Car taxation in a horizontal differentiation model*

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Abstract

Sport utility vehicles (SUVs) emit more pollutants than sedans do. I use a horizontal differentiation model modified to include externality. Externality production depends on firms' locations. There are two firms producing each a car model. In the first part, the travelled distance per car is fixed. In an extreme locations setting, as the per-distance tax increases, the demand for that car and the pollution level decrease. When product selection is allowed, firm A always produces the less pollutant car. There is always some differentiation. Pollution always decreases with a tax. In a second part, I use quasilinear preferences in an extreme locations setting. The per-distance tax does necessarily induce a decrease in the demand for the SUV. The impact of the tax depends on the difference between the fuel consumption and emission rate parameters. Finally, there is a possibility that the pollution level increases following a tax: people may purchase dirtier vehicles or people may purchase cleaner vehicles but travel more.

1 Introduction

Worldwide governments aware about the Earth's climate change and air pollution in big cities but the production of pollutant emissions do not seem to alleviate. An important part of polluting emissions comes from the transportation sector.

A sport utility vehicle (SUV) consumes as much as 40% more fuel than a sedan. For example, a typical SUV driven 20,000 km a year produces about 6 tonnes of CO_2, compared to 4 tonnes for a sedan[12]. Figure 1 shows the gas consumption for different SUVs and sedans on highways and in urban areas. In addition to the fuel consumption, the SUVs produce more pollutant per kilometre than a sedan. They are usually classified in a more pollutant bin than the sedans.

Previously designed for rural activities, SUVs seem now to invade urban regions and are used in the same way as simple cars.

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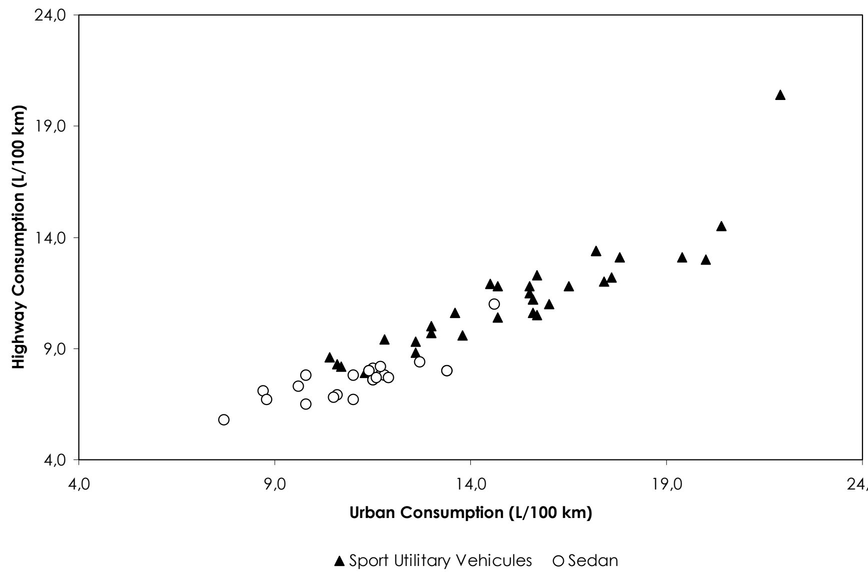

Figure 1: Gas consumption for different SUVs vs. Sedans. Source:[26]

This paper addresses the question of car pollution in a model of horizontal differentiation. Vertical models have been used to study the effect of taxation in the car market. I claim that this market presents characteristics of horizontal differentiation.

Myles and Uyduranoglu [19] use a vertical model in which the more performing car is ranked better by every consumer. Better performing cars are associated with higher fuel consumption. Surprisingly, they find that, in some circumstances, it may be optimal to subsidise larger cars. Also, in their model, everything on the supply side is exogenous: the models of cars produced are given, and the firms do not adjust their prices when a tax is announced.

This paper allows for the endogeneity of the production side. First, prices are endogenous. Second, an attempt is done to incorporate the product selection (models of cars) as a choice variable. Then, the impact of a car-specific tax on the market structure is studied. I am also interested in the impact of such a tax on the pollution level.

The fuel tax is the principal policy tool to control for car pollution now, I investigate under what conditions is such a tax optimal. The results of a horizontal model are also compared with the ones of a vertical model.

The paper develops a model in which the impact of a per-distance tax and fuel tax on car demands, distance demands, market structure and pollution level can be analysed.

There are two main variants of the model in this paper. In section 3, the travelled distance with a car is assumed to be fixed. I examine both cases of with and without product selection. In section 4, I use quasilinear preferences to endogenise the travelled distances and the case without product selection is examined. Then section 5 summarises the results. Before that, I justify the use of a horizontal model and give details on air pollution and regulations in the next section.

2 Horizontal characteristics, Air pollution and Regulations

2.1 A horizontal model

A vertical model is a model in which all the consumers have the same ranking over the goods offered. If all the goods have the same prices, the whole demand will be for only one of the goods. Cremer and Thisse [5] introduce a commodity tax in a vertical differentiation model with endogenous product selection. However, their model is general and does not include an externality. Myles and Uyduranoglu [19] examine a vertical differentiation model where the more performing car is ranked better by every consumer. Better performing cars are associated with higher fuel consumption. They find that, in some cases, it may be optimal to subsidise larger cars.

On the other side, we see that consumers are willing to pay a higher price for environment-friendly products compared to standard versions of the same good. Cremer and Thysse [6] explain it as follows: environment-friendliness for a product is considered as a quality attribute from every consumer. In this context, they use a model of vertical differentiation to study the production of environmental quality. They also study how the taxation can affect the market structure. Moraga-Gonzalez and Padron-Fumero [18] examine the impact of policy on aggregate emissions and welfare impact in the same kind of model.

We can easily see the conflict between the two vertical characteristics here. On one side, performance (which implies higher pollution) is seen as a quality attribute (as in [19]). On the other side, environment-friendliness (which implies low pollution from cars) is also seen as a quality attribute (as in [6]). These two assumptions on preferences cannot survive together in a same vertical model.

