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A DISAGGREGATE BOX-COX LOGIT MODE CHOICE MODEL OF INTERCITY PASSENGER TRAVEL IN GERMANY AND ITS IMPLICATIONS FOR HIGH SPEED RAIL DEMAND FORECASTS

by

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ABSTRACT

We show that enriching logit mode choice model specification by mode attributes, socio economic variables and trip purpose characteristics significantly improves model quality, and that Box-Cox transformations applied to model attributes imply an asymmetry of the reaction curve, as well as more reasonable properties (diminishing marginal values of time savings, elasticities and values of time that differ among the modes) than those of the linear logit model. Moreover, the model yielded very different high speed rail market shares for Germany than results obtained with the usual linear utility functions.

Keywords: LINEAR LOGIT, BOX-COX LOGIT, VALUE OF TIME, ELASTICITY, INTERCITY MODE CHOICE, FEDERAL REPUBLIC OF GERMANY, HIGH SPEED RAIL, FORECASTS.

ZUSAMMENFASSUNG

Wir haben gezeigt, daβ durch die Anreicherung der Modellspezifikation mittels verkehrsträgerspezifischen, sozioökonomischen und reisespezifischen Variablen eine erhebliche Verbesserung der Modellgüte zu erreichen ist, sowie durch die Box-Cox Transformation der verkehrsträgerspezifischen Variablen ein asymmetrischer Verlauf der Reaktionskurve vorliegt. Die dabei erzielten Ergebnisse basieren auf besseren realitätsnahen Modelleigenschaften (“diminishing marginal values of time savings”, verkehrsträgerspezifische Elastizitäten und “values of time”) gegenüber den linearen Modellen und die prognostizierten Marktanteile des Verkehrsträgers Schiene in der Bundesrepublik Deutschland differieren in signifikanter Weise von denen, die mittels gebräuchlichen linearen Nutzenfunktionen erzeugt werden.

Stichworte: LINEAR LOGIT, BOX-COX LOGIT, “VALUE OF TIME”, ELASTIZITÄT, INTERCITY MODALWAHLMODELL, BUNDESREPUBLIK DEUTSCHLAND, SPURGEFÜHRTER HOCHGESCHWINDIGKEITS-VERKEHR, PROGNOSEN.

RÉSUMÉ

Nous montrons qu’il est possible d’améliorer de façon significative la qualité des modèles logit du choix du mode de transport en leur ajoutant des caractéristiques reliées aux modes de transport et aux motifs des déplacements, en plus des caractéristiques socioéconomiques des voyageurs. Nous montrons aussi que la présence de non-linéarités dans les fonctions d’utilité représentative des modèles Logit établit clairement que la fonction de réaction des consommateurs comporte un seuil et est asymétrique. Nous trouvons par ailleurs des résultats plus raisonnables (utilité marginale décroissante des économies de temps, élasticités et valeur du temps de transport qui varient entre les modes) que les résultats linéaires et qui impliquent des prévisions de parts de marché des trains à grande vitesse en Allemagne qui diffèrent considérablement des résultats obtenus avec les fonctions d’utilité linéaires habituelles.

Mots-clés : LOGIT LINÉAIRE; LOGIT BOX-COX; VALEUR DE TEMPS; ÉLASTICITÉ; MODÈLE DE CHOIX DU MODE DE TRANSPORT INTERURBAIN; RÉPUBLIQUE FÉDÉRALE D’ALLEMAGNE, TRAINS à GRANDE VITESSE, PRÉVISIONS.

1. INTRODUCTION*

In this paper, we want to study the choice of transport mode which is probably one of the most important issues in transport planning: mode choice affects the general efficiency with which one can travel in urban and inter-urban areas, the amount of space devoted to transport functions, and wether a range of choices is available to travellers. Mode choice analysis is the third step of the classical four-step transport planning process, coming after trip generation, which explains the level of trip-making, and trip distribution, which explains the relative frequency of trip lengths. Mode choice analysis requires information from the fourth step of the process: the assignment stage, or representation of itinerary choices within the networks, and the resulting values of prices and service levels by origin-destination pair. However, mode choice analysis tends to be the decisive step in the evaluation of transport scenarios because the "diversion" effects arising from network service modification typically dominate the effects on total trip making.

1.1. Mode choice embedded in a generation-distribution framework

This is best understood by reminding ourselves that mode choice models are normally embedded in some sort of model explaining total trip making by all modes. It is convenient to postulate, for the sake of discussion, the existence of an aggregate generation-distribution model: this corresponds to frequent practice and the points that should be made about an ideal specification also hold when disaggregate generation-distribution specifications are used.

The basic intent of generation-distribution models is to explain T_{ij}, the travel flow from $\underline{i}$ to $\underline{j}$ in terms of two classes of functions. Some, the A_{ij_a}, refer to activity levels at the origin or destination and others, the U_{ij_m}, represent the utility of the $\underline{m}$ travel modes:

$$T_{ij} = g^d(\{A_{ij_a}\}, \{U_{ij_m}\}) \ , \qquad a = 1, ..., A \ ; \ \ m = 1, ..., M \ . \tag{1}$$

A frequent specification, for the first type, is the geometric mean of values of the activity at the origin and destination, for instance

$$A_{ij_a} \equiv \left[S_{i_a}^{1/2} S_{j_a}^{1/2} \right] \ , \tag{2}$$

where S_a is a socioeconomic variable, such as population, or income.

A frequent specification for the second type is

$$U_{ij_m} = e^{V_{ij_m}} \tag{3}$$

where V_{ij_m} is primarily a function of network variables N_{ij_n}, such as cost, time, distance or "impedance", but also frequently of socioeconomic variables S_a as well.

Because of the complexity of the problem, it is practical to aggregate the modal utilities and define an index

$$U_{ij} = \sum_m e^{V_{ij_m}} \tag{4}$$

* This work is based on research directly financed by the German Ministry of Transportation and benefited indirectly from Marc Gaudry's tenure as an Alexander von Humboldt research prize awardee, as well as from Transport Canada and from National Sciences and Engineering Research Council of Canada funding.

which in effect constitutes the denominator of a logit model, and makes it possible to write the usual form

$$T_{ij} = \beta_o \, A_{ij_1}^{\beta_1} \ldots A_{ij_A}^{\beta_A} \, U_{ij}^{\beta_U} \, u_{ij} \quad , \tag{5}$$

where u_{ij} is an error term. Note that the coefficient β_U is expected to be positive, as an increased utility of travel modes should lead to higher flows, and that the signs of the activity coefficients will depend on the exact specifications used for each activity variable: in a simple model using, say Population, the coefficient of that term should be positive; however, if variables like the proportion of jobs belonging to a particular sector are used, their coefficients may well be negative.

An important subproblem in generation-distribution models is, dropping subscripts, whether the modal utility index U enters multiplicatively or not. The reason for this is that the natural logarithm of the denominator of the logit model, $\ell n[U] = \ell n \left[\sum_m e^{V_m} \right]$, is known to represent the expected value of the maximum utility available to the consumer over all travel modes or alternatives, and is often called the inclusive value or price of the modes.

Over the last 20 years, there has been a steady drift of variables from the first part of (1), which includes socioeconomic variables, to the second part of (2), which uses estimates of a logit model. As a consequence, the mode choice models that in 1975 included only network variables now routinely include dozens of socioeconomic variables that may be more related to total trip making than to mode choice. This has increased the tendency mentioned above for "diversion" effects to dominate total trip making effects, at least since Domencich and McFadden (1975).

1.2. The representative utility function of the logit model

Our emphasis is on the specification of the utility functions, both in terms of contents and mathematical form. In terms of contents, the products of interest are modal trips supplied by the the different carriers (train, plane and car in our case) that offer different characteristics (namely price, time, frequency, number of transfers, etc.): it is of practical importance to examine the impact of a model specification that is "rich" in the sense that it includes, in addition to those network variables, socioeconomic and trip purpose characteristics as well. In terms of functional form, models used for mode choice modelling are typically based on the assumption that the utility function has a linear form: it may well be that, if this assumption is wrong and one forces the utility function to be linear, an error in introduced that cannot easily be compensated by an additional term in a linear utility function.

The standard utility function of the logit model is linear and very few systematic departures from linearity are observed in the literature. Of course *ad hoc* nonlinearities are used (for instance the natural logarithm of the number of cars is frequently found in the car utility function), but systematic nonlinearities of variables are exceptional. The principal reason for this is the unimodality of the log likelihood function garanteed by linearity in parameters, and the added computational burder associated with estimation of nonlinear forms on variables. This is the case despite the strong evidence on the influence of functional form (e.g. Oum, 1989): indeed, it is known (Gaudry and Wills, 1978) that incorrect specification of functional form not only modifies elasticities – an obvious point – but can reverse regression signs!

In this context, it is frequent practice to split the sample by socioeconomic group and trip characteristic, a practice called segmentation (to be discussed below), that may be a partial substitute for nonlinearity, as segment-specific linear parameters are estimated.

We will show the importance of using a rich, as opposed to a sparse, specification of the utility functions, and show as well that nonlinear utility functions obtained by using Box-Cox transformations yield different and more reasonable results than those obtained with the usual linear form on both rich and sparse specifications. The problem of interest occurs in many areas of consumer choice and applied economics where the logit is in increasing use.[2]

2. THE MODEL

2.1. Theoretical framework

The logit discrete choice model can be derived from random utility theory, along the following lines (Domencich and McFadden, 1975; Ortuzar and Willumsen, 1990):

i) There is a set $C_n = \{1, ..., M\}$ of available alternatives $i, j \in C_n$.
ii) There is a set $X_k = \{1, ..., K\}$ of vectors of measured attributes of the individuals and their alternatives.
iii) The individuals $n = \{1, ..., N\}$ select the alternative i which maximizes their personal utility $(U_{in} > U_{jn}, \forall\, i \neq j \in C_n)$ subject to their individual constraints. The value of utility itself is based on a comparison and individual evaluation of the different characteristics which describe the attractivity of the alternatives.
iv) It is not possible to possess complete information about all elements that determine this choice. Errors can arise for specification or observational reasons. For example, instead of the true modal characteristic X^*_{kin}, only X_{kin} (or a functional $f(X_{kin})$) may be available. To take into account the unobserved measurement error, X^*_{kin} is effectively replaced by $X_{kin} + \epsilon_{kin}$ (or $f(X_{kin}) + \epsilon_{kin}$), where ϵ_{kin} designates the unknown error. For individuals who have the same set of alternatives and who face the same constraints, it can be assumed that the residuals ϵ_{in} are random variables with mean 0 and a certain distribution.

