIPEG

ABSTRACT

In this paper, we study the empirical and numerical properties of one member of the IPT class, the Linear IPT-Logit model. This models makes it possible to deal with three very practical problems of mode choice modelling: asymmetric response effects, modal captivity and consistency with the independence from irrelevant alternatives axiom (IIA), which excludes the possibility of complementarity among alternatives. In our analysis, a completely full form of the utility functions is specified and estimated, seemingly for the first time in mode choice analysis. Furthermore, the interactions among such enlarged specifications of utility functions, asymmetry of the reaction functions and modal captivity are analyzed. Statistical tests are carried out on a binomial case using urban data from Winnipeg in order to examine both the theoretical properties of estimators and the empirical gains of the Linear IPT Logit model over its simpler linear Logit root. All three additional dimensions are found to be of practical use.

Key-words: PROBABILISTIC CHOICE, CAPTIVITY, ASYMMETRY OF RESPONSE, MODE SPECIFICITY, LOGIT, INVERSE POWER TRANSFORMATION, IPT, WINNIPEG

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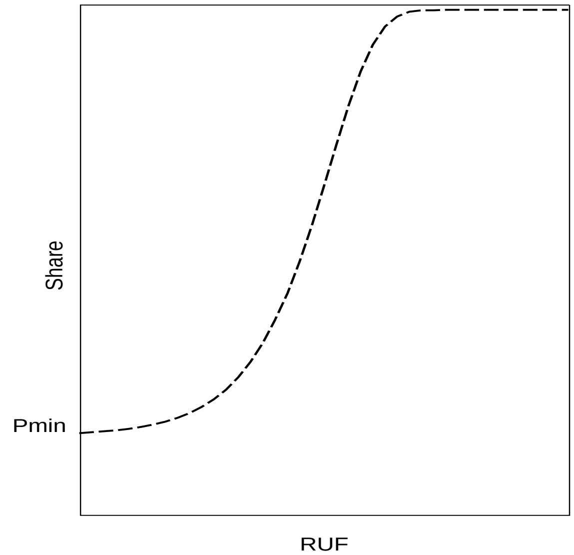
1.INTRODUCTION

This paper deals with three very practical problems of mode choice modelling: asymmetric response effects, modal captivity and consistency with the independence from irrelevant alternatives axiom (IIA), which excludes the possibility of complementarity among alternatives. The first two are illustrated in Fig. 1, where the share of the mode is on the y-axis and the Representative Utility Function (RUF) corresponding to this mode is on the x-axis. An asymmetric curve is a sigmoïd shape where the upper and lower parts of the curve are not mirror images of each other. This property is not taken into account with well known Logit and Probit models.

Figure 1
Asymmetry and Captivity: The Intuition



Another practical issue is the possibility of having non zero observed shares for very unattractive modes. In Fig. 1 this means that the minimum share is P_{min} no matter how poor the level of service is. So far, the only model that handles captivity or loyalty towards a mode is the dogit model (Gaudry and Dagenais, 1979). The IPT model developed by Gaudry (1981) takes into account both captivity and threshold effects. In this paper, we study the empirical and numerical properties of one member of the IPT-Logit class, the Linear IPT-Logit model.

Because of the way in which the IPT model deals with asymmetry of response and captivity effects, it allows as a by-product the use of full or complete representative utility functions for <u>each</u> mode: not only can the characteristics of <u>all</u> modes be entered in <u>each</u> utility function, but the socioeconomic variables as well. The enriched specifications of representative utility functions are made possible by either the asymmetric shape or the variable tail thickness of the reaction function. The enriched specification enables complementarity among alternatives, is unconstrained by the IIA assumption and, lastly, permits the estimation of a generic socioeconomic specification unaffected by the choice of a reference mode.

In this paper, a completely full form of the utility functions is specified and estimated, seemingly for the first time in mode choice analysis. Furthermore, the interactions among such enlarged specifications of utility functions, asymmetry of the reaction functions and modal captivity are analyzed. Statistical tests are carried out on a binomial case using urban data in order to examine both the theoretical properties of estimators and the empirical gains of the Linear IPT Logit model over its simpler linear Logit root. All three additional dimensions are found to be of practical use.

2.THE LIN-IPT-LOGIT MODEL

Formally, the Linear IPT-Logit model is written

$$P_i = \frac{\Psi_i}{\sum_{j=1}^{M} \Psi_j} \tag{1}$$

$$\Psi_i = (\lambda_i \exp(V_i) + 1)^{(1/\lambda_i)} + \mu_i, \qquad \lambda_i \neq 0 \tag{2-A}$$

$$= \exp(\exp(V_i)) + \mu_i, \qquad \lambda_i \to 0 \tag{2-B}$$

with

$$\lambda_i \geq 0 \text{ and } \mu_i \geq -1 \quad , \tag{3}$$

where P_i is the share of the i^{th} of M alternatives and V_i is called the representative utility function (RUF) of alternative *i*. The Logit model is a nested special case and is obtained when $\lambda_i = 1, \mu_i = -1, \forall i$. It is useful to insure by (3) that computed shares be non-negative and smaller than one for all potential values of V_i's .

2.1 Full representative utility functions, IIA and complementarity

In general, the representative utility function (V_i) contains two categories of variables:

- service level or network variables (N_i) that differ across the M modes or alternatives;
- socioeconomic variables (S) that do not vary across alternatives.[1]

The Lin-IPT-L model makes it possible to enlarge the representative utility function of <u>each</u> mode with all the variables: level of service variables of all modes as well as socioeconomic variables. The reason for this is that a term common to all utility functions cannot be factored out any more: this is obvious in (2) when the λ_i differ from 1 but is also true if they are equal to one because of the remaining 1's and μ's . The application of the Lin-IPT-L form therefore solves the under-identification problem of the Logit.

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[1] Vectors N_i and S are both column vectors, that is one column and as many rows as the number of variables. The constants are counted as socioeconomic variables.

We now define various specifications of the representative utility function which exploit this property, starting with the simplest Logit format. For simplicity, the number of modes is assumed equal to three.

2.1.1 Alternative-Specific Logit (A-S, L)

The alternative-specific Logit specification of the representative utility functions ($\lambda_i = 1, \mu_i = -1$) is given in Section 1 of Table 1. The RUF are clearly incomplete since network variables only appear once and measure the direct effect of network variables of a specific mode on the corresponding utility function. Moreover, socioeconomic variables only appear in the first two utility functions, because $\beta_1^1, \beta_2^2, \beta_3^3, \beta_4^1,$ and β_4^2 are the only parameters that can be identified: here the third mode is assumed to be the reference mode.

2.1.2 Alternative-Specific Logit Extended (A-S, LE)

When asymmetry and/or captivity are present, the Lin-IPT-L model enables to "extend" the above specification by including socioeconomic variables in the utility functions of all the alternatives, as indicated in Section 2 of Table 1. We shall refer to this specification as the alternative-specific "Logit Extended" (LE) specification. The Logit specification is restrictive since it means that β_4^3 is zero or that only the difference of $(\beta_4^1 - \beta_4^3)$ and $(\beta_4^2 - \beta_4^3)$ can be estimated.

The estimation of the extended utility function is interesting since it renders possible to identify the effects of socioeconomic variables on each utility function. For instance, as income increases, the utility of each mode may increase but, since the shares must add-up to one, at least one share must increase and another must decrease. Therefore, income effects on the shares may be misleading as one could conclude that modes are inferior. We argue that a more appropriate measure of the effect of one socioeconomic variable on one mode is through its effects on the RUF. This means that it could be possible for a transportation mode to be a normal or non-inferior good, when measured through its effect on the RUF, even though its share decreases as income increases.

2.1.3 Alternative-Specific Universal Logit (A-S, UL)

Again, if asymmetry and/or captivity are present, the Lin-IPT-L model enables one to use network variables in each utility function as shown in Section 3 of Table 1. We call this specification of the RUF the "Universal Logit" (UL) specification[2]. The UL specification ensures that the Lin-IPT-L model is no longer consistent with the IIA assumption since relative shares now depend upon service levels of all modes.