In the present paper, I use a horizontal differentiation model to describe the car market. A horizontal model is one in which all the consumers do not agree over the ranking of the goods. If all the goods have the same price, the demand of each good will be positive. There are some reasons that make a horizontal model appropriate. First, cars would be poorly described by the use of homogeneous products. Since different models of car pollute differently and we are interested in pollution policy, this setting is not appropriate. Second, vertical models have already been developed and it would be interesting to compare results from both models. Finally, this market presents important horizontal characteristics.

Of course a compact car is not comparable to a high-quality large car. In that case, a vertical model would probably describe the market in a better way. That is why the present horizontal model cannot describe the whole market. I see this model like a part of the whole market where, first, a consumer chooses a category of car allowed by his income. That corresponds to a vertical choice. Second, the consumer is now in a horizontal model where he has to choose between more or less polluting and higher or lower performing cars. The characteristics of the vehicles are as follows.

Trade-off between design, performance and environmental-friendliness – As mentioned, vertical models are used in two distinct ways in this literature. On one side, car performance (associated with higher pollution) is evaluated as a quality attribute. On the other side, environment-friendliness (which implies low pollution) is also seen as a quality attribute. These two assumptions on preferences cannot survive together in a same vertical model.

A horizontal model allows to incorporate these two attributes together. People then face a trade-off between the importance that they give to those two preferences. Consumers who put a higher weight on environment-friendliness are located differently on the Hotelling line than consumers who prefer high performance. This trade-off, when all characteristics are known and evaluated, leads the consumer to locate on the Hotelling line according to his preferences.

Comfort and Security – Since the cars in the model are assumed to be in the same quality bin, the comfort of both vehicles are about the same.

Some people may argue that a SUV is safer than a sedan. Gayer [10] estimates that, in a crash, a light truck (SUV) driver is 0.29 to 0.69 times as likely to die than is a car driver. However, the crash frequency estimates suggest that light trucks are 2.3 times as likely to get in a crash than cars are. When crash frequency is taken into account, a light truck is as dangerous as a car is[1]. Then, it seems there would be no major difference in the safety of a SUV as compared to a sedan.

Since comfort and security are about the same for the two vehicles, no vertical characteristics seem to apply here.

2.2 Air pollution

To study car pollution, it is important to understand the pollution created. There are mainly two broad categories of pollutant emissions from transportation: the pollutants that contribute to poor air quality and the pollutants that contribute to climate change.

The first category is composed of smog-forming emissions (e.g. volatile organic compounds (VOCs), oxides of nitrogen (NO_x), particulate matter (PM), sulphur oxides (SO_x), and carbon monoxide (CO)) and toxic pollutants (e.g. benzene, 3-butadiene, formaldehyde, acetaldehyde and acrolein). These gases contribute to

[1] The author controls for the heterogeneity of the samples.

health diseases such as respiratory troubles. Vehicles are equipped with emission controlled technologies that substantially reduce the release of these pollutants. Therefore, there is no specific relation between fuel consumption and emission rate of a vehicle.

Greenhouse gases (GHG) that contribute to climate change include carbon dioxide (CO_2 – the most important one), nitrous oxide(N_2O) and methane (CH_4). The Kyoto Protocol has as objective to reduce international emissions of greenhouse gases by 5.2% below the 1990 levels by the period 2008-2012. Canada's target is to reduce its own emissions by 6%[13]. In Canada, more than one quarter of GHG emissions comes from the transportation sector. It is more than 40% for the British Columbia and territories[13]. Unlike the pollutants from the first category, GHG emissions from vehicles are not controlled by emission control equipment. The carbon dioxide emissions are proportional to fuel consumption and can be estimated by assuming that all carbon in the fuel consumed is emitted as CO_2.

2.3 On-Road Vehicle and Engine Emission Regulations

The Canadian Environmental Protection Act (see [11]) regulates on-road pollution in Canada. The purpose of these regulations is to reduce emissions of diverse pollutants and establish standards that are aligned with the United States Environmental Protection Agency (EPA).

Table 1 presents the average oxides of nitrogen (NO_x) standard that a company's fleet, composed of all its light-duty vehicles and light light-duty trucks, cannot exceed. Values are in grams per mile (g/mi).

Table 1. NO_x standard for Light-duty vehicles and Light light-duty trucks

Model Year	Fleet Average NO_x (g/mi)
2004	0.25
2005	0.19
2006	0.13
2007	0.07
2008	0.07

The average NO_x value is computed as follows:

$$\left[\sum \left(A \times B\right)/C\right]$$

where A is the NO_x emission rate for each full useful life emission bin; B is the number of vehicles in the fleet that conform to that rate; C is the total number of vehicles in the fleet. It means that two models in a same fleet can produce different emission rates (in grams per mile). For example, say a company produces a sedan and a SUV in the same proportion. If the sedan emits 0.2 g/mi and the SUV, 0.3 g/mi, the average emission of the fleet will be 0.25 and will respect the regulation.

2.4 The insufficiency of a fuel tax

A standard economic approach to externality is a pigovian tax. In the transportation sector, fuel burning is responsible for the emissions. In a complete and perfect information setting, we would know exactly how much of pollutants each vehicle produces. The government then sets a tax in a way such that the reduction goal is met.

The main policy tool used now is the fuel tax. However, such a tax cannot be sufficient to reproduce a pigovian tax. The next example shows easily this point.

Take two cars that have different fuel consumptions but the same emission rate. Say that the environmental fuel tax is 0.20$/litre and that a gram of NO_x produces an externality of a value 1$/gram. Table 2 shows fictitious data for a travelled distance of one kilometre.

Table 2. Example

Cars	Fuel cons.(l/100km)	Emis.rate(g/km)	Tax	Externality	Extern. not paid
A	8	0.2	1.6	2	0.4
B	10	0.2	2	2	0

It is obvious that since the emission rate is not directly related to the fuel consumption, a fuel tax is not sufficient to perfectly internalise the externality cost.

3 Inelastic demand for distance

In the first part, I assume that all consumers are homogeneous in their demand for distance; they travel all the same fixed distance. Graham and Glaister [14] report a range of estimates in the literature, and the short-run fuel price elasticity is around -0.25. They get the same estimates. With such estimates, the demand for fuel is inelastic enough in the short-run and the assumption that the travelled distance is fixed seems reasonable.