More precisely, the utility U_{in} can be represented by two components:

- an observed representative component V_{in} which is a function of measured mode specific and socioeconomic attributes X_{kin} and
- an unknown random component ϵ_{in} which represents unobserved attributes, taste variations, and measurement or observational errors.

Since the utilities $U_{in} = V_{in} + \epsilon_{in}$ are random across the individuals, this event is associated with a probability:

$$P(i)_n = P(U_{in} > U_{jn};\ \forall\, i \neq j \in C_n), \tag{6}$$

or more explicitly:

$$P(i)_n = P(V_{in} + \epsilon_{in} > V_{jn} + \epsilon_{jn};\ \forall\ i \neq j \in C_n). \tag{7}$$

Depending on the assumption made about the joint probability distribution of the set of disturbances $\{\epsilon_{in}, i \in C_n\}$, a specific random utility model can be obtained. For the multinomial

[2] As noted by one of the referees, the argument that nonlinearity yields a superior model, so that no model of unprobed functional form should be accepted, irrespective of its author's *a priori* preference for a particular form, "could be made for virtually any regression model".

Logit model, each error ϵ_{in} is assumed to be independently and identically distributed over the population and for each individual according to the Gumbel or Type 1 extreme value distribution which has the following cumulative distribution function:

$$P(\epsilon_{in} \leq \epsilon) = \exp\left[-\exp\left(-\sqrt{\pi^2/6\delta^2}\epsilon\right)\right] \tag{8}$$

with mode zero and variance δ^2 for each alternative $i \in C_n$.

The probability that individual n chooses alternative i can now be expressed as:

$$P(i)_n = \frac{\exp\left(V_{in}\right)}{\sum\limits_{j \in C_n} \exp\left(V_{jn}\right)} , \tag{9}$$

where the representative unobserved component V_{jn} is a linear function of the K variables X_{kin}:

$$V_{jn} = \sum_{k=1}^{K} \beta_k X_{kjn} . \tag{10}$$

To estimate the parameters, the likelihood function is used. For a random sample of size N the likelihood function can be viewed as the product of the choice probabilities associated with M subsets of observations, in which the first subset includes N_1 individuals observed to have chosen alternative 1, the next one N_2 individuals to have chosen alternative 2, etc., all observations being independent:

$$\mathcal{L} = \prod_{n=1}^{N_1} P(1)_n \prod_{n=N_1+1}^{N_1+N_2} P(2)_n \cdots \prod_{n=N_1+N_2+\ldots+N_{M-1}+1}^{N} P(M)_n . \tag{11}$$

This expression can be simplified by defining a dummy variable d_{in} such that $d_{in} = 1$ if individual n has chosen alternative $i \in C_n$ and $d_{in} = 0$ otherwise:

$$\mathcal{L} = \prod_{n=1}^{N} \prod_{i \in C_n} P(i)_n^{d_{in}} . \tag{12}$$

The corresponding log-likelihood function can now be written as:

$$L = ln\, \mathcal{L} = \sum_{n=1}^{N} \sum_{i \in C_n} d_{in}\, ln\, P(i)_n . \tag{13}$$

2.2. The classical linear and the Box-Cox Logit models

Properties of linearity . The “classical” linear Logit model specification normally assumes (Gaudry, 1992):

i) linearity in variables;
ii) the exclusion of characteristics of other alternatives $j \in C_n$ from the representative utility of the i$^{\text{th}}$ one $(i \in C_n, i \neq j)$;
iii) equal “abstract” or “generic” coefficients for the network characteristics, a constraint that is not necessary but is frequently imposed.

These assumptions lead to unrealistic properties. Because of (ii), the model implies:

a) equal cross elasticities of demand: this means that setting up a bicycle path between two cities will draw the same percentage of travellers from the plane, car and train. Furthermore (iii) implies identical values of time across the modes: this means that representative train and plane users value time identically;
b) the exclusion of complementarity among alternatives;
c) that only differences in the level of characteristics matter, or that the function is not homogeneous of degree zero: in consequence doubling all fares and income will change the market shares.

Because of (i) the model further implies that:

d) the effect of a given difference in transport conditions is independent of the service level characteristics so that the response curve to changes in service characteristics is symmetric with respect to its inflection point (see Figure 1). For instance, a 20 minute train service improvement has the same impact on choice probabilities for the Hamburg-Hannover origin-destination pair as for the Hamburg-München pair. Similarly, adding an amount of 10 DM to the price of travelling by plane will have the same impact as adding 10 DM to the price of travelling by train. Generally speaking, symmetry with respect to the inflection point, implies that potential asymmetry of behavior, where consumers/travellers suddenly start to react and then change their behavior, cannot be detected;
e) coefficients for the constants and for the variables common to all alternatives are underidentified, which means that, for these variables, only differences with respect to an arbitrarily chosen reference can be identified.

We also note in passing that the logit form requires that

f) the choice probabilities go to zero (one) when the representative utility V_i goes to $-\infty$ ($+\infty$) so that (see Figure 1) one cannot model thick tails due to specification error, modeler ignorance, compulsive consumption or captivity to modes;

The Box-Cox device. To bypass most of these constraints (generally speaking, only (c) and (f) will remain), the Box-Cox transformation is used:

$$X_{kjn}^{(\lambda_{kj})} = \begin{cases} \dfrac{\left(X_{kjn}^{\lambda_{kj}} - 1\right)}{\lambda_{kj}} & \text{if} \quad \lambda_{kj} \neq 0, \\ ln\, X_{kjn} & \text{if} \quad \lambda_{kj} = 0. \end{cases} \tag{14}$$

Now the model (4)-(5) can be rewritten as:

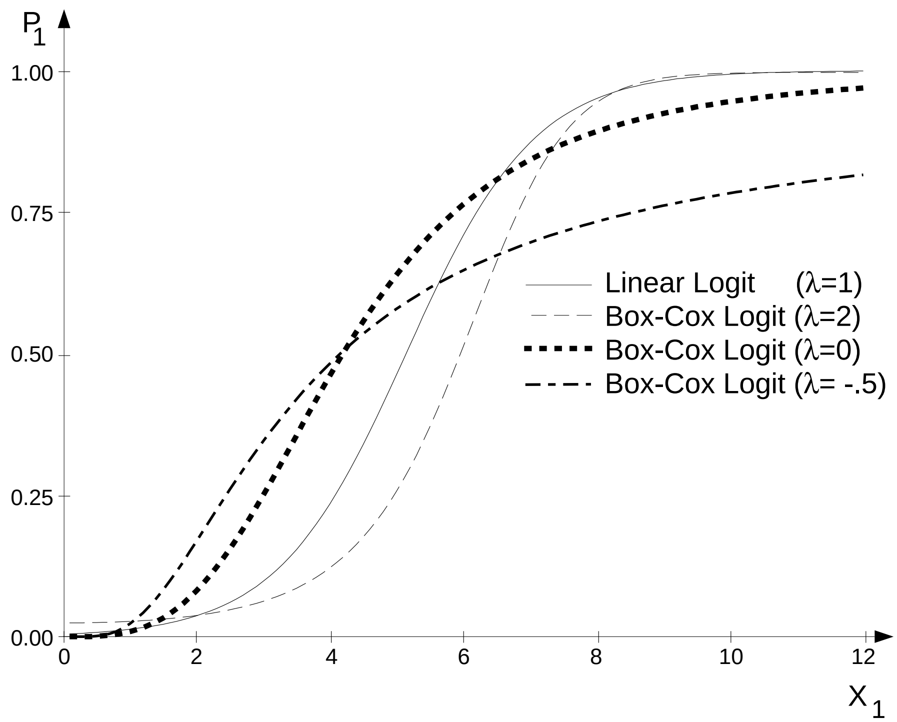
$$P(i)_n = \frac{\exp\left(\beta_i X_i + \sum_{k=1}^{K} \beta_{ki}\, X_{kin}^{(\lambda_{kj})}\right)}{\sum_{j \in C_n} \exp\left(\beta_j\, X_j + \sum_{k=1}^{K} \beta_{kj}\, X_{kjn}^{(\lambda_{kj})}\right)} \quad , \tag{15}$$

where $X_i = X_j = 1$ are regression constants.

If λ_{kj} is equal to one (or zero), then the variable is entered in its linear (or logarithmic) form. Since the transformation is continuous for all possible values of the λ-parameter, but defined only for a positive variable, it is clearly understood that in (15) some of the X_{kjn}'s cannot be transformed: the constant, the dummies and the ordinary variables that contain negative observations. Variables that contain positive and null values can be transformed as long as a compensating dummy variable is created. In this model, we will not transform one such variable (number of transfers by train) even if the computer program used (Gaudry et al., 1993) would allow it.

This device makes it possible to do in a continuous fashion what researchers often do on a trial-and-error basis when they explore many *ad hoc* specific transforms (e.g. the natural logarithm of a variable, or the square of a variable) in their search for “reasonable” signs and better statistical fits. Naturally, the benefit of this continuous search for an optimal form determined by the data themselves is not free. Besides the computational costs which have to be payed for this complexity, one also loses the certainty of a unique maximum if more than one variable is transformed in its nonlinear form. In the case of different λ parameters, one has to make sure that the regression does not reach a local maximum by starting the regression procedure from different points.