As a consequence of the UL specification, complementarity among modal shares may arise, which means that improvement of the level of service of one mode may not necessarily reduce the share of the other modes, as occurs with the Logit model. To see this, let X_k be an attribute of mode *i* included not only in the vector of network variables N_i, but in the others as well ($N_j\ , j=1,..,M, j\neq i$); the elasticity of the mode *i* and *j* with respect to X_k are for own and cross elasticities, respectively:[3]

$$\eta_k^i \quad \equiv \quad \frac{\partial P_i}{\partial X_k}\frac{X_k}{P_i} \quad = \quad \left(\beta_k^i \Psi_i^* \left(\sum_{l=1}^{M} \Psi_l \right) - \Psi_i \left(\sum_{l=1}^{M} \Psi_l^* \, \beta_k^l \right) \right) X_k \quad / \quad \left(\left(\sum_{l=1}^{M} \Psi_l \right) \Psi_i \right) \qquad (4-A)$$

$$\eta_k^j \quad \equiv \quad \frac{\partial P_j}{\partial X_k}\frac{X_k}{P_j} \quad = \quad \left(\beta_k^j \Psi_j^* \left(\sum_{l=1}^{M} \Psi_l \right) - \Psi_j \left(\sum_{l=1}^{M} \Psi_l^* \, \beta_k^l \right) \right) X_k \quad / \quad \left(\left(\sum_{l=1}^{M} \Psi_l \right) \Psi_j \right) \qquad (4-B)$$

$$\text{where, } \Psi_i^* = (\Psi_i - \mu_i)^{(1-\lambda_i)} \exp(V_i). \qquad (5)$$

First, notice how, if $\lambda_i = 1, \text{ and } \mu_i = -1 \text{ for } \forall i$, then $\Psi_i^* = \Psi_i = \exp(V_i)$. If moreover $\beta_k^j = 0$, $\text{for } i \neq j$, then $\eta_k^i \text{ and } \eta_k^j$ reduce to the well-known formulas for the Logit model:

$$\eta_k^i = \quad \beta_k^i (1 - P_i) X_k \quad \text{for the own elasticities and} \qquad (6-A)$$

$$\eta_k^j = \quad -\beta_k^i P_i X_k \qquad \text{for the cross elasticities} \qquad (6-B)$$

From (6), modes i and j are necessarily substitutes in the Logit model, given the sign of β's : the sign of (6-B) is necessarily opposite to that of (6-A). Moreover, the cross-elasticity is the same for all *j* modes. Now two properties are of special interest.

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[2] This term comes from McFadden(1980).
[3] Complementarity and substitution could be dicussed with respect to the value of the representative utility function V_i, but we confine ourselves to effects on the modal share.

Complementarity. In the real world, cross elasticities may be positive or negative and may differ among mode pairs considered. Two modes may even be substitutes or complements depending on the level of the variables. For instance, bus and car may be substitutes if travel time by car is relatively low, but may be complements as travel time by car becomes relatively high. Depending upon the conditions in a specific market, complementarity or substitution may arise. The same applies to computed shares: this will be explained in detail in Section 4.2 and shown in Figure 5.

All of the above cases are possible with the Lin-IPT-L model. From (4-B), if the number of modes is greater than two, η_k^j may be positive or negative and may vary with i or j. Moreover, η_k^j may be negative (positive) for low values of X_k but positive (negative) for high values of X_k since Ψ and Ψ^* functions are not, in general, independent of X_k . Note that no general property of the elasticities can be inferred from the signs of the β_k coefficients.

Rejection of the IIA property. One important limitation of the Logit model is its inability to yield a differentiated reaction from existing modes following the introduction of a new mode. A new mode gaining 5% of the market means that the shares of the existing modes have all been reduced by 5%. This arises as shown by equation (7)[4] if the RUF does not have a UL specification: in this case then the RUF will be unaffected by the introduction of the new mode M+1. In contrast, if the service variables of all modes are in each RUF, then the RUF are modified by the introduction of a new mode and a differentiated reaction of modes is possible, as in equation (8). This is not surprising in view of the fact that cross elasticity patterns are different for each mode pair considered.

$$\frac{(P_i^B - P_i^A)}{P_i^B} = 1 - \frac{P_i^A}{P_i^B} = \Psi_{M+1} \;/\; \left(\sum_{j=1}^{M+1} \Psi_j\right) = P_{M+1} \qquad ,\text{if } \Psi_i^A = \Psi_i^B, \quad \forall i \tag{7}$$

$$= 1 - \left(\Psi_i^A \;/\; \left(\sum_{j=1}^{M+1} \Psi_j^A\right)\right)\left(\sum_{j=1}^{M} \Psi_j^B \;/\; \Psi_i^B\right) \qquad ,\text{if } \Psi_i^A \neq \Psi_i^B, \quad \forall i \tag{8}$$

2.1.4 Alternative-Specific Universal Logit Extended (A-S, ULE)

Combining the two preceding generalizations, one obtains the ULE specification shown in Section 4 of Table 1, which admits of complements, is generally not consistent with IIA, and clearly does not exhibit underidentification of parameters associated with socioeconomic variables or mode-

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[4] The superscripts B and A refer to before and after the introduction of the new mode.

specific constants.

2.1.5 Abstractness and specificity of the utility function

So far, we have defined the utility function by assuming that explanatory variables N_i and S have alternative-specific influences on the RUF. If one is interested in the potential effect of a new mode, an abstract or generic specification may be required. Section 5 of Table 1 describes the general form of a generic spefication for the Logit model. It is readily observable that socioeconomic variables are not "really" generic. One well-known problem related with this specification is that estimated γ coefficients are not invariant with respect to the mode of reference, in this example the third mode. This problem did not occur in alternative-specific logit formats in Sections 1 and 3. Here, the underidentification of parameters associated with variables common to all alternatives, combined with the further requirement that the β_4^1 and β_4^2 of Table 1 be equal, yields γ_3 parameters that depend on the mode of reference (except in the two-alternative case).

Again, if asymmetry and/or captivity are present, the Lin-IPT-L model enables to define an abstract or generic version of the Universal Extended Logit of the RUF as shown in Section 6 of Table 1.[5] The generic version of the ULE is completely free from an *ad hoc* choice for the reference mode.

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5 In order to keep the notation simple, the cross effects have been assumed to take a linear form. Of course, it is conceivable to have other forms for cross network variables such as min($N_j, \forall j \neq i$), mean($N_j, \forall j \neq i$), etc.

Table 1. Specifications of the Representative Utility Function

Section 1. Alternative-Specific Logit (A-S, L)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1 +$		$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2 +$		$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3$		

Section 2. Alternative-Specific Logit Extended (A-S, LE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1 +$		$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2 +$		$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3 +$		$\beta_4^3 S$

Section 3. Alternative-Specific Universal Logit (A-S, UL)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1 +$	$\beta_2^1 N_2 + \beta_3^1 N_3 +$	$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2 +$	$\beta_1^2 N_1 + \beta_3^2 N_3 +$	$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3 +$	$\beta_1^3 N_1 + \beta_2^3 N_2 +$	

Section 4. Alternative-Specific Universal Logit Extended (A-S, ULE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1 +$	$\beta_2^1 N_2 + \beta_3^1 N_3 +$	$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2 +$	$\beta_1^2 N_1 + \beta_3^2 N_3 +$	$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3 +$	$\beta_1^3 N_1 + \beta_2^3 N_2 +$	$\beta_4^3 S$

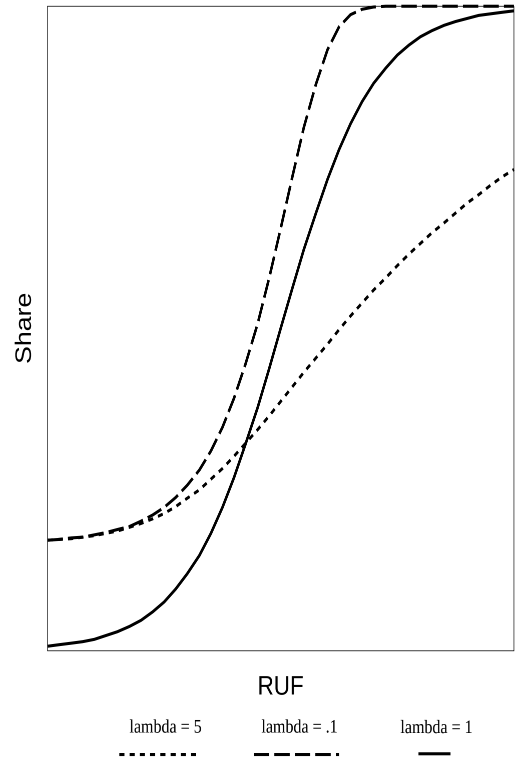
Section 5. Generic Logit (G, L)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\gamma_1 N_1 +$		$\gamma_3 S$
	$V_2 =$	$\gamma_1 N_2 +$		$\gamma_3 S$
	$V_3 =$	$\gamma_1 N_3$		