By the regulation requirement, we know the emission rate for each vehicle, the externality per car can be computed,

$$E = e \cdot r \cdot d$$

where e is the **externality cost per gram** released (in \$/gram)[2]; r is the **emission rate** of the vehicle (in grams/kilometre); d is the travelled **distance** (in kilometres). The product of those variables, E, represents the **externality cost per car** (in \$).

[2] In the case of the emissions contributing to the poor air quality, the externality cost is a function of the population who bear that externality and of the initial pollution level. Then, the cost would be higher in urban areas thant in rural ones. In the fixed distance setting, the model can be slightly modified: a certain distance assumed to be travelled in each area. For greenhouse gases, they contribute to the climate change wherever emissions are done.

The market is defined as follows. A continuum of consumers is assumed to be located on a Hotelling line of unit length. I assume uniform distribution with density 1 along the line[3]. Consumers are located on the Hotelling line according to their fundamental preferences towards design, performance and environmental-friendliness. Some consumers will value more the performance than the environmental-friendliness of the vehicle, and consequently, will be located more to the right. And conversely, some will be more to the left.

There are two firms in the market. Each firm produces one model; firm A produces a sedan and firm B produces a SUV. As a firm locates more right on the Hotelling line, the vehicle that it produces is closer to the design of a SUV, which is more powerful, but also is less environmental-friendly. I assume that there is full coverage of the market by the two firms.

The two vehicles are different in emission rate and fuel consumption. The emission rates for cars A and B are respectively r and $(1+\gamma)\,r$, where $\gamma > 0$. The fuel consumption of cars A (sedan) and B (SUV) are, respectively, fc and $(1+\delta)fc$, where $\delta > 0$.

3.1 Extreme locations

I first analyse the situation where the two firms are located at each extremity of the product spectrum. Firms choose only their price. That can be the case when the government implements a new regulation and the firms' production possibilities are fixed.

3.1.1 Consumer's utility and demands for the cars

Each consumer has a unit demand for cars. The utility that the consumer derives from the consumption (or the use) of the car is:

$$\begin{aligned} u_A &= s - p_A - tx^2 - F_A - T_A \\ &= s - p_A - tx^2 - (p_f fc + t_d)\,\overline{d} \end{aligned}$$

for car A, and:

$$\begin{aligned} u_B &= s - p_B - t\,(1-x)^2 - F_B - T_B \\ &= s - p_B - t\,(1-x)^2 - ((1+\delta)p_f fc + (1+\gamma)t_d)\,\overline{d} \end{aligned}$$

for car B.

s is the (gross) **surplus** enjoyed from the purchase of a car. It is assumed to be large enough that consumers do not drop the market. p_i is the **price of the**

[3] That means the number of consumers in the market is fixed and does not vary.

car produced by firm i ($i = A, B$). t is the quadratic[4] **"transportation" cost**[5]. It represents the "cost" born by a consumer who cannot purchase the model that he would like. Farther from the consumer on the line is the purchased model, higher is the "transportation" cost for that consumer. x is the **location** of the consumer on the Hotelling line.

F_i is the present total fuel cost. It can be expressed as:

$$F_i = \sum_{y=0}^{Y} \frac{1}{(1+\rho)^y} (p_f \cdot fc_i \cdot d_i / Y)$$

The parenthesis contains the total fuel payment per year. fc_i is the **fuel consumption** for the car i (in litres/km). Y is the number of years that the car can be used. ρ is the discount factor. However, to simplify the problem, I assume that the discount factor is equal to 0 and that d is the total distance travelled. The **fuel cost** simplifies to $F_i = p_f fc_i d$. Myles and Uyduranoglu [19] use the term "running costs" instead of fuel cost to include insurance and registration fees. It can be included here or ignored without much loss of generality.

T_i is the vehicle tax. We want a tax that will cover the externality. In that setting, we know exactly the externality created by each car,

$$E_i = e \cdot r_i \cdot d$$

No measures of fuel consumption appear in the car externality cost, a fuel tax is not necessary to internalise the externality; a vehicle tax is sufficient. Since we know the pollution per unit of distance by knowing the rate emission, the policy tool chosen is a **per-distance tax**, t_{di}. In that case, the vehicle tax is $T_i = t_{di} d$.

Optimally, t_{di} is equal to er_i[6]. I can already compare that result with the one of Myles and Uyduranoglu in a vertical environment. Unlike them, a subvention on the bigger car is not optimal. With exogenous product selection, the optimal tax is equal to the externality and the externality is bigger with the SUV than in the sedan.

Even if we know the optimal tax, I keep the expression t_d for further comparative statics, to evaluate situations when the government sets the tax differently than at its optimal level. The per-distance tax is car-specific and depends on the emission rate, $t_{dA} = t_d$ and $t_{dB} = (1+\gamma)\, t_d$.

The consumer $\widetilde{x}$ is indifferent between car A and car B if and only if $u_A = u_B$:

$$s - p_A - tx - (p_f fc + t_d)\, \overline{d} = s - p_B - t\,(1-x) - ((1+\delta) p_f fc + (1+\gamma) t_d)\, \overline{d}$$

[4] Quadratic costs ensure the existence of an equilibrium in the case of product selection (see [7]).

[5] The use of the term "transportation" cost may be confusing here since we are talking about tranport. But that "transportation" cost has to do only with preferences. That is why I put transportation in quotation marks.

[6] It is the case in an exogeneous product selection setting. However, when product selection is endogeneous, private product choices may differ than social optimal ones and in that case the optimal tax may not be equal to the externality.

Solving for $\widetilde{x}$:

$$\widetilde{x} = \frac{(p_B - p_A) + (\delta p_f fc + \gamma t_d)\,\overline{d} + t}{2t}$$

Demands (market shares) are found using the above equation,

$$D_A(p) = \widetilde{x} = \frac{(p_B - p_A) + (\delta p_f fc + \gamma t_d)\,\overline{d} + t}{2t} \tag{1}$$

$$D_B(p) = (1 - \widetilde{x}) = \frac{(p_A - p_B) - (\delta p_f fc + \gamma t_d)\,\overline{d} + t}{2t} \tag{2}$$

3.1.2 Profit maximisation problem

Each firm maximises its profit given the demand that it faces. I assume that the production cost for both firms is constant and is denoted c_A and c_B. I further assume that the production cost of a SUV is not lower than the sedan's : $c_B \geq c_A$.