Figure 1. Linear Logit *versus* Box-Cox Logit



Visual and economic significance. Figure 1 clearly shows the difference between the linearity and nonlinearity of a variable. The asymmetric curve (in respect to its inflexion point) given by the Box-Cox transformation of the strictly positive variable X_1 illustrates the error which will occur when a nonlinear variable is forced to be linear. For example, assume X_1 is denotes total travel time: in the linear case, the value X_1 equal to 4 is associated to the probability P_1 equal to 0.25; in the nonlinear case, the probability is higher if $\lambda < 1$ and smaller if $\lambda > 1$. Hence, if one forces a nonlinear variable or in equality the utility function to be linear, this will result in an over- or underestimation of the probability related to this variable. In addition to asymmetry of the response function ($\lambda \neq 1$), reaction thresholds, defined in terms of acceleration of the rate of change of the probability, can be identified as we shall soon explicate.

A more formal statement of the Box-Cox properties is given in Table 1. The Box-Cox transformation of the strictly positive variables of the linear Logit model leads to the Box-Cox Logit model with an asymmetry of response, as shown in Figure 1, because the effect of a unit change in the service will depend on the level of the variables X_{kj} for all values of λ_{kj} not equal 1. This can be seen by examining the partial derivatives of the representative utility function V_j of the j^{th} mode. In Table 1, one notes that the effect of additional service will be smaller at higher service levels than at lower ones if λ_{kj} is smaller than 1. These diminishing returns mean that given absolute reductions in total travel time have more impact when total travel times are low than high: a gain in travel time of 10 minutes means less on a long trip than on a short one. Conversely, increasing returns exist if λ_{kj} is larger than 1.

Clearly, if one is considering very small changes in the service levels of a mode, the mathematical form used does not matter very much because one is forecasting in the immediate neighbourhood of current sample values. However, if one is considering significant changes in service levels, such as reducing train travel time by one half with ICE trains, then curvature is decisive. What database allowed us to shed some light on this issue?

Table 1. Non constant returns in a Box-Cox Logit model

Partial derivatives:		$\partial V_j/\partial X_{kj} = \beta_{kj} X_{kj}^{(\lambda_{kj}-1)}$	$\partial V_j^2/\partial X_{kj}^2 = \beta_{kj}(\lambda_{kj}-1) X_{kj}^{(\lambda_{kj}-2)}$
Returns:			
• Decreasing	$\lambda_{kj} = -1$	β_{kj}/X_{kj}^2	$-2\beta_{kj}/X_{kj}^3$
	$\lambda_{kj} = 0$	β_{kj}/X_{kj}	$-\beta_{kj}/X_{kj}^2$
• Constant	$\lambda_{kj} = 1$	β_{kj}	0
• Increasing	$\lambda_{kj} = 2$	$\beta_{kj} X_{kj}$	β_{kj}

2.3. Database

The socioeconomic variables and the trip purposes, as well as the origin-destination pairs, were given by the KONTIFERN database, which was developed under the auspices of the German Ministry of Transportation in the year 1979/1980. This database long distance passenger traffic (defined as trips longer than 50 km) contains 62 982 observations, and each of these observations includes 84 informations. It was been checked for consistency, validity and completeness with respect to variables that are important for modelling, leaving 49 399 available observations. To reduce computing time, maximization of the log likelihood function was carried out with a random sample of 6 000 observations drawn to respect to the distribution of the trip purposes. Detailed information can be found in the appendices A, B and C of Mandel (1992), where procedures to obtain network data are also described.

Networks for each of the three modes (train, plane and car in our case) were developed for the year 1979/1980. The size of the networks, as well as the calculated modal characteristics, differ among modes. The network comprises about 7 500 nodes, 18 000 links and 5 characteristics for the car mode, about 900 nodes, 2 600 links and 6 characteristics for the train mode and about 200 nodes, 6 500 links and 22 characteristics for the plane mode. Network itinerary values were obtained under the assumption that the individuals choose the shortest time path from origin to destination, using an algorithm by Ford modified to take into account several link and node dimensions like transfer time, minimum-connecting time, delays and car driving pauses. In addition, the information on the shortest path of each mode can be split in its elements (14 for car, 15 for train and 22 for plane).

3. RESULTS

3.1. Chosen specifications and associated likelihood values

Sparseness or richness? Four model series, ranging from the simplest to the richest, were specified to study the benefits of greater use of available information. Table 2 summarizes this by showing the variables used (exept for constants, which are required in the presence of Box-Cox transformations to guarantee the invariance of the estimates of the λ to changes in units of observations):

- MKs : represents the minimal model, as in many passenger analyses done until recently and in freight mode choice models due to the relative difficulty of collecting adequate data;
- MK0 : adds to the first series two network variables. In the detailed analysis (available in the appendix or from the authors), the number of transfers by train is not very significant and, with an unexpected positive sign, is probably correcting the travel time variable (constructed as a sum of time components on each path). It was not retained in the next series;
- MKI : adds socioeconomic variables to the network variables. As the last two variables shown were highly correlated with other variables (the first with age classes 18-24, 25-44 and 45-65; the second with age class 18-24), they were not retained in the next series;
- MKII : includes two significant variables describing the trip purpose.

Breaking up the travel time between some of its components (in-vehicle and out-of-vehicle time, adding elements (number of driving pauses) or introducing additional socioeconomic variables (driver's annual trip frequency or cumulative trip length; type of education, type of occupation, autonomous business activity) either did not increase the log likelihood value or introduced multicollinearity. The database did not include a direct measure of the consumer's income.

In addition, the results reported here for all specifications have the property that the identifiable coefficients of explanatory variables (other than the regression constants) were constrained equal across alternatives; i.e. are "generic" coefficients as in Equation (10). Attempts at relaxing these constraints either did not involve significant increase in the log likelihood values or caused multicollinearity. As the reference mode was taken to be the plane, these results generally imply that socioeconomic factors contribute to the explanation of the relative use of the plane but not to the relative use of the train and the car. They also imply that, once due account is taken of the network characteristics and functional form, it is difficult with these data to stray from a mode-abstract view of the travel modes.

Functional form. For each model series, three functional forms were studied, namely the linear, logarithmic and the optimal form obtained by using a Box-Cox transformation on the network variables. The log likelihood values are shown in Table 3. Further tests with up to four Box-Cox transformations did not imply significant gains in log likelihood or robust results. These were clearly diminishing returns in trying to get a better idea of different nonlinearities for fares and travel time, for instance, and no return to attempts at detection of nonlinearity specific to frequency. These results are not surprising if one thinks that modal pricing is distance based and to this day without meaningful price discrimination, or that service frequencies are already very high.

3.2. Selected principal results[3]

Sparseness and functional form. An examination of the log likelihood values shown in Table 3 makes it clear that, irrespective of the functional form used, massive gains are involved first in adding the frequency of service variable and then in adding further socioeconomic factors to the explanation of mode choice. It is also clear that the linear form is always rejected strongly in favour of the logarithmic form in sparse models and in favour of the power one quarter ($\widehat{\lambda}$ = 0,24) as the specification is enriched. This optimal value is, interestingly, different from the logarithmic value (λ = 0) that would imply a product form sometimes used in the past due to its convenience, namely, dropping the index for the individual and imposing generic β_k and without loss of generality:

$$P(i) = \frac{\prod_{k=1}^{K} X_{ki}^{\beta_k}}{\sum_j \left(\prod_{k=1}^{K} X_{kj}^{\beta_k} \right)} \, . \qquad (16)$$

Elasticities and values of time. Of practical importance are the elasticities of demand. In Table 4, the elasticities of the Box-Cox estimates are lower in the rich model than in the sparse model; the Box-Cox estimates differ strongly from the linear estimates. Another difference of interest is that of the values of time: the Box-Cox estimates imply that train users have lower values of time than car users who themselves have lower values of time than plane users — a ranking that is more reasonable than the equal values built into the linear form. In interpreting the absolute levels of these revealed trade-offs, it is important to remember that, in a mode choice model, the trip frequency decision is assumed to be given. In consequence, the computed value is in effect a conditional value that does not reflect a trade-off involved in not making the trip but in choosing a particular mode given that the trip will be made. That is the reason why trade-offs that one could derive, conditionnally upon the mode choice itself, among competing itineraries would be expected to be higher still.

However, it is important to note that values of time are, technically speaking, marginal rates of substitution between two variables holding the level of utility constant. As these marginal rates are simply ratios of partial derivatives, the Box-Cox logit results will have a significative build-in advantage over the linear logit results because they will vary not only with each mode (as the derivatives will not be evaluated at the same point for each mode on average) but will also depend on how much time is saved, as the value of the marginal minute depends on trip length. The contrast with the linear case is illustrated in Figure 2 where the curves denote, respectively,the value of the marginal minute (in 2a) and the cumulative value of additional minutes saved (in 2b) in the Box-Cox model and the straight lines indicate the behaviour of the same two values in the linear model.

[3] Full results found in Appendix A “Linear, Logarithmic and Box-Cox Logit MK Model Comparison” are available from Marc Gaudry.

Table 2. Model specification: variables

Model Series*		MKs	MK0	MKI	MKII
Network variables**					
• travel cost		X	X	X	X
• travel time		X	X	X	X
• frequency per week		—	X	X	X
• number of transfers (train only)		—	X	—	—
Socioeconomic variables					
• age class 0 to 9 years	[65 or more]	—	—	X	X
• age class 10 to 14 years	[65 or more]	—	—	X	X
• age class 15 to 17 years	[65 or more]	—	—	X	X
• age class 18 to 24 years	[65 or more]	—	—	X	X
• age class 25 to 44 years	[65 or more]	—	—	X	X
• age class 45 to 64 years	[65 or more]	—	—	X	X
• sex: male	[female]	—	—	X	X
• employee	[independent]	—	—	X	X
• single-person household	[multiple person]	—	—	X	—
• trip abroad		—	—	X	—
Trip purpose					
• business	[vacation]	—	—	—	X
• private	[vacation]	—	—	—	X

* In the case of categorical variables, the implicit reference class is indicated in square brackets.
** In addition to the above, the variables "trip distance" and "access/egress distance to the airport were used, but only for the adjustments to the MKII LIN and MKII BC models reported in Table 5.