Section 6. Generic Universal Logit Extended (G, ULE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\gamma_1 N_1 +$	$\gamma_2 (N_2 + N_3) +$	$\gamma_3 S$
	$V_2 =$	$\gamma_1 N_2 +$	$\gamma_2 (N_1 + N_3) +$	$\gamma_3 S$
	$V_3 =$	$\gamma_1 N_3 +$	$\gamma_2 (N_1 + N_2) +$	$\gamma_3 S$

2.2 Asymmetry and modal captivity

The role of the λ and μ parameters can be analyzed with the reaction curve. The parameters λ and μ determine the asymmetry of the reaction function and the captivity level, respectively. In Figure 2, three different reaction functions are drawn. The continuous line denotes the reaction curve of the Logit model: its sigmoid shape has an inflection point where modal shares are equal. The reaction curve of a mode will be asymmetric if its λ parameter differs from 1; for instance, if the λ parameter is smaller than one, the reaction curve becomes steeper or introduces the threshold effect mentioned above. The share is less sensitive to the RUF if λ is greater than one, but the reaction curve is still asymmetric. The minimum share of a mode is different from zero, and a captivity effect is present, if μ is greater than -1. We now present a way to measure both the asymmetry of the reaction curve and the level of captivity.

Figure 2
Asymmetry and Captivity: The Role of the Inverse Box-Cox Transformation



2.2.1 Modal captivity

The modal captivity of mode *i* is defined as the modal share (P_{min}) with a very poor level of service ($V_i \to -\infty$) or, more formally, as a non-zero limit of the share:

$$P_{\min}(i) \quad = \quad (1+\mu_i) \quad / \quad \left(1+\mu_i+\sum_{\substack{j=1 \\ j\neq i}}^{M} \Psi_j\right) \quad . \tag{9}$$

It is clear that, in practice, captivity is a limit concept and that its presence can be explained by the absence of various explanatory variables in the V_i functions.[6]

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[6] Such an explicitation for the Dogit model can be found in Swait and Ben-Akiva (1987).

2.2.2 Asymmetry measure

Although the presence of asymmetry is quite intuitive, a more formal definition is useful. First note that, on the x-axis in Fig.2, we show the RUF and are therefore "in utility space". We might as well have shown any variable contained in RUF, for instance a service level, and presented the curve "in characteristics space". Had we shown the service level, however, it would not have been true that asymmetry is not possible with a Logit or Probit model: indeed, non linear transformations, such as the natural logarithm of a variable present in a linear-in-parameters RUF, imply an asymmetric response for these models in characteristics space -but not in utility space.

Note that an asymmetric reaction curve of one mode does not imply a complementary asymmetric reaction curve for any other mode. This is possible since the reaction curve is defined with the utility function and the share of the same mode. In consequence, the complementary share of a mode is not the reaction curve of that other mode. For instance, with 2 modes, and assuming that $\lambda_1 = 15$ and $\lambda_2 = 1$, the reaction curve of mode 1 is asymmetric, and the complementary curve $1 - p_1$ is also asymmetric, but the latter refers to the effect of V_1 on P_2 as opposed to the effect of V_1 on P_1, which is used in the definition of asymmetry: the complement $1 - p_1$ is similar to a cross effect.

Let (RUF_f) be the value of the RUF associated with the inflection point of the reaction curve, i.e. the point at wich its curvature changes from concave to convex. This point can be found by equating to zero the second derivative of the Lin-IPT-L mode share,

$$\partial(P_i(V_i = RUF_f))^2 \quad / \quad \partial^2 V_i \quad = \quad 0 \quad . \tag{10}$$

A reaction curve is symmetric if the lower (upper) part of the reaction function perfectly explains the upper (lower) part. This suggests υ_i, a definition of asymmetry of the reaction function of mode *i*, in terms of an R^2 measure:

$$\upsilon_i \quad \equiv \quad 1 - \rho[\, P_i(RUF_{i,U}), 1 - P_i(RUF_{i,L})\,]^2 \tag{11}$$

where
$$RUF_{i,U}(t) \quad = \quad RUF_f \quad + \quad \Delta_t \quad ,$$
$$RUF_{i,L}(t) \quad = \quad RUF_f \quad - \quad \Delta_t \,, \quad t = 1,..,T \quad , \tag{12}$$

and ρ is the partial correlation coefficient, with Δ_t corresponding to a small increment away from the inflection point. The length of the domain has to be defined in such a way that one just reaches the lower and upper limits of the reaction function.

The υ_i measure is therefore comprised between 0 and 1: it is equal to zero if there is no asymmetry, as occurs when $\lambda_i = 1$, and equal to 1 if the lower part does not relate at all with the upper part of the reaction function.

Of course, the computation of the asymmetry measure (11) for a certain mode is conditional upon the RUF values of the other modes: for instance, sample average values are used to compute these measures in the empirical example reported in Section 4.

3.NUMERICAL AND STATISTICAL ISSUES

3.1 DATA BASE

Our empirical model is a variant of Cléroux *et al.* (1981), the ULE version of which is:

$$V_{bus} = \beta_1^1 TZIMP1 + \beta_2^1 TFBUS + \beta_3^1 AUTIM1 + \beta_4^1 TAUT + \beta_5^1 OINC + \beta_6^1 OCAR + \beta_7^1 MEN1 + \beta_8^1$$
$$V_{car} = \beta_1^2 TZIMP1 + \beta_2^2 TFBUS + \beta_3^2 AUTIM1 + \beta_4^2 TAUT + \beta_5^2 OINC + \beta_6^2 OCAR + \beta_7^2 MEN1 + \beta_8^2 \qquad (13)$$

where the variables and their sample characteristics are defined in Table 2. The data come from a 1976 origin-destination survey (20% of households) in Winnipeg. The calibration is carried out using work trips during the 7:30-8:30 AM peak period. The origin-destination pairs were chosen according to two criteria: that there should be trips by car and by bus, and that the total numbers of trips by the two modes be greater than 60. There are 211 pairs which satisfy those two criteria.

Table 2. Averages and Variances of the Regression Variables for the Winnipeg Data Base 1976, 211 O-D Pairs

DEFINITION	CODE	AVERAGE	VARIANCE
bus share		0.37	0.04
Bus Network Variables (N_{bus})			
• transit impedance time = [(3(access, egress, waiting time) + travel time)]	TZIMP1	76.91	575.51
• bus price ($)	TFBUS	0.25	0.00
Car Network Variables (N_{car})			
• road travel time (minutes)	AUTIM1	20.92	91.62
• car cost ($)	TAUT	0.53	0.08
Socioeconomic variables (S)			
• average income/household in the origin zone ($/year)	OINC	16355.13	15535537.00
• average car ownership/household in the origin zone	OCAR	0.96	0.04
• proportion of men travelling at the peak hour	MEN1	0.61	0.03

3.2 ESTIMATION PROCEDURE

3.2.1 Log-Likelihood function

In order to define the log-likelihood function, we randomize the Lin-IPT-L model by associating to each Ψ_{it} term a random term ε_{it} pertaining to the share *i* for the O-D pair *t*. The statistical Lin-IPT-L model is then defined as

$$P_i \quad = \quad \Psi_{it} \exp(\varepsilon_{it}) \quad / \quad \left(\sum_{j=1}^{2} \Psi_{jt} \exp(\varepsilon_{jt}) \right) \quad , \text{ where} \qquad (14)$$

$$\Psi_i \quad = \quad (\lambda_i \exp(V_{it}) + 1)^{(1/\lambda_i)} + \mu_i, \qquad \lambda_i \neq 0$$

$$\quad = \quad \exp(\exp(V_{it})) + \mu_i, \qquad \lambda_i \to 0 \qquad (15)$$

$$\text{with} \quad \mu_i \geq -1 \quad , \text{ and } \lambda_i \geq 0. \qquad (16)$$

Assuming that each random term is normally distributed with constant variance, the log-likelihood function (L) corresponding to (14) can be written for our bimodal case as:[7]

$$\mathrm{L} \quad = \quad - \quad \frac{1}{2} \sum_{t=1}^{T} \left(\ln\left(\frac{P_{1t}}{P_{2t}} \right) - \Psi_{1t} + \Psi_{2t} \right)^2 \quad - \quad \frac{T}{2} \ln(2\pi) \quad . \qquad (17)$$

The maximization of (17) was done with the SHARE program (Liem *et al.* 1993) which uses the Davidon-Fletcher-Powell algorithm (Fletcher and Powell 1963).