Firm i's problem is:

$$\Pi_i\,(p) = \max_{p_i}\,(p_i - c_i)\,D_i(p)$$

Using Equations 1 and 2 in this problem, the FOCs lead to the reaction functions,

$$p_A = \frac{p_B + (\delta p_f fc + \gamma t_d)\,\overline{d} + t + c_A}{2} \tag{3}$$

$$p_B = \frac{p_A - (\delta p_f fc + \gamma t_d)\,\overline{d} + t + c_B}{2} \tag{4}$$

Using Equations 3 and 4, I solve for p_A and p_B.

$$p_A^* = t + \frac{2}{3}c_A + \frac{1}{3}\left(c_B + (\delta p_f fc + \gamma t_d)\,\overline{d}\right)$$

$$p_B^* = t + \frac{2}{3}c_B + \frac{1}{3}\left(c_A - (\delta p_f fc + \gamma t_d)\,\overline{d}\right)$$

For given optimal prices, demands (Equations 1 and 2) are:

$$D_A(p^*) = \frac{1}{6t}\left(3t - c_A + c_B + (\delta p_f fc + \gamma t_d)\,\overline{d}\right)$$

$$D_B(p^*) = \frac{1}{6t}\left(3t + c_A - c_B - (\delta p_f fc + \gamma t_d)\,\overline{d}\right)$$

3.1.3 Comparative statics

The inefficiency of a fuel tax – Now that I have the demands for cars, I can see how they vary when the vehicle tax[7] or the fuel price change as,

$$\frac{\partial D_A(p^*)}{\partial t_d} = \frac{\gamma}{6t}d > 0$$
$$\frac{\partial D_A(p^*)}{\partial p_f} = \frac{\delta fc}{6t}d > 0$$

An increase in either the per-distance tax or fuel price would lead to an increase in the demand for car A. Since, the demand for B is the opposite of car A, this increase is exactly equal to the decrease in the demand for car B.

Proposition 1 *A fuel tax is more likely to be suboptimal.*

Proof. A marginal change in the fuel price can reproduce a marginal change in the per-distance tax if and only if $\delta fc = \gamma$. Since the per-distance tax is the optimal tool to internalise completely the externality, a fuel tax is suboptimal if $\delta fc \neq \gamma$. ∎

A production tax on Firm B (or production subsidy on Firm A) would have the following marginal impact,

$$\frac{\partial D_A(p^*)}{\partial t_B} = \frac{1}{6t} > 0$$

The marginal effect of such a tax would be the same as the per-distance tax if and only if $\gamma d = 1$.

A change in the "transportation" cost – The differential of demand with respect to the "transportation" cost is,

$$\frac{\partial D_A(p^*, T)}{\partial t} = \frac{1}{6t^2}\left(c_A - c_B - \left(\delta p_f fc + \gamma er\right) d\right) < 0$$

As the "transportation" cost decreases, the demand for car A increases. A decrease in the "transportation" cost can be seen as the fact that consumers become more indifferent towards the heterogeneity of the goods. A non fiscal way for the government to encourage the purchase of car A would be to decrease the "transportation" cost. This result shows that it is not necessary to encourage the purchase of cleaner cars and denounce the purchase of dirtier ones[8], increasing the homogeneity of the goods

[7] Note that when I talk about the vehicle tax (or the tax per unit of distance), this is not a flat tax; this is a differentiated tax according to the emission rate. The tax is then t_d for car A, but $(1+\gamma)\,t_d$ for car B. An increase of 1 in the tax for car A implies an increase of $(1+\gamma)$ for car B.

[8] On this point, in the last year in the United States, the Evangilical Environmental Network launched the campaign "What would Jesus drive?" arguing that Jesus would not drive an "amoral" vehicle such as a SUV that destroys the Earth's environment. The SUVs Owners of America responded by a campaign in which they showed a man named Jesus (Jesus Riviera) who drives, like a lot of Americans, a SUV.

in the consumer's mind is enough[9].

Reduction in pollution − The pollution level is equal to the number of cars A bought times their externality plus the number of cars B times their externality. That is,

$$\begin{aligned} P &= D_A \cdot rd + D_B \cdot (1+\gamma)\, rd \\ &= (1+\gamma D_B)\, rd \end{aligned}$$

The externality cost can be computed as follows,

$$EC = e \cdot P$$

The differentials with respect to tax t_d are,

$$\begin{aligned} \frac{\partial P}{\partial t_d} &= \frac{\partial D_B(p^*)}{\partial t_d} rd = -\frac{1}{6t}\left(\gamma d\right)^2 r \\ \frac{\partial EC}{\partial t_d} &= \frac{\partial P}{\partial t_d} e = -\frac{1}{6t}\left(\gamma d\right)^2 er \end{aligned}$$

Since the distance is fixed, the reduction in pollution following is directly proportional to the shift in demands.

The higher e, γ or r are, higher is the tax; and therefore, bigger are the reduction in pollution and externality cost. Again, the lower is t, the less impact has the taxation on the pollution reduction.

3.2 Product selection

In this section, I examine the case where the two firms can choose their location on the product spectrum. Knowing that the government will implement a tax, firms may respond by producing cleaner vehicles. So far, in previous literature on car pollution, the market structure is given and exogenous. This section is an attempt to endogenise the market structure and to study how firms' responses to fiscal incentives may affect the pollution level.

To endogenise the product selection, I have to change the previous model in two ways: the parameters δ and γ have to be flexible. Remember that the sedan (less pollutant) is located at zero and the SUV (more pollutant) at one. Any type of car between locations zero and one should be in the range of fuel consumption and emission rate between the two base types. Hence, the fuel consumption and the rate emission are function of the location of the firm l (where $l = [a, (1-b)]$ and $a, b \in [0,1]$). These functions are denoted $\Delta(l)$ and $\Gamma(l)$ for the fuel consumption

[9] This could be done by an advertising campaign by example.

and the rate emission respectively. As a firm locates more to the right, the vehicle produced is more performing, so more pollutant, that is[10],

$$\frac{\partial \Delta(l)}{\partial l} > 0 \text{ and } \frac{\partial \Gamma(l)}{\partial l} \geq 0.$$

For the sake of computations, I impose a functional form to these functions. I assume that the parameters are increasing monotonically[11],

$$\begin{aligned}\Delta(l) &= 1 + (\delta \cdot l) \\ \Gamma(l) &= 1 + (\gamma \cdot l)\end{aligned}$$

3.2.1 Consumer's utility and demands for the cars

In this setting, the utility that the consumer derives is:

$$u_A = s - p_A - t(x-a)^2 - \Gamma(a)\, t_d d - \Delta(a) F$$

for car A, and:

$$u_B = s - p_B - t(1-b-x)^2 - \Gamma(1-b)\, t_d d - \Delta(a) F$$

for car B.