Table 3. Model specification: functional form and log likelihood values

MKs			MK0			MKI			MKII		
LIN	LOG	BC	LIN	LOG	BC	LIN	LOG	BC	LIN	LOG	BC
-1392.96	-1309.08	-1309.04	-1383.65	-1306.090	-1306.22	-1292.35	-1234.64	-1231.69	-1230.66	-1195.20	-1189.35

Table 4. Selected linear and Box-Cox Logit results for MK0 and MKII series

Models for Germany 1979/1980			MK0		MKII	
			LIN	BC	LIN	BC
Characteristics						
• Own elasticity*	- travel cost	plane	-0.67	-0.69	-0.99	-0.62
		train	-0.12	-0.38	-0.13	-0.24
		car	-0.02	-0.05	-0.04	-0.04
		(t-statistic)**	(-7.07)	(-6.46)	(-8.39)	(-5.48)
	-travel time	plane	-0.74	-1.79	-0.75	-1.69
		train	-0.72	-1.14	-0.63	-1.00
		car	-0.07	-0.15	-0.08	-0.14
		(t-statistic)**	(-17.15)	(-15.60)	(-14.92)	(-15.14)
	-frequency	plane	0.15	0.16	0.12	0.10
		train	0.29	0.11	0.19	0.08
		car	—	—	—	—
		(t-statistic)**	(4.55)	(2.31)	(3.70)	(1.72)
• Value of time (DM/minute)		plane	1.10	2.50	0.93	3.18
		train	1.10	0.48	0.93	0.73
		car	1.10	1.00	0.93	1.09
• Power transformation λ			1.00	0.07	1.00	0.24
• Final log likelihood value			-1383.65	-1306.22	-1230.66	-1189.35
• Rho-squared			0.16	0.21	0.26	0.28

* Weighted aggregate elasticity of choice probability.

** Student's t-statistic of underlying β_k coefficient computed conditionnally upon the estimated value of the Box-Cox transformation.

LIN = Linear; BC = Box-Cox.

Figure 2.a. Value of Time with Respect to the *m*th Units of Travel Time Saved

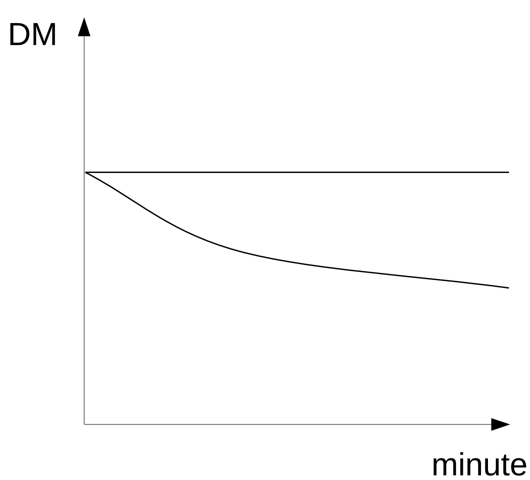
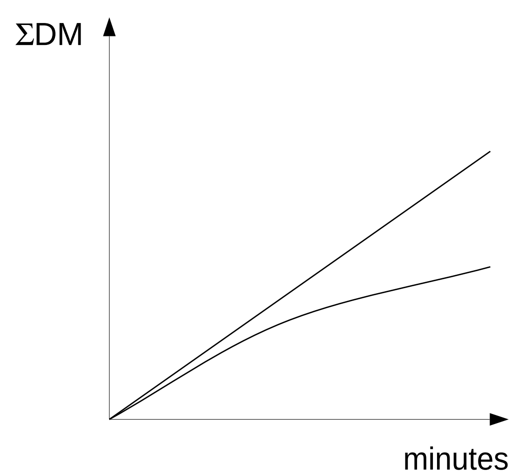


Figure 2.b. Value of Time with Respect to the *m* Units of Travel Time Saved



4. FORECASTING

4.1. Adjustment to Observed Aggregate Flows of 1985

Formally, the forecasting exercise involves two distinct steps. When the parameter values for the two MK-II variants, derived from the 1979 (disaggregate) survey, are used to predict 1985 market share values, it is clear that they differ from "true" values for that year.

This can be understood if the 1979/1980 survey data systematically underreport all trips, and specially trips of shorter distances as may occur if forgetfulness yields a systematic bias in the reporting of trips by distance. Furthermore, the aggregate flow matrices of 1985 are considered very reliable because the base of this survey is given by simulation processes which are controlled by direct observations for the car mode, by the tickets sold by the German Bundesbahn for the train mode and by the public air transportation statistics and internal statistics of Lufthansa for the plane mode. The possible bias in this set of data is assumed to be small although, to complete the matrix, the traffic flows between some origin-destination pairs had to be estimated or computed by simulation processes. The necessary modal variables for the MK-II models had been added to the 1985 survey data by using the shortest path algorithms on networks developed for that year.

The biases mentioned above were reflected in the difference between the forecasted and the "true" values, which was strongly and systematically related to the travel distance and differed across the modes. In a first step, correction factors are therefore incorporated with parameter values that are in keeping with the functional form of each variant.

The new utility functions V_{jn}^{*} include the complete old utility functions V_{jn} as well as the variable trip distance for the alternatives car and train and the variable access/egress distance to the airport for the plane alternative. If there is no difference between the forecasted and "true" values the new utility functions V_{jn}^{*} will be equal to the old utility functions V_{jn}. Moreover, Box-Cox transformations on the distance variable were used to examine whether the difference between the forecasted and the "true" values mentioned above is related to this variable in a nonlinear way or not.

We found again that power functions give a much better adjustment than linearity of the distance correction factor: this implies that the 1979 survey data systematically underreport all trips, but in a way that decreases rapidly with the distance (trip length) as the value of 0.52 shown for the λ in Table 5 requires. An incomplete comparison of the linear and nonlinear adjustments applied to the distance variables of the Linear and Box-Cox Logit models is given in Table 5.

Table 5. Comparison of the Linear and Nonlinear Adjustments of the MK-II Models

Models for Germany, 1985:	Linear Logit MK-II LIN		Box-Cox Logit MK-II BC	
Characteristics:	Linear	Nonlinear	Linear	Nonlinear
• Coefficient of V_j	1.016	1.013	0.999	0.995
• Power transformation λ*	1.000	3.482	1.000	0.521
• Final Log-Likelihood	-1795.683	-1639.144	-15201.524	-15184.223
• Rho-squared bar	0.314	0.313	0.492	0.514
* Applied to distance variable				

4.2. Results of the Linear and Box-Cox Logit MK-II Variants

In the second step, the variants, ”augmented” with functional correction factors, are used to forecast market shares for various scenarios involving major changes in travel speeds in 2010. The reference scenario R and the planning scenario V differ only by three origin-destination high speed rail links, namely Hamburg-Berlin (286 km, 250/160 km/h, 127 minutes), Frankfurt-(Halle)Leipzig-Dresden ((383 km) 465 km, 160/200/250 km/h, (145 minutes) 173 minutes), and Nürnberg-Berlin (616 km 200/250 km/h, 226 minutes), which are only included in the scenario V. All other basic input data and constraints are the same for the linear Logit model with linear adjustment (MK-II LIN) and the Box-Cox Logit model with the nonlinear adjustment (MK-II BC). For more details concerning the developed networks and databases used for the year 2010, see Mandel (1992).

4.2.1. General Differences

In Table 6 the major forecast results give the market share of the train for some regional origin-destination pairs which are highly influenced by the high speed rail links mentioned above. The size and geographical location of these regions is shown in Figure 3 by thick edges and numbers. Furthermore, the differences of the forecasts for the two scenarios R and V are noted according to model type: ΔL for MK-II LIN and ΔB for MK-II BC. In the right-hand side column, the ratio of ΔB and ΔL is computed as an indicator of difference between the forecasts.

Obviously, Table 6 shows that the Box-Cox Logit variant MK-II BC produces forecasted values that differ significantly from those of the linear Logit variant MK-II LIN, as the existence of an asymmetric modal response implies. In interpreting the results, one should remember that the forecasts for the different scenarios have been made under the *ceteris paribus* condition: the train is the only modified mode.[*]

A comparison of the forecasted shares shows that the linear results are higher than, or roughly equal to, the nonlinear ones if the distance between the origin-destination pair is quite low, or have a common regional border (Bayern-NFC SW / SE, Ext. a. Berlin-NFC N, Coast OFC-NFC N, Rhein/Main-NFC SW / SE). They also show that the difference increases with distance (Rhein/Ruhr-NFC SW / SE / N). Note also that the regions are large and no additional access/egress services, like Interregio trains, were included in scenario V. The very high increasing market shares are all related to origin-destination pairs with agglomeration points like Berlin, Hamburg, München, Dresden, etc.

However, if we aggregate these individual values over the Federal Republic of Germany as a whole, we obtain a higher train market share for the linear than for the nonlinear model, which is logical because the number of travellers decreases with trip length.

The difference between the linear and nonlinear forecasts is also obvious if one looks at the ratio of ΔB and ΔL given in the last column of Table 5. On average, the impact of high speed rail links on the market behavior is predicted 1.78 times higher with the nonlinear MK-II BC model than with the linear one.

[*] It has to be remarked that it is not possible to compare the results shown in this paper directly with the results in Mandel (1992) because this paper uses an updated traffic flow matrix for the year 2010 and a slightly different demarcation of regions.