3.2.2 Numerical constraints

In order to insure that no numerical problems arise during the estimation of the Lin-IPT-L model (14)-(16), several numerical constraints are needed over and above the theoretical constraints (16).

Constraints against underflow. Very small V_{it} may cause problems related to the computation of the exponential function and the computation of the inverse Box-Cox transformation. We need,

- $V_{it} \quad > \quad B_I \quad ,$ (18)
- $\Psi_{it} \quad > \quad \mu_i + 1 \quad .$ (19)

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[7] The reader should keep in mind that the log-likelihood function in (17) is valid only with two modes. When more than two modes are present a covariance matrix has to be included in the log-likelihood function. A full discussion on that matter is contained in Laferrière (1993).

Constraint (18) is meant to guarantee that the computation of the exponential function is above the inferior limit (B_I) of the computer. Constraint (19) guarantees that the inverse Box-Cox transformation is numerically meaningful.

Constraints against overflow. Reciprocally, very high V_{it} may cause problems when computing the exponential function or the inverse Box-Cox transformation. Indeed the argument of the exponential function should be smaller than the superior limit (B_S) of the computer,

- $V_{it} \quad < \quad B_S \quad .$ (20)

The inverse Box-Cox transformation is numerically meaningful if the addition of the number one to the base is numerically significant, namely if

- $(\lambda_i \exp(V_{it}))^{(1/\lambda_i)} \quad < \quad (\lambda_i \exp(V_{it}) + 1)^{(1/\lambda_i)} \quad .$ (21)

Note that eq. (21) must be tested only with positive values of V_{it} , since it will always hold when $V_{it} < 0$ and its computation with negative values may lead to underflow problems.

Finally, the last constraint prevents the result of the inverse Box-Cox transformation from going beyond the numerical capacity of the computer:

- $(\lambda_i \exp(V_{it}) + 1)^{(1/\lambda_i)} \quad < \quad \exp(B_S) \quad ,$ (22-A)

which may also be applied after a logarithmic transformation

- $\ln(\lambda_i \exp(V_{it}) + 1)/\lambda_{it} \quad < \quad B_S \quad .$ (22-B)

We did not meet any particular problems of maximization, which leads us to believe that the log likelihood is well behaved. However, the presence of constraints limits the usefulness of any plot of the function since the constraints produce local maxima. To clarify this point, we now proceed to a study of the distribution of the parameter estimates, in order to focus the analysis on the purpose of estimation: unbiased parameter estimates.

3.3 A MONTE CARLO STUDY

As the computational burden for the estimation of the Lin-IPT-L model is relatively high, a better understanding of the properties of the estimates is in order. For this reason, we decided to perform a Monte Carlo study. Instead of using synthetic data, the true model is based on a subset of variables contained in the Winnipeg data set: the variables *TAUT*, *TFBUS*, and *OINC* defined in Table 2 were left out. The use of real data is intended to reduce the inherent arbitrariness of a Monte Carlo study and at the same time provide a better understanding of the empirical results presented in the next section. The random term (ε_{it}) used to generate samples is assumed to be normally identically distributed with a standard error equal to 0.2 . Table 3 contains the true values and average values for the β , λ , μ parameters and the η , computed according to (4).

Table 3
Monte Carlo Results for all Coefficients and Elasticities (1000 replications)

	TRUE VALUE	AVERAGE	STANDARD DEVIATION		TRUE VALUE	AVERAGE	STANDARD DEVIATION
β_1^1	3.900	6.195	5.581	η_1^1	1.637	1.527	0.410
β_2^1	-0.020	-0.032	0.050	η_2^1	-0.577	-0.576	0.053
β_3^1	0.002	-0.009	0.113	η_3^1	0.144	0.145	0.031
β_4^1	-1.140	-1.387	4.047	η_4^1	-0.543	-0.548	0.055
β_5^1	-1.050	-0.960	4.827	η_5^1	-0.325	-0.323	0.037
β_1^2	-1.000	-1.531	4.334	η_1^2	-1.742	-1.650	0.447
β_2^2	0.003	0.002	0.042	η_2^2	0.614	0.621	0.056
β_3^2	-0.015	-0.037	0.095	η_3^2	-0.153	-0.156	0.033
β_4^2	0.500	1.109	3.305	η_4^2	0.578	0.592	0.060
β_5^2	0.500	1.478	4.154	η_5^2	0.346	0.349	0.041
λ_1	2.150	4.851	7.847	μ_1	-0.900	-0.358	2.596
λ_2	0.500	3.758	15.906	μ_2	-0.700	1.100	6.880

Low forecast errors despite biased parameter estimates. The true values of the β's were obtained from an estimation after having decided on true values for the λ and μ parameters. As shown in Table 3, the estimated values of these parameters (β , λ , μ) are biased. For instance, the average values for the estimated β_4^2 and β_5^2, 1.109 and 1.478, are more than twice as large as the true values, 0.5. The biases are even more important for the λ and μ parameters. Furthermore, the standard deviations of these estimates suggest that they have very large distributions.

Figure 3 shows the distributions of the estimated parameters. The parameters associated with OCAR (β_4^1 , β_4^2) and MEN1 (β_5^1 , β_5^2) have much wider distribution than the others β's . This is not entirely surprising since the variances of OCAR and MEN1 are smaller than those of the two other variables (see Table 2), thus reducing the precision of the estimates.

On the basis of these results, one could question the appropriateness of estimating the Lin-IPT-L model. Firstly, it should be said that, despite these biases, the predictive capability of the Lin-IPT-L model is very reasonable as the average forecast error[8] is only 0.0014 . This suggest that some compensations take place among parameters, notably across shares; a high value of λ_1 may be offset by a correspondingly high value of λ_2 and/or high values for the β parameters.

Unbiased elasticities as proper model output. As Equation (4) indicates, the computation of the elasticities involves all the parameters of the model. Clearly, if the parameters are offsetting one another, this should be revealed by the elasticities. Except for the constant terms, the estimated elasticities are unbiased as shown in Figure 4. Moreover, the distributions of the estimated elasticities are relatively small: more than 10 times smaller than the true parameters in the case of elasticites related to variables TZIMP1, OCAR and MEN1 and 5 times smaller than the true parameters in the case of elasticities η_3^1 and η_3^2 related to variable AUTIM1. Generally speaking, the standard deviations of all the estimated elasticities are about 5 times smaller than that of the residuals (0.2). Despite the appearances, the distributions of the elasticities do not follow a normal distribution according to the Kilmogoroff-Smirnoff test. After an examination of (4), the opposite would have been surprising.

We therefore argue on the basis of unbiased results for elasticities that the estimation of the Lin-IPT-L model is certainly relevant and appropriate. With a very nonlinear model, the moments of the distributions for the estimated parameters (β's , λ's and μ's) are far from being obvious

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[8] This is defined as the average over the observations in the sample of the absolute value of the difference between the observed share and the expected value of the predicted share ($\overline{|\, p_{it} - E(\hat{p}_{it})|}$).

and to some extent are not so relevant. In our mind, it seems more interesting to have good distributions for the elasticities, which are, after all, the most meaningful model coefficients. One way to understand this result is to remember that the share format makes it easy for some parameters to behave as close substitutes to others -to "offset" them-, even if all parameters are strictly identified.

A question arises as to wether the λ and μ parameters are dependent on units of measurement. Indeed, we have shown elsewhere that the direct Box-Cox transformation (Gaudry and Laferière, 1987) is invariant to a power transformation of the variable or of the function to which it is applied. In this case however, the reader should note that issue does not arise because the V_i are without units of measurement.