As shown in the utility functions, the vehicle tax is now equaled to $\Gamma(l)\, t_d d$. Again, the tax is specific to the vehicle emission rate. The basic per-distance tax is t_d (for the vehicle that would be produced at location 0) and increases as the emission rate increases[12]. Similarly, F is the basic fuel cost associated with the vehicle that would be located at 0. The fuel cost is increasing as the fuel consumption increases.

Plugging the functional forms for $\Delta(l)$ and $\Gamma(l)$ in the utility functions,

$$\begin{aligned}u_A &= s - p_A - t(x-a)^2 - (1 + a\gamma)\, t_d d - (1 + a\delta) F \\ u_B &= s - p_B - t(1-b-x)^2 - (1 + ((1-b)\gamma))\, t_d d - (1 + (1-b)\delta) F\end{aligned}$$

The consumer $\widetilde{x}$ is indifferent between car A and car B if and only if $u_A = u_B$,

$$\begin{aligned}&s - p_A - t(x-a)^2 - (1 + a\gamma)\, t_d d - (1 + a\delta) F = \\ &\qquad s - p_B - t(1-b-x)^2 - (1 + ((1-b)\gamma))\, t_d d - (1 + (1-b)\delta) F\end{aligned}$$

[10] The emission rate is assumed to be non-decreasing in l (instead of increasing) because a better performant vehicle can be equipped of emission control technology and have the same emission rate of a less performant vehicle.

[11] I assume that these functions are known to the consumers when choosing their location on the Hotelling line.

[12] Here, unlike the previous section with extreme locations, the optimal tax is not necessarily equal to the marginal damage. By locating at each extreme of the line, there are loss in "transportation" costs. The society would be better off (when there is no externality) with the firms less separated – formally at 1/4 and 3/4, to minimise the average transportation cost –. Then, the optimal tax depends, in addition to the marginal damage, the incentive that it gives to the Firms to move their location to increase competitiveness. I come to that point latter.

Solving for $\widetilde{x}$,

$$\widetilde{x} = a + \frac{(1-a-b)}{2} + \frac{p_B - p_A}{2t(1-a-b)} + \frac{\gamma t_d d + \delta F}{2t}$$

The demands are found using the last equation:

$$\begin{aligned} D_A(p) &= \widetilde{x} = a + \frac{(1-a-b)}{2} + \frac{p_B - p_A}{2t(1-a-b)} + \frac{\gamma t_d d + \delta F}{2t} && (5) \\ D_B(p) &= (1-\widetilde{x}) = b + \frac{(1-a-b)}{2} + \frac{p_A - p_B}{2t(1-a-b)} - \left(\frac{\gamma t_d d + \delta F}{2t}\right) && (6) \end{aligned}$$

3.2.2 Profit maximisation problem

Each firm maximises its profit given the demand that it faces. I assume a two-stage game in which the firms choose simultaneously their location, and then, their price. I solve the problem by backward induction.

Second stage of the profit maximisation game : Choice of prices - The firm i's problem is,

$$\Pi_i(p) = \max_{p_i} (p_i - c_i) D_i(p)$$

Using Equations 5 and 6, the FOCs with respect to p_i are for firms A and B, respectively,

$$\begin{aligned} p_A &= \frac{1}{2}\left(c_A + p_B + (1-a-b)(\gamma t_d d + \delta F) + t - ta^2 - 2tb + tb^2\right) \\ p_B &= \frac{1}{2}\left(c_B + p_A - (1-a-b)(\gamma t_d d + \delta F) + t - tb^2 - 2ta + ta^2\right) \end{aligned}$$

The solutions for that two-equation system are,

$$\begin{aligned} p_A^*(l) &= \frac{1}{3}(2c_A + c_B) + t(1-a-b)\left(1 + \frac{a-b}{3}\right) + \frac{1}{3}(1-a-b)(\gamma t_d d + \delta F) && (7) \\ p_B^*(l) &= \frac{1}{3}(2c_B + c_A) + t(1-a-b)\left(1 + \frac{b-a}{3}\right) - \frac{1}{3}(1-a-b)(\gamma t_d d + \delta F) && (8) \end{aligned}$$

Using them in the demand functions and working out some algebra, I get,

$$\begin{aligned} D_A(p^*(l), l) &= \frac{1}{2} + \frac{a-b}{6} + \frac{(c_B - c_A)}{6t(1-a-b)} + \frac{\gamma t_d d + \delta F}{6t} && (9) \\ D_B(p^*(l), l) &= \frac{1}{2} + \frac{b-a}{6} + \frac{(c_A - c_B)}{6t(1-a-b)} - \left(\frac{\gamma t_d d + \delta F}{6t}\right) && (10) \end{aligned}$$

First stage of the profit maximisation game : Choice of locations - In the first stage, both firms maximise their profit by choosing their location given optimal prices.