Table 6. Train Share Forecasts at the Regional Level for the Year 2010

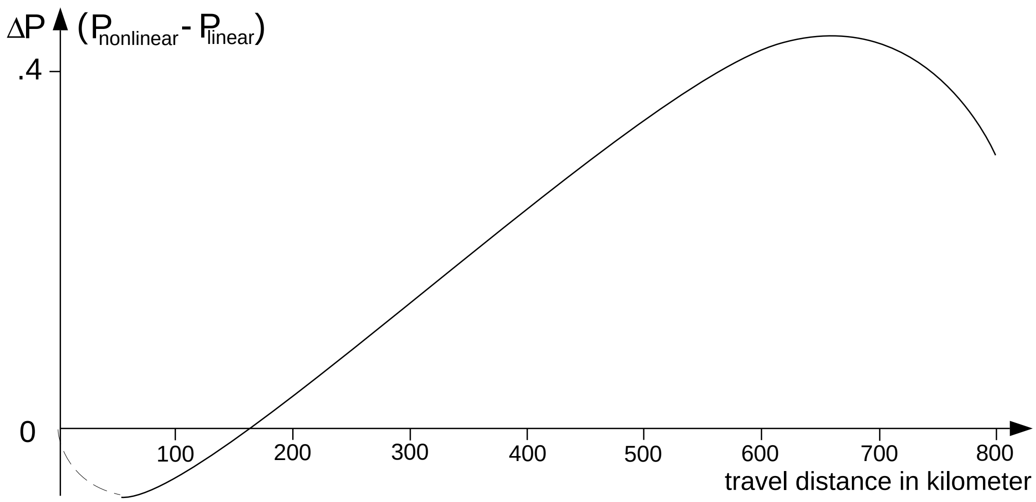
No. - Regional origin - destination pairs		MK-II LIN			MK-II BC			
		R	V	ΔL	R	V	ΔB	ΔB / ΔL
1 - Coast OFC	Ext. a. Berlin	10.7	15.1	4.4	11.6	18.0	6.4	1.45
	NFC N	9.5	9.6	0.1	7.6	7.8	0.2	2.00
	NFC SW	11.0	11.2	0.2	11.9	13.0	1.1	5.50
	NFC E	11.9	13.5	1.6	15.2	16.9	1.7	1.06
2 - North OFC	Ext. a. Berlin	15.7	16.1	0.4	18.8	19.7	0.9	2.25
	NFC N	10.6	11.1	0.5	11.8	12.7	0.9	1.80
	NFC SW	9.6	9.7	0.1	6.7	6.9	0.2	2.00
	NFC SE	10.4	12.0	1.6	11.3	14.1	2.8	1.75
3 - Rhein / Ruhr	Ext. a. Berlin	25.4	25.4	0.0	32.5	32.5	0.0	—
	NFC N	13.7	13.7	0.0	19.4	19.4	0.0	—
	NFC SW	11.4	12.6	1.2	12.6	14.7	2.1	1.75
	NFC SE	13.7	20.3	6.6	15.6	26.2	10.6	1.60
4 - Rhein / Main	Ext. a. Berlin	25.0	26.9	1.9	29.7	33.0	3.3	1.73
	NFC N	17.7	17.7	0.0	25.9	25.9	0.0	—
	NFC SW	9.4	10.5	1.1	8.1	10.4	2.3	2.09
	NFC SE	10.3	16.2	5.9	10.6	21.5	10.9	1.84
5 - Saar / Rhein /	Ext. a. Berlin	23.9	26.0	2.1	27.1	30.2	3.2	1.52
Neckar	NFC N	20.5	20.6	0.1	28.0	28.1	0.1	1.00
	NFC SW	11.2	13.0	1.8	12.5	15.9	3.4	1.88
	NFC SE	9.7	14.5	4.8	10.9	19.0	8.1	1.68
6 - Bayern	Ext. a. Berlin	12.0	18.7	6.7	13.0	23.5	10.5	1.56
	NFC N	11.7	13.6	1.9	16.0	18.8	2.8	1.47
	NFC SW	7.0	8.4	1.4	4.9	7.2	2.3	1.64
	NFC SE	6.0	7.0	1.0	3.9	5.4	1.5	1.50
7 - Ext. a. Berlin	NFC N	11.7	11.7	0.0	8.2	8.2	0.0	—
	NFC SW	9.6	13.4	3.8	6.5	10.0	3.5	0.92
	NFC SE	9.6	10.6	1.0	7.5	8.0	1.5	1.50
8 - NFC N	NFC SW	10.2	10.2	0.0	9.0	9.0	0.0	—
	NFC SE	9.3	9.3	0.0	9.6	9.6	0.0	—
9 - NFC SW	NFC SE - 10	9.0	9.8	0.8	5.1	6.1	1.0	1.25

OFC: Old Federal Countries | NFC: New Federal Countries | Ext. a.: Extended area
SW: South-West | SE: South-East | N: North

Figure 3. German High Speed Rail Network for the Year 2010

This difference ΔP between the nonlinear and linear forecast is visualized in Figure 4 as an example which specially takes into account the new high speed rail links. The curvature varies in a small range for different origin - destination pairs but always keeps the shape shown so that it can be stated that the linear form tends to predict higher market shares on short distances than the nonlinear form, and the reverse for longer distances.

Figure 4. Difference of Nonlinear and Linear Forecasts Related to Travel Distance



4.2.2. Asymmetry

But, independently from the level of market shares, the difference (ΔL and ΔB) between the forecasts for scenario R and V is even more interesting from the perspective of asymmetry: do asymmetric modal response functions influence high speed rail demand forecasts?

To illustrate the asymmetry of the response functions due to the inflexion point of the curve and the threshold effect mentioned before, Figure 5 shows for different origin-destination pairs the response curves with respect to the variable travel time for the train alternative while all other conditions (characteristics and modes) remain unchanged.

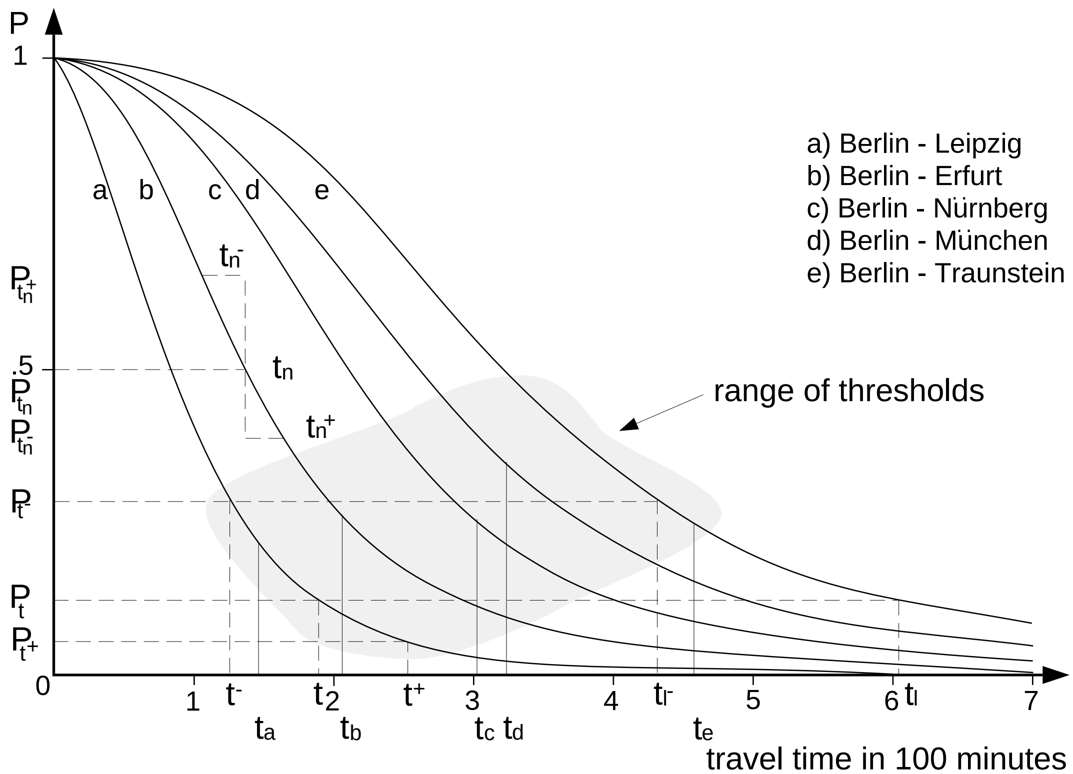
To describe asymmetry more formally one first has to define the inflexion point of the curve. At this point the curvature changes its functional shape from convex to concave and one can compute the value of the inflexion point (P_{t_n}, t_n) by equating to zero the second derivative of the MK-II BC mode share in respect to the travel time. The response curve can be called asymmetric with respect to its inflexion point if equidistant reductions and increments of travel time t_n by Δt [that is, $t_n^+ = t_n + \Delta t$ and $t_n^- = t_n - \Delta t$] will give different absolute values, namely $\Delta P^+ = \left| P_{t_n} - P_{t_n^+} \right|$ and $\Delta P^- = \left| P_{t_n} - P_{t_n^-} \right|$: otherwise the curve has to be called symmetric. More formally, one can define asymmetry, as in Laferrière and Gaudry (1993) in terms of the partial correlation ξ of $P_{t_n^+}$ and $\left(1 - P_{t_n^-}\right)$: this yields an indicator that is necessarily between 0 and 1.

4.2.3. Thresholds

A threshold effect occurs when the travel time reaches a critical value of t beyond which any further reduction of t to $t^- = t - \Delta t$ provokes a more substantial growth of the mode share P_t

than an equidistant increment of t to $t^+ = t + \Delta t$, so that the absolute difference of the mode shares $|P_t - P_{t^-}|$ is higher than $|P_t - P_{t^+}|$.

Figure 5. Asymmetry, Thresholds and the Impact of Travel Distance (MK-II BC Model)



The word threshold implicitly involves an individual evaluation of the preception of change; hence it is up to the decision-maker to define his threshold by exploring the percentage of mode share increment which he will consider as a threshold i. e. which will satisfy his opinion about a threshold. More formally a critical value η has to be defined so that the absolute difference of $|P_t - P_{t^-}| = (1 + \eta)|P_t - P_{t^+}|$. Alternatively, $\Delta P_{t^-} = (1 + \eta)\Delta P_{t^+}$ and hence $\eta = (\Delta P_{t^-}/\Delta P_{t^+}) - 1$. From a visual point of view, one would intuitively expect to find the thresholds to be in the range given by the grey zone in Figure 5, where a reduction of one unit would increase the probability of choosing the alternative train by additional 10 to 50 percent ($\eta = 0.1$ to 0.5), so that ΔP_{t^-} would be equal to 1.1 to 1.5 times ΔP_{t^+}.

It is obvious that the given results in Figure 5 are based on a *ceteris paribus* assumption: consequently a variation of other mode specific characteristics like frequency or travel fare would imply a change of the location of the response curve so that the threshold would have to be relocated.