Figure 3. Empirical Distribution of the Parameters

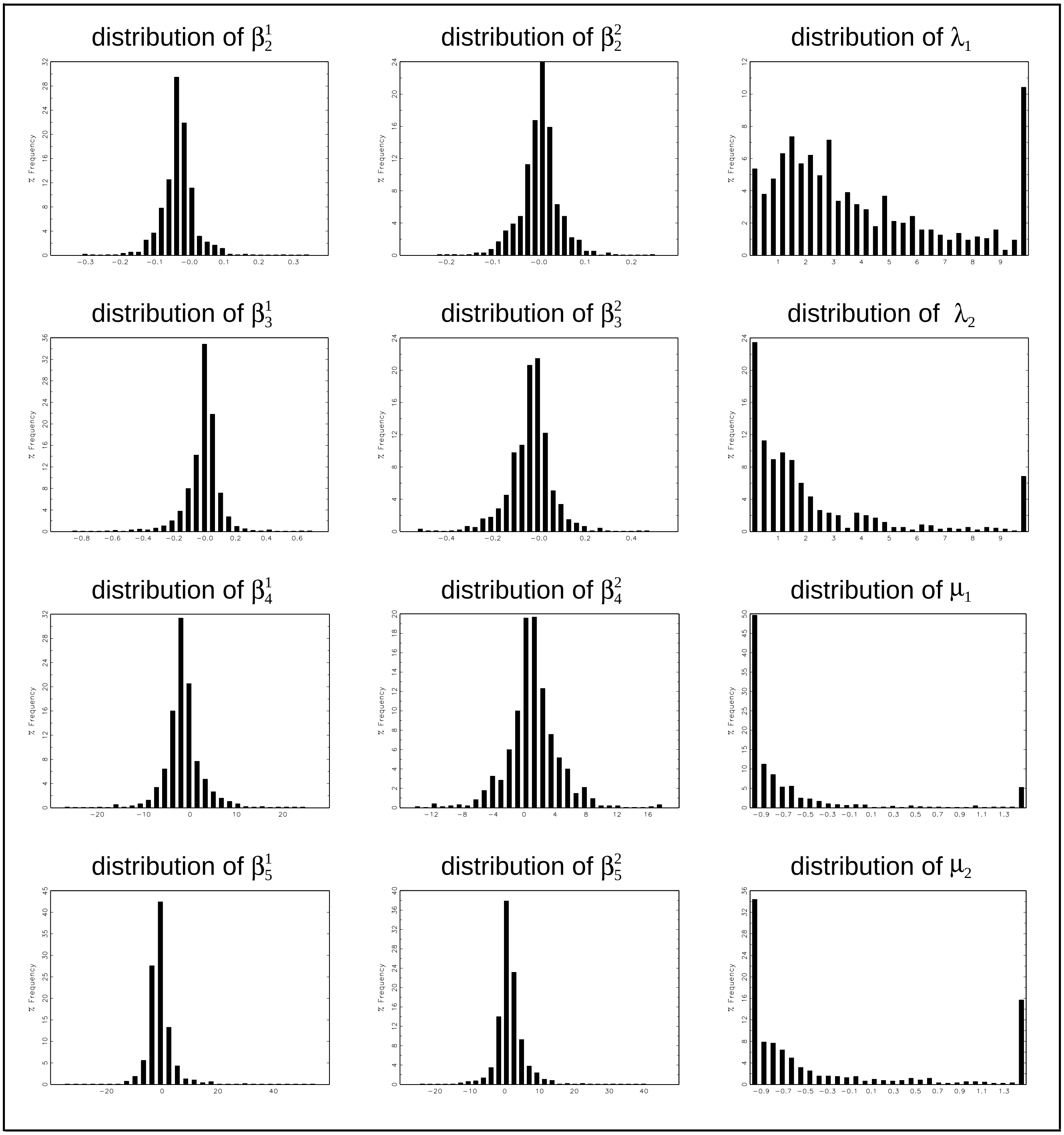
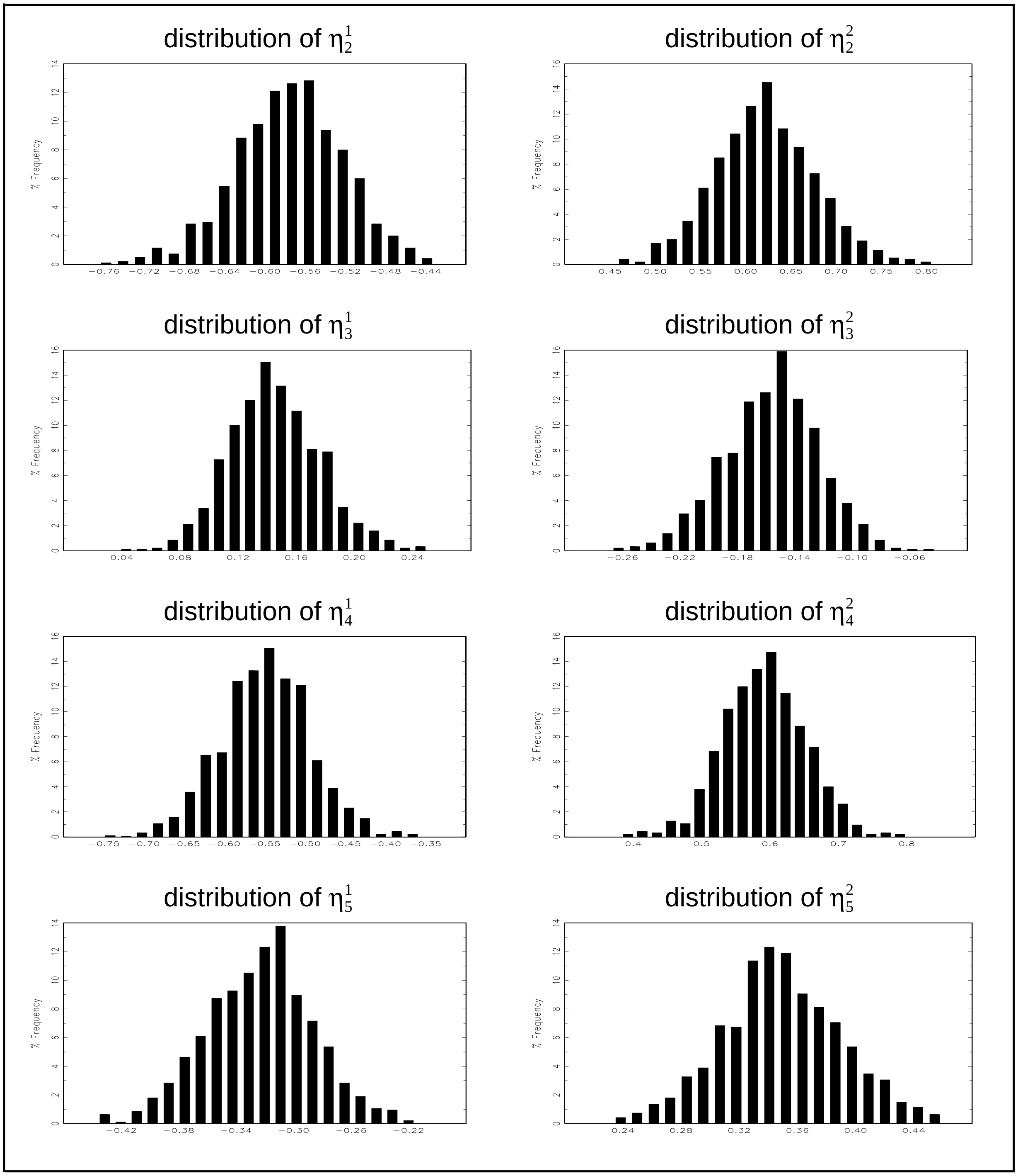


Figure 4. Empirical Distribution of the Elasticities



Model misspecification. What are the consequences of estimating a Logit model if the true model is a Universal Extended Logit model? The results of the estimation of a Logit model are presented in Table 4. Clearly, wrongly assuming Logit values for the λ and μ parameters yields major biases for the elasticities: overestimation for the first mode elasticities and underestimation for the second mode elasticities.

Table 4. Monte Carlo Results with the Logit Model (1000 replications)

	TRUE VALUE	AVERAGE	STANDARD DEVIATION		TRUE VALUE	AVERAGE	STANDARD DEVIATION
β_1^1	3.900	1.8122	0.138	η_1^1	1.637	1.772	0.135
β_2^1	-0.020	-0.018	0.002	η_2^1	-0.577	-1.337	0.138
β_3^1	0.002	0.000	....	η_3^1	0.144	0.513	0.086
β_4^1	-1.140	-3.019	0.164	η_4^1	-0.543	-2.837	0.154
β_5^1	-1.050	-3.041	0.178	η_5^1	-0.325	-1.801	0.105
β_1^2	-1.000	0.000	....	η_1^2	-1.742	-0.041	0.003
β_2^2	0.003	0.000	....	η_2^2	0.614	0.031	0.003
β_3^2	-0.015	-0.025	0.004	η_3^2	-0.153	-0.012	0.002
β_4^2	0.500	0.000	....	η_4^2	0.578	0.065	0.004
β_5^2	0.500	0.000	....	η_5^2	0.346	0.041	0.002
λ_1	2.150	1.000	....	μ_1	-0.900	-1.000	....
λ_2	0.500	1.000	....	μ_2	-0.700	-1.000	....