The firm A's problem is:

$$\Pi_A(p,l) = \max_a \left(p_A^*(l) - c_A\right) D_A(p^*(l), l)$$

Using Equation 9, the FOC with respect to a is:

$$\left(-t\left(1+\frac{a-b}{3}\right)+\frac{t(1-a-b)}{3}-\left(\frac{\gamma t_d d+\delta F}{3}\right)\right)\left(\frac{1}{2}+\frac{b-a}{6}+\frac{(c_B-c_A)}{6t(1-a-b)}+\frac{\gamma t_d d+\delta F}{6t}\right)$$
$$+\left(\frac{1}{3}(c_B-c_A)+t(1-a-b)\left(1+\frac{a-b}{3}\right)+\frac{1}{3}(1-a-b)(\gamma t_d d+\delta F)\right)$$
$$*\left(-\frac{1}{6}+\frac{c_B-c_A}{6t(1-a-b)^2}\right)=0$$

Similarly, the firm B's FOC with respect to b is,

$$\left(-t\left(1+\frac{b-a}{3}\right)+\frac{t(1-a-b)}{3}+\frac{\gamma t_d d+\delta F}{3}\right)\left(\frac{1}{2}+\frac{b-a}{6}+\frac{(c_A-c_B)}{6t(1-a-b)}-\left(\frac{\gamma t_d d+\delta F}{6t}\right)\right)$$
$$+\left(\frac{1}{3}(c_A-c_B)+t(1-a-b)\left(1+\frac{b-a}{3}\right)-\frac{1}{3}(1-a-b)(\gamma t_d d+\delta F)\right)$$
$$*\left(\frac{1}{6}+\frac{c_A-c_B}{6t(1-a-b)^2}\right)=0$$

Solving for these FOCs gives four possible solutions for a and four for b. It is more instructive to proceed by marginal analysis.

3.2.3 Marginal analysis in product selection

D'aspremont et al. [7] show that for quadratic "transportation" costs, the equilibrium exists and is obtained at maximal differentiation ($a = b = 0$). The firm A locates where $\frac{\partial \Pi_A}{\partial a} = 0$ and B, where $\frac{\partial \Pi_B}{\partial b} = 0$. Without loss of generality, I use the firm A as an example. By the envelope theorem, we know that $\partial \Pi_A / \partial p_A^* = 0$. Then, $\partial \Pi_A / \partial a$ simplifies to,

$$\frac{\partial \Pi_A}{\partial a} = \left(p_A^* - c_A\right)\left(\underbrace{\frac{\partial D_A}{\partial a}}_{\text{(a)}} + \underbrace{\frac{\partial D_A}{\partial p_B}\frac{\partial p_B^*}{\partial a}}_{\text{(b)}}\right) \quad (11)$$

Terms (a) and (b) represent the demand effect and the strategic effect respectively. The fact that a firm may want to move to the centre to increase its market share is the demand effect. But, by moving away from the centre, the price competition decreases and the firm increases its market power. It is the strategic effect. These two effects provide in opposite incentives for the firm. In the standard case, the strategic effect dominates the demand effect: the firms want to move away from each other and we get maximal differentiation as a result.

However, there are two differences in this model compared to the standard case: the fuel cost and the vehicle tax are increasing along the Hotelling line (since the fuel consumption and the rate emission are increasing).

Following D'aspremont et al., Firm A should be located at $a = 0$. Moving to the right – closer to firm B –, firm A, in addition to the standard results, increases costs (fuel and tax) associated with its good. However, the optimal location for firm B is not that clear. Firm B decreases its differentiation by moving to the left – closer to firm A – but costs associated with its good decrease as well. There is a trade-off to consider between those effects.

I verify these intuitions algebraically.

Using Equations 5, 7 and 8 in 11, the term (a) is,

$$\begin{aligned}\frac{\partial D_A}{\partial a} &= \frac{1}{2} + \frac{p_B^* - p_A^*}{2t\left(1-a-b\right)^2} \\ &= \frac{3-5a-b}{6\left(1-a-b\right)} + \frac{\left(c_B - c_A\right)}{6t\left(1-a-b\right)^2} - \frac{\left(\gamma t_d d + \delta F\right)}{3t\left(1-a-b\right)} \qquad (12)\end{aligned}$$

Using Equations 5 and 8, the term (b) is,

$$\begin{aligned}\frac{\partial D_A}{\partial p_B} &= \frac{1}{2t\left(1-a-b\right)} \\ \frac{\partial p_B^*}{\partial a} &= \frac{t}{3}\left(-4+2a\right) + \frac{1}{3}\left(\gamma t_d d + \delta F\right) \\ \frac{\partial D_A}{\partial p_B}\frac{\partial p_B^*}{\partial a} &= \frac{-2+a}{3\left(1-a-b\right)} + \frac{\left(\gamma t_d d + \delta F\right)}{6t\left(1-a-b\right)} \qquad (13)\end{aligned}$$

Adding Equations 12 and 13, I get,

$$\frac{\partial D_A}{\partial a} + \frac{\partial D_A}{\partial p_B}\frac{\partial p_B^*}{\partial a} = \frac{-\left(1+3a+b\right)}{6(1-a-b)} + \frac{\left(c_B - c_A\right)}{6t\left(1-a-b\right)^2} - \frac{\left(\gamma t_d d + \delta F\right)}{6t\left(1-a-b\right)}$$

If the difference in production costs is not too big[13], the expression is negative[14]. Derived from the two standard effects, there is, what I call, the "clean-effectiveness"

[13] Formally, if $\left(c_B - c_A\right) < \left(1-a-b\right)\left(t\left(1+3a+b\right) + \gamma t_d d + \delta F\right)$.

[14] The point is that if the firm A's production cost is low compared to firm B's cost, firm A can move to the right and have market power through low costs. However, there is a limitation to the model here: production costs are exogenous. But if firm A chooses to move to the right, it means that it produces better performing cars and then production costs are probably not the same. But, the production cost to the firm is more likely to be function of the location on the product spectrum. For that reason, in the product selection section, I assume that the costs are the same.

effect that provides an incentive to move to the left. The "clean-effectiveness" effect is the fact that consumers have to pay less tax and fuel when a firm is located more left (the car is then more fuel-efficient and has a lower emission rate), leaving more income to pay a higher price for the good itself.

Knowing that the mark-up $(p_A^* - c_A)$ is non-negative, $\partial \Pi_A / \partial a < 0$. Firm A locates at 0. The original strategic effect and the "clean-effectiveness" effect dominate the original demand effect for firm A.