If one compares the different response curves in Figure 5 from left to right, the impact of travel distance on consumer behavior in respect to travel time can be seen. By increasing the distance, the asymmetric shape of the functions become more flat: this corresponds to the statement that a reduction of travel time by 15 minutes on short distances i. e. short trips has an higher impact on consumer behavior than on long trips. Conversely, one could say that to have the same impact on consumer behavior on long trips, as on short trips, one has to decrease the travel time by much more ($|t_l - t_{l^-}| \ggg \left|t - t^-\right|$).

The t_α values marked on the x-axis represent the actual travel time on the different origin-destination pairs which are directly influenced by the high speed rail links mentioned before. One can say that on short distances and on long North-South origin-destination relations, the actual travel times are in the range of the thresholds, while on long West-East origin-destination relations an additional reduction of travel time would be necessary to catch the threshold effect.

It has to be mentioned that in general by interpreting the results shown in Figure 5 one has to take into account that the travel time represents the time of a door to door trip and that origin-destination regions (see thin edged areas in Figure 3) are relatively large, which means that the average access/egress time to the main mode is quite high. Therefore a change of the access/egress services can have an important impact on the choice of an alternative.

With respect to these points, one can detect in Table 6, by comparing a range of origin-destination pairs (Rhein/Main-NFC SW / SE, Rhein/Ruhr-NFC SW / SE, Bayern-SW / Ext. a. Berlin), a threshold related to distance, or total travel time, which has to be crossed so to obtain a strong change of demand. This market behavior is shown in column ΔL as well as in ΔB but, in the linear case, this threshold occurs at shorter distance than in the nonlinear one.

4.2.4. Other Considerations

The purpose of our example is to visualize the asymmetry of the response functions, the existence of thresholds and the impact of travel distance on consumer behavior: it is not to analyze in detail the different investment projects on high speed rail links mentioned before. If one wanted to do this, it would of course be necessary to consider in addition the impact of travel fare, frequency, access/egress characteristics, etc..

Moreover the formal statements made here on the topics of asymmetry and threshold are given for our particular example of a decreasing function (with respect to travel time) and have to be reversed if one examines characteristics like frequency, which imply an increasing reaction curve.

The examination shown in Figure 5 also can be done in reverse direction where one first defines the probability of choosing the mode and then computes the necessary characteristics which satisfy this condition. Different kinds of modal services, which are related to different technologies like magnetic-, Intercity-express-, Pendolino-train/service, etc., are therefore represented by the underlying travel time. Implicitly we have the possibility to verify the optimal investment by relating it to the mode specific characteristics that maximize revenue.

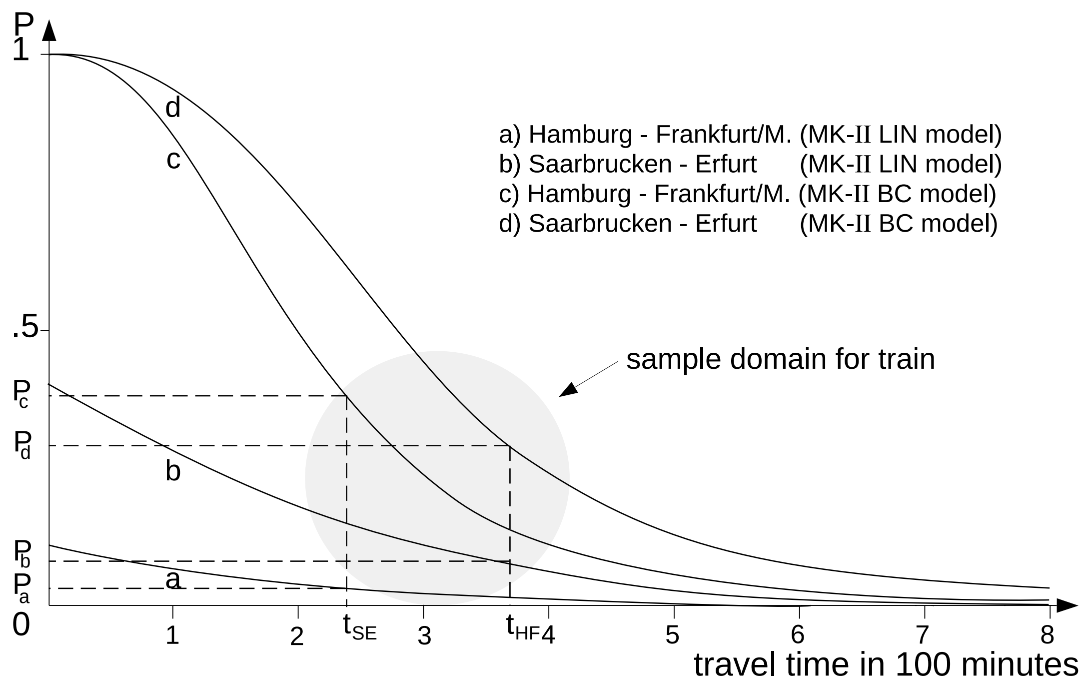
4.2.5. Nonlinearity and Segmentation

One of the great benefits of nonlinearity is to reduce dramatically the need for certain kinds of segmentation — e. g. by fare or distance classes — typically done in order to obtain adequate representation or fit in domains other than that of the sample mean. Were one to forecast well outside of the sample domain (represented in Figure 6 by the standard derivation away from the mean), it is clear that a linear form would behave very differently from a nonlinear form. As high speed rail involves large changes in service levels, one would certainly not forecast with a linear form without segmenting the sample — in effect doing piecemeal linear approximation of the underlying nonlinear form.

In our model, the need for segmentation may still remain for different categories of trips — e. g. trip purposes. A convenient approach would consist in splitting the sample (if a dummy variable had indicated heterogenity of the full sample with respect to trip purpose) in order to see whether the curvatures differ by trip purpose or are adequately represented by a (constant) shift of the response curve obtained with the full sample.

We have noted in the introduction the tendency for mode choice models to contain increasing numbers of socioeconomic variables and suggested that a reason for this drift – a part from the fact that mode choice is easier to model than trip generation – may be that mode choice models capture effects that should be dealt with in the generation-distribution part of the modelling effort. In this context, it is clear that the need for segmentation should only be partially reduced by nonlinear forms because nonlinearity – whether applied to network variables or to socioeconomic variables like income or age – is distinct from market heterogeneity. It would be very useful to study the extent to which market segmentation is a partial substitute for nonlinearity, as one of the present authors is currently involved in doing with excellent Swedish data (Algers and Gaudry, 1997).

Figure 6. Segmentation and the Sample Domain



5. CONCLUSION: THE IMPACT OF ASYMMETRY

We have shown that, irrespective of whether the model was sparse or rich, nonlinearity made both more theoretical and practical sense in a logit model. Although our disaggregate sample pertained to travel mode choice in Germany, there is little doubt that similar structures — summarized visually by the asymmetry of the response curve that they imply — could be present in many choice situations where the value of the marginal unit of a factor can be expected to depend on the total number of units. There is also little doubt that one should not accept without questioning linear logit results where the linearity has not been probed.

Using linear and Box-Cox Logit variants of an mode choice model, we have shown that the presence of asymmetry of the reaction function implies quite different forecasts than the forecasts made with a linear form. In particular, the linear form tends to overpredict market shares for short distances and to underpredict market shares for longer distances. In particular, the influence of high speed rail links on consumer behavior is much stronger in the nonlinear case than in the linear one.

Furthermore we show that the existence of thresholds and asymmetric behavior of the response curve means that customers will significantly change their modal choice behavior if the threshold can be reached. Generally speaking, the nonlinear functional form found establishes that a 15 minute reduction of travel time for short trips has a much higher impact on mode choice than for long trips.

In addition, the benefits of nonlinearity can be seen in the different values of time and elasticities for each alternative (which are more reasonable than in the linear case) and in the reduction of the necessity to segment the sample in order to make credible forecasts involving significant changes in service attributes.

Because of the significant differences we found between the forecasts, we assume that this has impacts for the optimal sequence of investments.

ACKNOWLEDGMENTS

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APPENDIX A

Table A shown in the Appendix contains more complete results for the twelve models specified in Tables 2 and 3 and is produced by the Tablex feature found in the TRIO software (Gaudry et al., 1993). The first part of the the table contains marginal rates of substitution (i.e. values of time in this case), t-statistics (of the underlying β_k regression coefficients) computed conditionnally upon the value of the Box-Cox transformation, as well as own and cross elasticities. The symbols (GE) or (SP) denote a generic or a specific coefficient and the symbol L1 denotes the associated Box-Cox transformation shown in the second part of the table (along with their t-statistics computed with respect to 0 and 1, successively). A variable with an underlined code name is a dummy variable. The third part of the table contains general statistics.