4. A BIMODAL URBAN APPLICATION

4.1 STATISTICAL RESULTS

As mentionned in Section 2, the Lin-IPT-L model allows asymmetry of the reaction function, modal captivity and the enlargement of the representative utility function of each mode. We now proceed to test whether or not those three dimensions appear, and whether they interact, using model (13). The statistical test on which all the tests in the present subsection are based on is the likelihood ratio test, namely that twice the change in the log-likelihood function follows a chi squared distribution with n degrees of freedom. We will use in the forthcoming discussion a confidence level of 95%.

Testing asymmetry. Clearly, the asymmetry of the reaction curve may depend upon the assumption made on the captivity level or the specification of the RUF. For instance, one may obtain strong evidence of asymmetry if no modal captivity is assumed but symmetry might prevail if modal captivity is allowed to be detected. Therefore, we choose to test asymmetry unconditionally upon any specific assumptions regarding modal captivity. For this reason, all cases presented in Section 1 of Table 5 involve the estimation of the two μ captivity parameters. The most general case is denoted as unconstrained asymmetry. One interesting special case is that of identical asymmetry. The IPT model implies that if λ parameters are all equal but different from 1, then all reaction functions are identically asymmetric. We refer to this case as constrained asymmetry.

It could further be argued that our tests should pertain only to the ULE specification of the RUF because it is the most general. However, because of the importance of the Alternative-Specific Logit in the transportation litterature we have decided to test also for the presence of asymmetry with this specification.

In Section 1 of Table 5, constrained and unconstrained asymmetry are both rejected with the Alternative-Specific Logit specification. Similarly, constrained asymmetry is rejected with the Alternative-Specific Universal Logit Extended. Interestingly enough, only unconstrained asymmetry with the Alternative-Specific Universal Logit Extended specification cannot be rejected. This suggests that asymmetry takes place only if some form of interaction is allowed among β, λ, and μ parameters. A more complete treatment of this issue is given below.

Testing modal captivity. As mentionned above, modal captivity depends on assumptions concerning the presence of asymmetry and the specification of the RUF. Thus, a meaningful test of modal captivity ought to be unconditional, notably with respect to specific assumptions regarding asymmetry. For this reason, all cases presented in Section 2 of Table 5 involve the estimation of the two λ asymmetry parameters. The most general case considered is denoted as unconstrained modal captivity. Constrained modal captivity denotes identical modal captivity for all modes, or μ parameters that are all equal and differ from -1.

In Section 2 of Table 5, constrained and unconstrained modal captivity cannot be rejected with the Alternative-Specific Logit specification. However, constrained and unconstrained modal captivity are both rejected with the more general specification of the RUF (A-S, ULE). This suggests that modal captivity does not show up when interactions between β and λ parameters is allowed. A more complete treatment of this issue is given below.

Testing the IIA assumption. Even with only two available modes, it is possible to test the IIA asumption with the approach suggested in Subsection 2.1.3 above. This is done by testing whether or not the coefficients $\beta_3^1, \beta_4^1, \beta_1^2, \beta_2^2$ in (13) are simultaneously significantly different from zero. In our bimodal case, rejection of the IIA assumption means that there would be a differentiated reaction from existing modes if a new mode were introduced.

As above, this test is performed with no *a priori* assumptions regarding asymmetry or modal captivity: λ and μ parameters are estimated. Furthermore, it is performed before and after extending the RUF with socioeconomic variables.

When the RUF are not extended with socioecomic variables (comparison of columns (1) and (2) in Section 3 of Table 5), the IIA assumption is rejected. The same conclusion is reached with the extended version (comparison of columns (3) and (4) in Section 3 of Table 5). The usefulness of the enriched specifications of the RUF is thus established.

Specificity and interaction effects. In logit models, modal specificity is naturally defined solely in terms of constraints on the β's. However, specificity in the Lin-IPT-L model may not be due solely to the β parameters but could result from the λ or μ parameters as well. Therefore, it might be interesting to know if specificity is brought about by asymmetric reaction functions and/or by modal captivity and/or by the enriched RUF.

Let us start by looking at the specificity produced by *asymmetry*. In Section 4 of Table 5, this can be tested with 4 different comparisons: assuming a Generic Universal Logit Extended specification and a constrained (column 1 vs column 3) or unconstrained (column 2 vs column 4) modal captivity;

or assuming an Alternative-Specific Universal Logit Extended specification and a constrained (column 5 vs column 7) or unconstrained (column 6 vs column 8) modal captivity. In all these cases, the alternative-specific (unconstrained) asymmetry assumption cannot be rejected.

From these comparisons it appears that specificity produced by asymmetry induced a smaller increase of the log-likelihood function with a Generic RUF than an Alternative-Specific RUF. That is, a positive interaction takes place among β's and λ's parameters. This result is consistent with what we found above when testing asymmetry with models that differed in number of β parameters considered: the freer the β's, the more asymmetry can be detected.

The alternative-specificity resulting from modal *captivity* may also be tested in 4 ways: assuming a Generic Universal Logit Extended specification and a constrained (column 1 vs column 2) or unconstrained (column 3 vs column 4) asymmetry; or assuming an Alternative-Specific Universal Logit Extended specification and a constrained (column 5 vs column 6) or unconstrained (column 7 vs column 8) asymmetry.

From all these cases, alternative-specific modal captivity assumption is rejected only with the last case: the μ's parameters do not add any specificity to modes if asymmetry and RUF are fully alternative-specific. This result is consistent with the absence of modal captivity that we found above -namely that a model with more β's exhibits less captivity- but only if the asymmetry parameters are unconstrained: if they are constrained, the generic model (i.e. the model with restrictions on the β's) contains less specific captivity than the alternative-specific model. This suggests that asymmetry has a powerful effect on captivity, as the graph in Fig. 2 implies: specific captivity can be increased by requiring identical asymmetry for each mode, because a different curvature of the reaction function strongly affects the "tail" needed to account for the observations.

By comparing each column from the left-hand side of Table 5, Section 4 with the corresponding one in the right-hand side provides tests for alternative-specific β's *coefficients*. For all these cases, we cannot reject the assumption that β's coefficients are alternative-specific.

Each new dimension of the Linear-IPT-L model does not support mode specificity in the same way: mode-specific β's are alway supported, mode-specific asymmetry parameters are supported in all specifications that use an enriched RUF, but mode-specific captivity tends to be rejected in very general models unless its "substitute" dimension, asymmetry, is restricted. There are therefore interaction among the three new dimensions.

Table 5. Statistical Results: Log-likelihood Values and Degrees of Freedom (in parentheses)

Section 1. Asymmetry Results

Alternative-Specific Logit (A-S, L) and unconstrained modal captivity (μ_1, μ_2)			Alternative-Specific Universal Logit Extended (A-S, ULE) and unconstrained modal captivity (μ_1, μ_2)		
1 Symmetry ($\lambda_1 = \lambda_2 = 1$)	2 Constrained Asymmetry ($\lambda_1 = \lambda_2$)	3 Unconstrained Asymmetry (λ_1, λ_2)	4 Symmetry ($\lambda_1 = \lambda_2 = 1$)	5 Constrained Asymmetry ($\lambda_1 = \lambda_2$)	6 Unconstrained Asymmetry (λ_1, λ_2)
-228.626 (0)	-226.935 (1)	-226.869 (2)	-219.272 (0)	-218.660 (1)	-213.297 (2)

Section 2. Modal Captivity Results

Alternative-Specific Logit (A-S, L) and unconstrained asymmetry (λ_1, λ_2)			Alternative-Specific Universal Logit Extended (A-S, ULE) and unconstrained asymmetry (λ_1, λ_2)		
1 No Modal Captivity ($\mu_1 = \mu_2 = -1$)	2 Constrained Modal Captivity ($\mu_1 = \mu_2$)	3 Unconstrained Modal Captivity (μ_1, μ_2)	4 No Modal Captivity ($\mu_1 = \mu_2 = -1$)	5 Constrained Modal Captivity ($\mu_1 = \mu_2$)	6 Unconstrained Modal Captivity (μ_1, μ_2)
-232.165 (0)	-230.525 (1)	-226.869 (2)	-215.273 (0)	-214.171 (1)	-213.297 (2)