Since the problem is not symmetric, I do the similar computations for firm B. Again, by the envelope theorem, we know that $\partial \Pi_B / \partial p_B^* = 0$. Then, $\partial \Pi_B / \partial b$ simplifies to,

$$\frac{\partial \Pi_B}{\partial b} = (p_B^* - c_B) \left(\underbrace{\frac{\partial D_B}{\partial b}}_{\text{(c)}} + \underbrace{\frac{\partial D_B}{\partial p_A} \frac{\partial p_A^*}{\partial b}}_{\text{(d)}} \right) \tag{14}$$

Using Equations 6, 7 and 8, the term (c) is,

$$\begin{aligned} \frac{\partial D_B}{\partial b} &= \frac{1}{2} + \frac{p_A^* - p_B^*}{2t\,(1-a-b)^2} \\ &= \frac{3-5b-a}{6\,(1-a-b)} + \frac{(c_A - c_B)}{6t\,(1-a-b)^2} + \frac{(\gamma t_d d + \delta F)}{3t\,(1-a-b)} \end{aligned} \tag{15}$$

Using Equations 6 and 7, the term (d) is,

$$\begin{aligned} \frac{\partial D_B}{\partial p_A} &= \frac{1}{2t\,(1-a-b)} \\ \frac{\partial p_A^*}{\partial b} &= \frac{t}{3}(-4+2b) - \frac{1}{3}(\gamma t_d d + \delta F) \\ \frac{\partial D_B}{\partial p_A}\frac{\partial p_A^*}{\partial b} &= \frac{-2+b}{3\,(1-a-b)} - \frac{(\gamma t_d d + \delta F)}{6t\,(1-a-b)} \end{aligned} \tag{16}$$

Adding Equations 15 and 16, I get,

$$\frac{\partial D_B}{\partial b} + \frac{\partial D_B}{\partial p_A}\frac{\partial p_A^*}{\partial b} = \frac{-(1+3b+a)}{6(1-a-b)} + \frac{(c_A - c_B)}{6t\,(1-a-b)^2} + \frac{(\gamma t_d d + \delta F)}{6t\,(1-a-b)}$$

Knowing that optimally $a = 0$, this expression reduces to,

$$\frac{\partial D_B}{\partial b} + \frac{\partial D_B}{\partial p_A}\frac{\partial p_A^*}{\partial b} = \frac{-(1+3b)}{6(1-b)} + \frac{(c_A - c_B)}{6t\,(1-b)^2} + \frac{(\gamma t_d d + \delta F)}{6t\,(1-b)} \tag{17}$$

In Firm B's case, the sign of the expression is not clear. The "clean-effectiveness" effect provides an incentive to move to the left – towards the other firm.

To simplify the analysis, from now on, I assume that $c_A = c_B$,

$$\frac{\partial D_B}{\partial b} + \frac{\partial D_B}{\partial p_A}\frac{\partial p_A^*}{\partial b} = \frac{1}{6t(1-b)}\left(\gamma t_d d + \delta F - t\left(1+3b\right)\right)$$

Using Equations 8 and 17, Equation 14 becomes:

$$\frac{\partial \Pi_B}{\partial b} = \frac{1}{6t}\left(t\left(1+\frac{b}{3}\right) - \frac{1}{3}\left(\gamma t_d d + \delta F\right)\right)\left(\gamma t_d d + \delta F - t\left(1+3b\right)\right) \qquad (18)$$

There are three theoretical possibilities.

CASE 1 - If $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=0} < 0$**,** firm B has an incentive to move to the right – to differentiate more –. Since it is not possible[15], the firm locates at 1 on the spectrum, at the complete opposite of firm A. There is maximal differentiation. This leads to the following proposition.

Proposition 2 *Under some conditions, a small pollution tax has no effect on the types of vehicles produced. Some consumers shift their consumption towards the cleaner vehicle, but the more pollutant vehicle is still produced.*

Proof. For the firm B to produce a cleaner vehicle, $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=0}$ has to be greater than 0. That happens iff $4t < (\gamma t_d d + \delta F)$. So, as long as the tax is not too high and $4t \geq (\gamma t_d d + \delta F)$, the firm B does not relocate. I already shown that the firm A locates at 0. For the consumers: $\frac{\partial D_A(p^*, a^* = b^* = 0)}{\partial t_d} > 0$. ■

Then, an increase in the per-distance tax (differentiated by vehicle type) or in the fuel tax would induce a change in the location of the firm B only if the inequality is violated. Also, a decrease in the "transportation" cost could induce a same change.

The firm B does not relocate here because the "clean-effectiveness" effect combined with the original demand effect are not strong enough to dominate the strategic effect. The firm B still wants to differentiate more its product, moving to the right, to enjoy a greater market power.

CASE 2 - If $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=0} > 0$ **and** $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=1} < 0$**,**

Firm B maximises its profits by locating somewhere between 0 and 1, at the point where $\partial \Pi_B / \partial b = 0$. There is some differentiation.

Proposition 3 *Under some conditions, a pollution tax leads the firm B to produce a cleaner vehicle.*

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[15] It is not possible because the firm is already at the extreme of the spectrum. This can be seen as the fact that moving to the right would induce an emission rate higher than allowed by the regulation.

Proof. By maximisation behavior: If $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=0} > 0$, firm B moves to the left until $\frac{\partial \Pi_B}{\partial b} = 0$. ■

In that case, the expense bill (tax and fuel cost) the consumers have to pay is high enough so that the "clean-effectiveness" effect (with the original demand effect) dominates the original strategic effect.

CASE 3 - If $\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=1} > 0$,

In that case, Firm B would locate at the same point as Firm A, both producing the same kind of car, and there would be perfect homogeneity. However,

Proposition 4 *The two vehicles produced cannot be homogeneous in terms of pollutant emissions.*

Proof. Taking the derivative of Equation 18 at $b = 1$, I get

$$\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=1} = -\frac{1}{18t}\left(\gamma t_d d + \delta F - 4t\right)^2 < 0$$

When located at 0, Firm B has an incentive to differentiate by moving to the right. ■

Perfect homogeneity in that case implies Bertrand competition, and then no profits for any of the firm.

3.2.4 Reduction in pollution level and externality cost

Under Case 1 ($\left.\frac{\partial \Pi_B}{\partial b}\right|_{b=0} < 0$ – Maximal differentiation), results in pollution level reductions are the same as in section 3.1.3. The differentials between the two cars in the vehicle tax and fuel costs are not sufficient to bring a change in the industry structure. The only way the pollution level is reduced is by the shift in the consumer demand from SUVs to sedans.

Under Case 2 (some differentiation), consumer demand for cars B may not decrease. However, reductions in pollution level are brought by the production of cleaner vehicles by Firm B.

3.2.5 The Social Planner

How does the competition differ from a situation in which a social planner would choose the locations?