Table A. Linear, Logarithmic and Box-Cox Logit MK Model Comparison

W.A. MRS : totrkf		TYPE =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
(COND. T-STATISTICS)		VARIANT =	MKs	MKsL	MKsB	MK0	MK0L	MK0B	MKI	MKIL	MKIB	MKII	MKIIL	MKIIB
W.A. ELASTICITIES		VERSION =	1	1	1	1	1	1	1	1	1	1	1	1
		DEP.VAR. =	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel
ALTERNATIVE 1														
Network Variables														
TOTAL PLANE FARE	totrkf		.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01
			(-7.57)	(-6.32)	(-6.37)	(-7.07)	(-6.30)	(-6.46)	(-7.05)	(-5.65)	(-5.83)	(-8.39)	(-4.80)	(-5.48)
	(1)	1	-.699	-.647	-.656	-.676	-.657	-.690	-.744	-.623	-.669	-.994	-.508	-.624
		2	.104	.086	.087	.101	.088	.092	.080	.064	.069	.101	.052	.063
		3	.011	.012	.012	.010	.012	.012	.016	.014	.015	.022	.011	.014
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
TOTAL PLANE	totrzf		.99E+00	.26E+01	.26E+01	.11E+01	.27E+01	.25E+01	.11E+01	.33E+01	.28E+01	.93E+00	.47E+01	.32E+01
TRAVEL TIME			(-17.13)	(-15.47)	(-15.48)	(-17.15)	(-15.47)	(-15.60)	(-12.64)	(-13.59)	(-13.80)	(-14.92)	(-15.05)	(-15.14)
	(1)	1	-.705	-1.798	-1.775	-.746	-1.887	-1.796	-.759	-1.984	-1.796	-.753	-1.953	-1.629
		2	.109	.238	.235	.116	.251	.241	.083	.204	.187	.083	.199	.168
		3	.010	.032	.032	.011	.033	.031	.016	.043	.039	.016	.043	.035
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
TOTAL PLANE AND	totfreqf					-.22E+01	-.43E+01	-.48E+01	-.18E+01	-.26E+01	-.37E+01	-.14E+01	-.17E+01	-.37E+01
ACCESS/EGRESS						(4.55)	(1.81)	(2.31)	(4.03)	(.92)	(1.75)	(3.70)	(.44)	(1.72)
FREQUENCY	(1)	1				.155	.131	.164	.135	.069	.122	.122	.032	.109
		2				-.026	-.017	-.022	-.015	-.007	-.013	-.014	-.003	-.011
		3				-.002	-.002	-.003	-.003	-.001	-.003	-.003	-.001	-.002
						(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
ALTERNATIVE 2														
Network Variables														
TOTAL TRAIN FARE	totrkb		.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01
			(-7.57)	(-6.32)	(-6.37)	(-7.07)	(-6.30)	(-6.46)	(-7.05)	(-5.65)	(-5.83)	(-8.39)	(-4.80)	(-5.48)
		2	-.132	-.406	-.403	-.128	-.404	-.388	-.110	-.331	-.291	-.131	-.279	-.246
		3	.012	.043	.043	.012	.043	.041	.010	.036	.031	.012	.031	.026
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
TOTAL TRAIN	totrzb		.99E+00	.42E+00	.43E+00	.11E+01	.44E+00	.48E+00	.11E+01	.47E+00	.61E+00	.93E+00	.58E+00	.73E+00
TRAVEL TIME			(-17.13)	(-15.47)	(-15.48)	(-17.15)	(-15.47)	(-15.60)	(-12.64)	(-13.59)	(-13.80)	(-14.92)	(-15.05)	(-15.14)
	(2)	1	.864	.902	.903	.923	.953	.970	.720	.773	.812	.748	.754	.784
		2	-.690	-1.127	-1.121	-.727	-1.160	-1.143	-.622	-1.054	-1.035	-.639	-1.072	-1.006
		3	.063	.120	.120	.065	.123	.120	.059	.115	.111	.060	.118	.108
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
TOTAL NUMBER	totumb					-.35E+02	-.44E+01	-.60E+01						
OF TRAIN TRANFERS						(2.34)	(1.41)	(1.66)						
	(2)	1				-.073	-.043	-.050						
		2				.059	.033	.039						
		3				-.005	-.003	-.003						
						(SP)	(SP)	(SP)						
TOTAL TRAIN AND	totfreqb					-.22E+01	-.17E+00	-.26E+00	-.18E+01	-.94E-01	-.27E+00	-.14E+01	-.54E-01	-.34E+00
ACCESS/EGRESS						(4.55)	(1.81)	(2.31)	(4.03)	(.92)	(1.75)	(3.70)	(.44)	(1.72)
FREQUENCY	(2)	1				-.222	-.066	-.091	-.131	-.027	-.057	-.117	-.012	-.055
		2				.292	.081	.112	.219	.037	.081	.198	.017	.084
		3				-.032	-.009	-.012	-.025	-.004	-.009	-.023	-.002	-.009

Table A. (Cont.) Linear, Logarithmic and Box-Cox Logit MK Model Comparison

W.A. MRS : totrkb		TYPE =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
(COND. T-STATISTICS)		VARIANT =	MKs	MKsL	MKsB	MK0	MK0L	MK0B	MKI	MKIL	MKIB	MKII	MKIIL	MKIIB
W.A. ELASTICITIES		VERSION =	1	1	1	1	1	1	1	1	1	1	1	1
		DEP.VAR. =	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel
Socioeconomic Variables														
AGE FROM 0 TO 9	age9								.10E+03	.31E+02	.41E+02	.70E+02	.30E+02	.39E+02
	====								(-.90)	(-1.30)	(-1.21)	(-.69)	(-1.01)	(-.87)
	(2)	1							.307	.508	.446	.205	.366	.280
		2							-.026	-.037	-.033	-.018	-.026	-.021
		3							-.002	-.005	-.004	-.002	-.003	-.003
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 10 TO 14	age14								.21E+02	.21E+02	.32E+02	.57E+02	.24E+02	.45E+02
	=====								(-.10)	(-.50)	(-.54)	(-.32)	(-.49)	(-.60)
	(2)	1							.073	.457	.488	.251	.446	.553
		2							-.003	-.018	-.020	-.010	-.018	-.022
		3							-.000	-.001	-.001	-.001	-.001	-.001
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 14 to 17	age17								-.27E+03	-.45E+02	-.68E+02	-.21E+03	-.48E+02	-.82E+02
	=====								(1.57)	(1.33)	(1.41)	(1.47)	(1.19)	(1.36)
	(2)	1							-.839	-.875	-.906	-.810	-.785	-.866
		2							.038	.045	.046	.038	.040	.044
		3							.002	.002	.002	.002	.002	.002
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 18 TO 24	age24								-.14E+03	-.12E+02	-.21E+02	-.98E+02	-.17E+02	-.31E+02
	=====								(1.55)	(.67)	(.84)	(1.38)	(.76)	(.99)
	(2)	1							-.414	-.222	-.268	-.397	-.256	-.322
		2							.037	.019	.023	.036	.021	.027
		3							.007	.004	.005	.006	.004	.006
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 25 TO 44	age44								-.44E+02	-.24E+01	-.56E+01	-.60E+02	-.16E+02	-.28E+02
	=====								(.63)	(.16)	(.27)	(1.05)	(.89)	(1.06)
	(2)	1							-.175	-.044	-.073	-.271	-.241	-.277
		2							.026	.006	.010	.042	.033	.039
		3							.005	.001	.002	.008	.007	.008
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 45 TO 64	age64								-.86E+02	-.13E+02	-.19E+02	-.74E+02	-.24E+02	-.37E+02
	=====								(1.20)	(.83)	(.91)	(1.27)	(1.30)	(1.41)
	(2)	1							-.334	-.230	-.247	-.335	-.351	-.368
		2							.044	.028	.030	.042	.041	.043
		3							.007	.005	.006	.008	.008	.009
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
TRIP ABROAD	au								-.58E+02	-.15E+02	-.27E+02			==
	(2)	1							-.152	-.243	-.293			
		2							.061	.074	.094			
		3							.016	.027	.032			
									(GE)	(GE)	(GE)			
WORKING PERSON	dberuf								.37E+03	.71E+02	.10E+03	.22E+03	.67E+02	.97E+02
	======								(-6.39)	(-5.97)	(-6.06)	(-4.57)	(-4.65)	(-4.62)
	(2)	1							1.448	1.261	1.272	.989	.967	.961
		2							-.294	-.222	-.229	-.196	-.167	-.170
		3							-.048	-.045	-.045	-.033	-.035	-.034
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
ONE-PERSON-HOUSEHOLD	dhh1								-.71E+02	-.14E+02	-.18E+02			
	====								(1.37)	(1.19)	(1.16)			
	(2)	1							-.232	-.215	-.205			
		2							.024	.021	.020			
		3							.004	.004	.004			
									(GE)	(GE)	(GE)			
SEX	sexo								.15E+03	.28E+02	.40E+02	.63E+02	.17E+02	.26E+02
	====								(-4.28)	(-3.60)	(-3.71)	(-2.02)	(-1.76)	(-1.89)
	(2)	1							.606	.511	.522	.292	.251	.269
		2							-.085	-.064	-.066	-.039	-.030	-.033
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
Trip Purpose														
BUSINESS TRIP	ge											.36E+03	.85E+02	.13E+03
	==											(-8.13)	(-7.17)	(-7.62)
	(2)	1										1.652	1.145	1.262
		2										-.429	-.261	-.289
		3										-.118	-.086	-.094
												(GE)	(GE)	(GE)
PRIVATE TRIP	pr											.68E+02	.62E+01	.19E+02
	==											(-1.54)	(-.52)	(-1.10)
	(2)	1										.404	.120	.253
		2										-.019	-.005	-.011
		3										-.002	-.001	-.001
												(GE)	(GE)	(GE)
REGRESSION CONSTANT	CONSTANT		-	-	-	-	-	-	-	-	-	-	-	-
			(11.85)	(7.99)	(7.97)	(7.32)	(6.31)	(6.07)	(7.92)	(8.04)	(7.81)	(7.04)	(7.79)	(7.45)
	(2)		-	-	-	-	-	-	-	-	-	-	-	-