Section 3. IIA Results

Unconstrained Asymmetry and Modal Captivity ($\lambda_1, \lambda_2, \mu_1, \mu_2$)			
1 Alternative-Specific Logit (A-S, L)	2 Alternative-Specific Universal Logit (A-S, UL)	3 Alternative-Specific Logit Extended (A-S, LE)	4 Alternative-Specific Universal Logit Extended (A-S, ULE)
-226.869 (0)	-219.898 (3)	-222.670 (0)	-213.297 (3)

Section 4. Alternative-Specificity Results

Generic Universal Logit Extended (G, ULE)				Alternative-Specific Universal Logit Extended (A-S, ULE)			
Constrained Asymmetry ($\lambda_1 = \lambda_2$)		Unconstrained Asymmetry (λ_1 , λ_2)		Constrained Asymmetry ($\lambda_1 = \lambda_2$)		Unconstrained Asymmetry (λ_1 , λ_2)	
1 Constrained Modal cap-tivity ($\mu_1 = \mu_2$)	2 Unconstrained Modal captivity (μ_1 , μ_2)	3 Constrained Modal cap-tivity ($\mu_1 = \mu_2$)	4 Unconstrained Modal captivity (μ_1 , μ_2)	5 Constrained Modal cap-tivity ($\mu_1 = \mu_2$)	6 Unconstrained Modal captivity (μ_1 , μ_2)	7 Constrained Modal cap-tivity ($\mu_1 = \mu_2$)	8 Unconstrained Modal captivity (μ_1 , μ_2)
-242.476 (0)	-235.253 (1)	-234.877 (1)	-231.660 (2)	-231.758 (6)	-218.660 (7)	-214.171 (7)	-213.297 (8)

4.2 ELASTICITY RESULTS FOR SEVERAL MODELS

Table 6 contains results presented in three sections. The first section lists elasticities of the modal share, defined by equation (4). Under each elasticity, the *t*-statistic of the regression coefficient is shown in parentheses. The second section contains estimated values for λ and μ parameters and *t*-statistics with respect to 1 for λ parameters and -1 for μ parameters. The third section contains general statistics, notably the log-likelihood and the number of observations and the number of estimated parameters, as well as the asymmetry measure defined by Equation (11).

As our objective here is not to discuss the specifics of the model results but to illustrate the characteristics of the IPT-Linear Logit with a real example, our comments will be selective.[9]

The first column in Table 8 corresponds to elasticities with the Logit model. In the three next columns, λ and μ parameters are estimated separately and then together. The estimation of the Universal Logit Extended is presented in the last column.

Introducing asymmetry and modal captivity (column 4) affects elasticities in a non trivial way. For instance, it makes Alternative 2 more elastic with respect to socioeconomic variables (OINC, OCAR, MEN1) and this is robust to a "Universal Extended" version of the RUF as indicated in column 5. Furthermore, in general, elasticities in column 4 reveal that both alternatives are more sensitive to network variables. However, this last result is not robust to filling the RUF because elasticities of network variables in column 5 tend to be similar to those in column 1.

As a general observation, the elasticities are not very affected by the model specifications. However one has to be very careful with elasticities evaluated at one point as they may hide many important aspects. Consider Figure 5 Part A, where reaction functions of alternative 2 with respect to the travel costs are drawn based on the (A-S, L) model without asymmetry and modal captivity (col.1) and on the (A-S, L) model with asymmetry and modal captivity (col.4). Even if both models yield the same elasticity (-0.25) of alternative 2 with respect to variable TAUT, they both imply very different reaction functions of a 50% reduction and a 100% increase of TAUT with respect to its mean (shown in Table 2).

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[9] The model presented in Table 6 is a variant of a model presented in Cléroux *et al.* (1981). The results shown in Table 6 should not be interpreted as substitute models to Cléroux *et al.* (1981): they are aimed at illustrating the usefulness of the IPT-Linear Logit generalizations.

Table 6
Logit and Universal Logit Extended Comparisons with the Lin-IPT Model

ELASTICITIES	1	2	3	4	5
	A-S,L	A-S,L	A-S,L	A-S,L	A-S,ULE
	$\lambda_1 = \lambda_2 = 1$	λ_1 , λ_2	$\lambda_1 = \lambda_2 = 1$	λ_1 , λ_2	λ_1 , λ_2
	$\mu_1 = \mu_2 = -1$	$\mu_1 = \mu_2 = -1$	μ_1 , μ_2	μ_1 , μ_2	μ_1 , μ_2
ALTERNATIVE 1 (Bus)					
Transit impedance time (TZIMP1)	-0.932	-1.455	-1.066	-1.068	-0.618
	(-5.17)	(-4.84)	(-4.93)	(-3.00)	(-1.15)
Bus price (TFBUS)	-0.243	-0.217	-0.387	-0.702	-0.120
	(-5.13)	(-5.06)	(-2.76)	(-1.14)	(-1.55)
Road travel time (AUTIM1)	0.098	0.071	0.347	0.366	0.061
	(-0.65)	(-0.79)	(-2.78)	(-1.22)	(-1.02)
Car cost (TAUT)	0.519	0.299	0.287	0.392	0.474
	(-5.13)	(-5.06)	(-2.76)	(-1.14)	(0.20)
Average income/household in the origin zone (OINC)	-0.528	-0.921	-0.555	-0.489	-0.486
	(-1.64)	(-1.86)	(-1.89)	(-1.49)	(0.23)
Average car ownership/household in the origin zone (OCAR)	-0.885	-1.144	-0.782	-0.904	-0.841
	(-2.72)	(-2.39)	(-2.47)	(-2.14)	(-0.55)
Proportion of men travelling at the peak hour (MEN1)	-0.706	-1.107	-0.697	-0.748	-0.685
	(-4.97)	(-3.48)	(-5.07)	(-2.68)	(-0.92)
Constant	3.311	5.509	3.165	3.157	20.347
	(10.81)	(3.64)	(6.98)	(0.72)	(0.95)
ALTERNATIVE 2 (Auto)					
Road travel time (AUTIM1)	-0.048	-0.001	-0.204	-0.238	-0.044
	(-0.65)	(-0.79)	(-2.78)	(-1.22)	(-1.41)
Car cost (TAUT)	-0.256	-0.006	-0.168	-0.255	-0.338
	(-5.13)	(-5.06)	(-2.76)	(-1.14)	(-1.55)
Transit impedance time (TZIMP1)	0.459	0.033	0.626	0.694	0.442
	(-5.17)	(-4.84)	(-4.93)	(-3.00)	(-1.25)
Bus price (TFBUS)	0.119	0.005	0.228	0.456	0.086
	(-5.13)	(-5.00)	(-2.76)	(-1.14)	(0.20)
Average income/household in the origin zone (OINC)	0.260	0.021	0.326	0.318	0.347
	(-1.64)	(-1.86)	(-1.89)	(-1.49)	(1.12)
Average car ownership/household in the origin zone (OCAR)	0.436	0.026	0.460	0.587	0.601
	(-2.72)	(-2.39)	(-2.47)	(-2.14)	(1.09)
Proportion of men travelling at the peak hour (MEN1)	0.348	0.025	0.410	0.486	0.490
	(-4.97)	(-3.48)	(-5.07)	(-2.68)	(0.74)
Constant	0.000	0.000	0.000	0.000	-1.402
					(-0.38)
EXTRA PARAMETERS					
λ_1	1.000	2.800	1.000	93.000	15.000
		(1.27)		(0.22)	(0.54)
λ_2	1.000	0.000	1.000	190.000	0.990
		(....)		(0.17)	(-0.00)
μ_1	-1.000	-1.000	-1.000	-0.990	-0.940
			(0.00)	(-0.21)	(-0.43)
μ_2	-1.000	-1.000	-0.890	-0.990	-0.750
			(-3.02)	(-0.23)	(-0.43)
GENERAL STATISTICS					
Log-likelihood	-234.71	-232.16	-228.62	-226.86	-213.29
Number of parameters estimated	7	9	9	11	18
Asymmetry measure υ_1	0.00	0.18	0.00	0.32	0.55
υ_2	0.00	0.01	0.00	.046	0.00

Another warning pertains to the expected sign of the elasticities. The elasticity of alternative 1 with respect to TZIMP1 with the most simple model (-.93 in column 1) and the most complex model (-.62 in column 5) both have the expected sign. Neither do the reaction functions depicted in Figure 5, Part B, indicate any specific problems: both are negatively sloped. However, there are 12 observations in the sample that yield positive elasticities. This is possible because the coefficient of the TZIMP1 variable in the RUF of alternative 2 has the "wrong" sign (see Table 7). It is not because elasticities and reaction functions evaluated with averages are as expected that this carries to each observation of the sample.

More to the point now, consider Figure 5, Part C, that shows a reaction function having a negative slope if AUTIM1 is smaller than 27 minutes and a positive slope otherwise. Own and cross elasticities of AUTIM1 in Table 8 have the expected sign. However, in the sample there are 79 observations with wrong signs for own and cross elasticities.

In the extra parameter section of Table 6, we note that the t-statistics suggest that the estimated asymmetry and captivity parameters are generally not different from 1: this may be caused by the approximation that we used (Berndt *et al.* 1974) and in no way invalidates the results and conclusions derived from the likelihood ratio test.

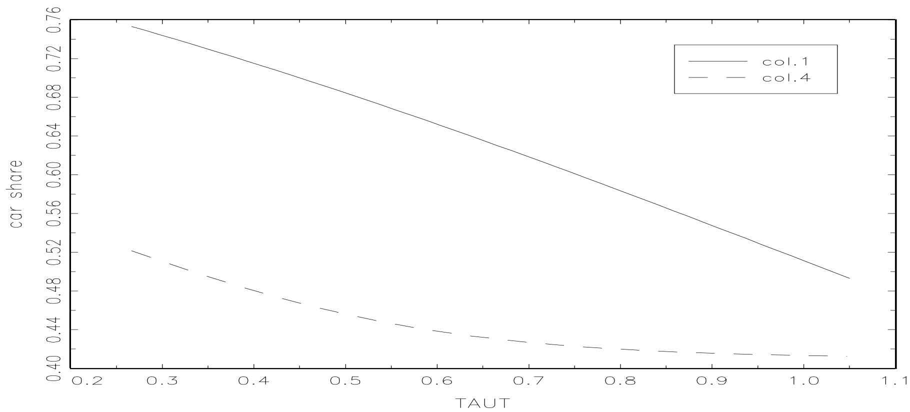
Finally, the reaction function of the bus mode with the (A-S, ULE) model is presented in Figure 6. It is interesting to see that the reaction function is really asymmetric as the asymmetry measure ($\upsilon_i = 0.55$) suggests. The continuous part of the curve is to the left of the inflection point.

Table 7
Coefficient Values with
the Universal Logit Extended Model

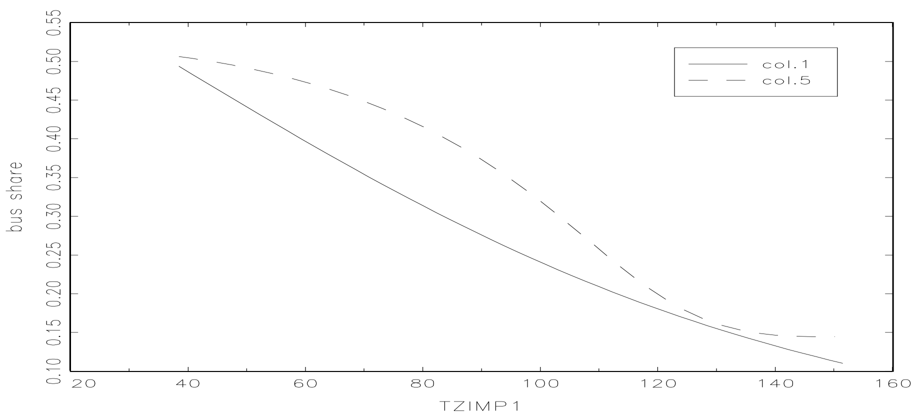
	TZIMP1	AUTIM1	TFBUS	TAUT	OINC	OCAR	MEN1
RUF_1	-0.11606	-0.16155	-2.37899	0.59568	0.00005	-2.24133	-6.85778
RUF_2	-0.01563	-0.06280	0.59568	-2.37899	0.00010	1.79417	0.97799

Figure 5. Reaction Functions in the Neighbourhood of the Mean for Selected Variables and Models

A) Car share reaction to car cost



B) Bus share reaction to transit time



C) Car share reaction to car time

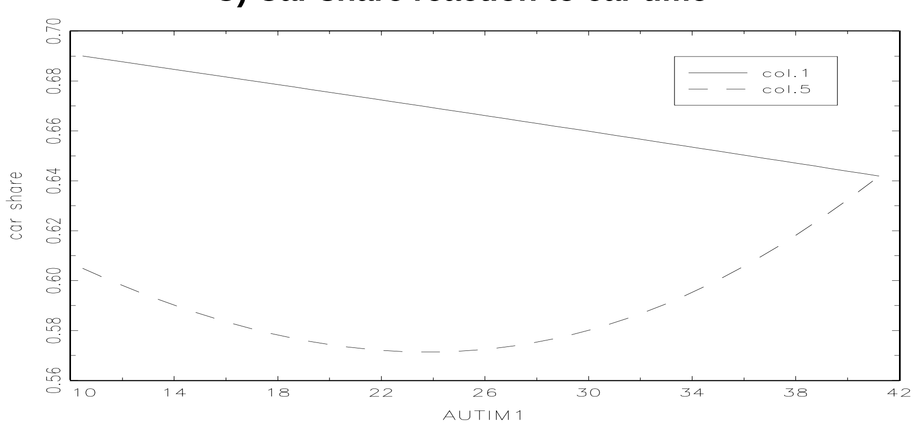
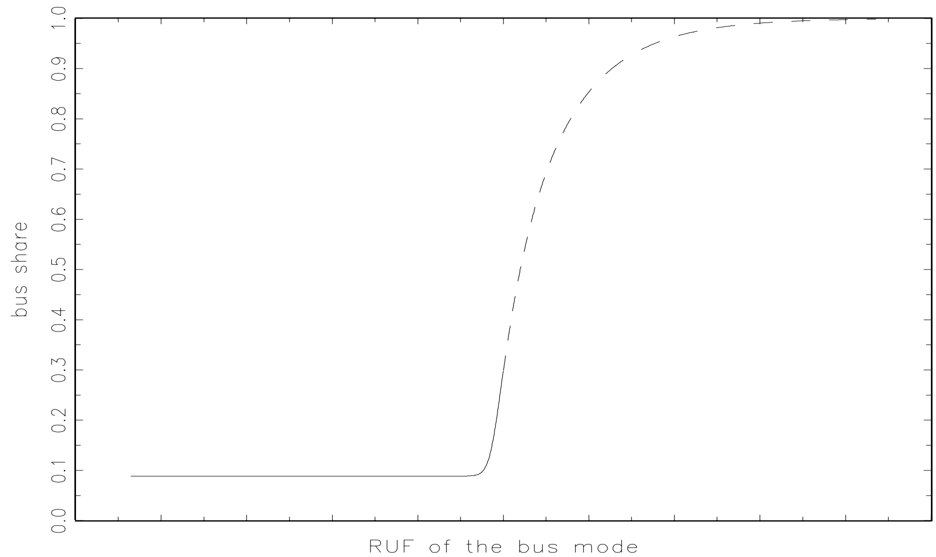


Figure 6
Reaction Function of the Bus Share



5.CONCLUSION

In this paper, we study the empirical and numerical properties of one member of the IPT class, the Linear IPT-Logit model. This models makes it possible to deal with three very practical problems of mode choice modelling: asymmetric response effects, modal captivity and consistency with the independence from irrelevant alternatives axiom (IIA), which excludes the possibility of complementarity among alternatives.

In our analysis, a completely full form of the utility functions is specified and estimated, seemingly for the first time in mode choice analysis. Furthermore, the interactions among such enlarged specifications of utility functions, asymmetry of the reaction functions and modal captivity are analyzed. Statistical tests are carried out on a binomial case using urban data from Winnipeg in order to examine both the theoretical properties of estimators and the empirical gains of the Linear IPT Logit model over its simpler linear Logit root. All three additional dimensions are found to be of practical use.

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