Table A. (Cont.) Linear, Logarithmic and Box-Cox Logit MK Model Comparison

W.A. MRS : totrka		TYPE =	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT	LOGIT
(COND. T-STATISTICS)		VARIANT =	MKs	MKsL	MKsB	MK0	MK0L	MK0B	MKI	MKIL	MKIB	MKII	MKIIL	MKIIB
W.A. ELASTICITIES		VERSION =	1	1	1	1	1	1	1	1	1	1	1	1
		DEP.VAR. =	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel	vmittel
ALTERNATIVE 3														
Network Variables														
TOTAL CAR FARE	totrka		.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01	.10E+01
			(-7.57)	(-6.32)	(-6.37)	(-7.07)	(-6.30)	(-6.46)	(-7.05)	(-5.65)	(-5.83)	(-8.39)	(-4.80)	(-5.48)
	(3)	1	.262	.322	.326	.251	.325	.336	.371	.381	.397	.545	.312	.377
		2	.158	.320	.320	.151	.316	.311	.134	.267	.248	.153	.227	.216
		3	-.031	-.055	-.055	-.029	-.054	-.054	-.031	-.050	-.048	-.040	-.042	-.043
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
TOTAL CAR TRAVEL TIME	totrza		.99E+00	.80E+00	.80E+00	.11E+01	.83E+00	.83E+00	.11E+01	.92E+00	.97E+00	.93E+00	.11E+01	.11E+01
			(-17.13)	(-15.47)	(-15.48)	(-17.15)	(-15.47)	(-15.60)	(-12.64)	(-13.59)	(-13.80)	(-14.92)	(-15.05)	(-15.14)
	(3)	1	.674	.896	.895	.704	.933	.939	.920	1.211	1.235	.923	1.200	1.193
		2	.367	.889	.880	.388	.909	.874	.346	.851	.794	.355	.873	.759
		3	-.074	-.152	-.151	-.078	-.156	-.152	-.080	-.158	-.152	-.081	-.161	-.145
			(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)	(GE)	L 1(GE)	L 1(GE)
Socioeconomic Variables														
AGE FROM 0 TO 9	age9								.29E+02	.13E+02	.14E+02	.19E+02	.12E+02	.12E+02
	====								(-.90)	(-1.30)	(-1.21)	(-.69)	(-1.01)	(-.87)
	(3)	1							.622	.980	.872	.461	.770	.608
		2							-.053	-.080	-.072	-.037	-.057	-.044
		3							-.002	-.005	-.004	-.002	-.003	-.003
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 10 TO 14	age14								.62E+01	.91E+01	.11E+02	.15E+02	.95E+01	.13E+02
	=====								(-.10)	(-.50)	(-.54)	(-.32)	(-.49)	(-.60)
	(3)	1							.228	1.729	1.854	.754	1.708	2.111
		2							-.010	-.069	-.073	-.035	-.067	-.083
		3							-.000	-.001	-.001	-.001	-.001	-.001
(GE)	(GE)	(GE)	(GE)	(GE)	(GE)									
AGE FROM 14 to 17	age17								-.79E+02	-.19E+02	-.23E+02	-.57E+02	-.18E+02	-.25E+02
	=====								(1.57)	(1.33)	(1.41)	(1.47)	(1.19)	(1.36)
	(3)	1							-3.681	-5.019	-5.311	-3.624	-4.681	-5.346
		2							.160	.196	.200	.164	.175	.192
		3							.002	.002	.002	.002	.002	.002
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 18 TO 24	age24								-.40E+02	-.52E+01	-.73E+01	-.26E+02	-.65E+01	-.93E+01
	=====								(1.55)	(.67)	(.84)	(1.38)	(.76)	(.99)
	(3)	1							-.779	-.435	-.520	-.752	-.505	-.626
		2							.158	.078	.095	.154	.089	.114
		3							.007	.004	.005	.006	.004	.006
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 25 TO 44	age44								-.13E+02	-.10E+01	-.19E+01	-.16E+02	-.64E+01	-.82E+01
	=====								(.63)	(.16)	(.27)	(1.05)	(.89)	(1.06)
	(3)	1							-.260	-.063	-.106	-.387	-.338	-.389
		2							.052	.012	.020	.085	.067	.078
		3							.005	.001	.002	.008	.007	.008
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
AGE FROM 45 TO 64	age64								-.25E+02	-.55E+01	-.67E+01	-.20E+02	-.94E+01	-.11E+02
	=====								(1.20)	(.83)	(.91)	(1.27)	(1.30)	(1.41)
		2							.134	.085	.093	.133	.128	.137
		3							.007	.005	.006	.008	.008	.009
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
TRIP ABROAD	au								-.17E+02	-.65E+01	-.93E+01			
	==								(1.37)	(1.91)	(2.38)			
	(3)	1							-.251	-.393	-.476			
		2							.169	.158	.208			
		3							.016	.027	.032			
									(GE)	(GE)	(GE)			
WORKING PERSON	dberuf								.11E+03	.30E+02	.34E+02	.59E+02	.26E+02	.29E+02
	======								(-6.39)	(-5.97)	(-6.06)	(-4.57)	(-4.65)	(-4.62)
	(3)	1							2.314	1.936	1.961	1.488	1.426	1.415
		2							-.549	-.418	-.430	-.378	-.325	-.330
		3							-.048	-.045	-.045	-.033	-.035	-.034
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)
ONE-PERSON-HOUSEHOLD	dhh1								-.21E+02	-.58E+01	-.63E+01			
	====								(1.37)	(1.19)	(1.16)			
	(3)	1							-.851	-.741	-.711			
		2							.200	.185	.176			
		3							.004	.004	.004			
SEX	sexo								.44E+02	.12E+02	.14E+02	.17E+02	.65E+01	.78E+01
	====								(-4.28)	(-3.60)	(-3.71)	(-2.02)	(-1.76)	(-1.89)
	(3)	1							.911	.739	.758	.407	.346	.370
		2							-.181	-.138	-.143	-.086	-.068	-.073
		3							-.017	-.015	-.015	-.008	-.008	-.008
									(GE)	(GE)	(GE)	(GE)	(GE)	(GE)

Table A. (Cont.) Linear, Logarithmic and Box-Cox Logit MK Model Comparison

W.A. MRS : totrka (COND. T-STATISTICS) W.A. ELASTICITIES			TYPE = VARIANT = VERSION = DEP.VAR. =	LOGIT MKs 1 vmittel	LOGIT MKsL 1 vmittel	LOGIT MKsB 1 vmittel	LOGIT MK0 1 vmittel	LOGIT MK0L 1 vmittel	LOGIT MK0B 1 vmittel	LOGIT MKI 1 vmittel	LOGIT MKIL 1 vmittel	LOGIT MKIB 1 vmittel	LOGIT MKII 1 vmittel	LOGIT MKIIL 1 vmittel	LOGIT MKIIB 1 vmittel
Trip Purpose															
BUSINESS TRIP	ge												.96E+02	.33E+02	.40E+02
	==												(-8.13)	(-7.17)	(-7.62)
	(3)	1											2.268	1.552	1.706
		2											-.758	-.472	-.515
		3											-.118	-.086	-.094
													(GE)	(GE)	(GE)
PRIVATE TRIP	pr												.18E+02	.24E+01	.58E+01
	==												(-1.54)	(-.52)	(-1.10)
	(3)	1											.987	.269	.578
		2											-.072	-.020	-.044
		3											-.002	-.001	-.001
													(GE)	(GE)	(GE)
REGRESSION CONSTANT	CONSTANT			-	-	-	-	-	-	-	-	-	-	-	-
				(34.77)	(25.30)	(25.27)	(32.69)	(13.05)	(14.25)	(17.12)	(12.50)	(14.06)	(14.22)	(12.49)	(14.04)
	(3)			-	-	-	-	-	-	-	-	-	-	-	-

P A R A M E T E R S UNCOND.[T-STATISTICS=0] UNCOND.[T-STATISTICS=1]		TYPE = VARIANT = VERSION = DEP.VAR. =	LOGIT MKs 1 vmittel	LOGIT MKsL 1 vmittel	LOGIT MKsB 1 vmittel	LOGIT MK0 1 vmittel	LOGIT MK0L 1 vmittel	LOGIT MK0B 1 vmittel	LOGIT MKI 1 vmittel	LOGIT MKIL 1 vmittel	LOGIT MKIB 1 vmittel	LOGIT MKII 1 vmittel	LOGIT MKIIL 1 vmittel	LOGIT MKIIB 1 vmittel
BOX-COX TRANSFORMATIONS														
LAMBDA(X) - GROUP 1	LAM 1			.000	.016		.000	.073		.000	.160		.000	.240
				FIXED	[.27]		FIXED	[1.15]		FIXED	[2.40]		FIXED	[3.31]
					[-16.69]			[-14.51]			[-12.59]			[-10.48]

GENERAL STATISTICS	TYPE = VARIANT = VERSION = DEP.VAR. =	LOGIT MKs 1 vmittel	LOGIT MKsL 1 vmittel	LOGIT MKsB 1 vmittel	LOGIT MK0 1 vmittel	LOGIT MK0L 1 vmittel	LOGIT MK0B 1 vmittel	LOGIT MKI 1 vmittel	LOGIT MKIL 1 vmittel	LOGIT MKIB 1 vmittel	LOGIT MKII 1 vmittel	LOGIT MKIIL 1 vmittel	LOGIT MKIIB 1 vmittel
LOG-LIKELIHOOD -FINAL VALUE		-1392.96	-1309.08	-1309.04	-1383.65	-1306.90	-1306.22	-1292.35	-1234.64	-1231.69	-1230.66	-1195.20	-1189.35
-INITIAL VALUE		-1392.96	-1309.08	-1309.04	-1383.65	-1306.90	-1306.22	-1292.35	-1234.64	-1231.69	-1230.66	-1195.20	-1189.35
-WITH CONSTANTS ONLY		-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56	-1663.56
-RATIO TEST		541.196	708.958	709.030	559.816	713.318	714.674	742.425	857.831	863.740	865.799	936.720	948.421
RHO-SQUARED		.163	.213	.213	.168	.214	.215	.223	.258	.260	.260	.282	.285
RHO-SQUARED BAR -AKAIKE		.160	.211	.210	.165	.211	.211	.214	.249	.250	.251	.273	.275
-HOROWITZ		.161	.212	.212	.166	.213	.213	.219	.253	.255	.256	.277	.280
-HENSHER AND JOHNSON		.162	.213	.213	.168	.214	.214	.222	.257	.259	.259	.281	.284
PER CENT RIGHT		93.933	93.867	93.867	93.917	93.850	93.867	94.050	93.967	94.017	94.250	94.317	94.217
SAMPLE -NUMBER OF ALTERNATIVES		3	3	3	3	3	3	3	3	3	3	3	3
-NUMBER OF OBSERVATIONS		6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
-FIRST OBSERVATION		1	1	1	1	1	1	1	1	1	1	1	1
-LAST OBSERVATION		6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
-AVAILABLE OBSERVATIONS		6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
1		6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
2		6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000	6000
3		5442	5442	5442	5442	5442	5442	5442	5442	5442	5442	5442	5442
NUMBER OF ESTIMATED PARAMETERS													
-BETAS .VARIABLES		2	2	2	4	4	4	13	13	13	13	13	13
.CONSTANTS		2	2	2	2	2	2	2	2	2	2	2	2
.ASSOCIATED DUMMIES		0	0	0	0	0	0	0	0	0	0	0	0
-BOX-COX TRANSFORMATIONS		0	0	1	0	0	1	0	0	1	0	0	1