In the standard case (with similar costs for both firms), the optimal locations, on a social point of view, are $\frac{1}{4}$ and $\frac{3}{4}$. Since the market is covered and every consumer gets a unit, the social surplus is maximised when the total "transportation" costs are minimised.

In the present model, the pollution externality is added. So we can no longer look at the "transportation" costs only. When the pollution externality is added to the social surplus[16], the equation is rather tedious to be solved. I however claim that the optimal social locations are pushed to the left compared to the standard case. This is for two reasons: the fuel cost and the pollution externality are both increasing when the location of a firm moves to the right. With similar production costs, the optimal location of the firm A, a, belongs to $[0, \frac{1}{4})$ and the location of the Firm B, b, belongs to $[0, \frac{3}{4})$, with $a \leq b$ (the equality being possible only when both locations are 0).

Then the firm A attains the optimal social location only when this one is 0. Optimal social locations can be attained by having a specific tax[17] on each vehicle. In that case, the optimal tax would not be equal to the marginal damage created by the pollution since it would consider the minimisation of the transportation costs as well.

4 Endogenous distance

In the previous part, travelled distances are fixed. One may find that assumption pretty restrictive since consumers may react when the price of travelling changes. In this section, the travelled distance is endogenous. To do so, I assume quasilinear preferences[18]. Each consumer has the following problem:

$$\begin{aligned} \max_{g,d,c} U(g,d,c) &= g + u(d,c) \\ \text{s.t.} \quad g + \omega_T &\leq \omega \\ \text{and} \quad g &> 0 \end{aligned}$$

where g is a bundle of goods not related to private transportation and is the numéraire. d is the travelled distance. c is the net surplus derived from the purchase of one car, either car A or car B. The total income is ω and the income spent on private transportation is ω_T.

The constraint $g > 0$ imposes the fact that each agent cannot consume only private transportation. Derived from that condition is the fact that the income spent on private transportation is fixed and noted $\overline{\omega_T}$.

[16] The social surplus is defined as the social surplus from the market minus the pollution externality,

$$\begin{aligned} SS &= \int_0^{\widehat{x}} \Big(s - c - t(a-x)^2 - (1 + a\gamma)T - (1 + a\delta)F\Big)\,dx \\ &+ \int_{\widehat{x}}^1 \Big(s - c - t(1-b-x)^2 - (1 + ((1-b)\gamma))T - (1 + (1-b)\delta)F\Big)\,dx \\ &+ (1 + \gamma a)\, erd\widehat{x} + (1 + \gamma(1-b))\, erd(1 - \widehat{x}) \end{aligned}$$

where the indifferent consumer is $\widehat{x} = \left(\frac{1+a-b}{2} + \frac{1}{2t}(\delta F + \gamma T)\right)$

[17] The tax can be negative, i.e. a subsidy.

[18] Quasilinear preferences do not allow for income effects.

The subutility $u(d,c)$ is assumed to be Cobb-Douglas. The maximisation problem on private transportation is,

$$\begin{aligned}\max u(g,c) &= d^{\frac{1}{2}}c^{\frac{1}{2}} \\ \text{where } c &= \max\left\{\begin{array}{c}(s-tx-p_A-(p_f f+t_d)\,d_A)\,, \\ (s-p_B-t(1-x)-((1+\delta)p_f fc+(1+\gamma)t_d)\,d_B)\end{array}\right\}\end{aligned}$$

The budget constraint is

$$\begin{aligned}p_i+F_i+T_i &\leq \omega_T \\ p_i+(p_f fc_i+t_{di})\,d_i &\leq \omega_T \quad \text{where } i=A,B\end{aligned}$$

Each consumer maximises his utility for the two kinds of cars subject to the budget constraint. The FOCs with respect to d are,

$$\frac{\partial u_A}{\partial d}=\frac{1}{2}d^{-\frac{1}{2}}(s-tx-p-(p_f fc+Ner)d)^{\frac{1}{2}}-\frac{1}{2}\frac{d^{\frac{1}{2}}(p_f fc+t_d)}{(s-tx-p-(p_f fc+t_d)d)^{\frac{1}{2}}}=0$$

for car A, and

$$\begin{aligned}\frac{\partial u_B}{\partial d} &= \frac{1}{2}d^{-\frac{1}{2}}\left(s-t(1-x)-p-((1+\delta)p_f fc+(1+\gamma)t_d)\,d\right)^{\frac{1}{2}} \\ &\quad -\frac{1}{2}\frac{d^{\frac{1}{2}}\left((1+\delta)p_f fc+(1+\gamma)t_d\right)}{\left(s-t(1-x)-p-((1+\delta)p_f fc+(1+\gamma)t_d)\,d\right)^{\frac{1}{2}}}=0\end{aligned}$$

for car B.

Using these two FOCs, the solutions for d are,

$$\begin{aligned}d_A &= \frac{1}{2}\left(\frac{s-tx-p_A}{p_f fc+t_d}\right) \\ d_B &= \frac{1}{2}\left(\frac{s-p_B-t(1-x)}{(1+\delta)\,p_f fc+(1+\gamma)\,t_d}\right)\end{aligned}$$

Substituting these optimal distances back in each utility function, I get the indirect utility functions:

$$\begin{aligned}V_A(p) &= \frac{1}{2}\left(\frac{1}{p_f fc+t_d}\right)^{\frac{1}{2}}(s-tx-p_A) \\ V_B(p) &= \frac{1}{2}\left(\frac{1}{(1+\delta)\,p_f fc+(1+\gamma)\,t_d}\right)^{\frac{1}{2}}(s-p_B-t(1-x))\end{aligned}$$

The consumer $\widetilde{x}$ is indifferent between car A and car B if and only if $V_A(p)=V_B(p)$:

$$\frac{1}{2}\left(\frac{1}{p_f fc+t_d}\right)^{\frac{1}{2}}(s-tx-p_A)=\frac{1}{2}\left(\frac{1}{(1+\delta)\,p_f fc+(1+\gamma)\,t_d}\right)^{\frac{1}{2}}(s-p_B-t(1-x))$$

Let $\alpha = (p_f fc + t_d)^{-\frac{1}{2}}$ and $\beta = ((1+\delta)\, p_f fc + (1+\gamma)\, t_d)^{-\frac{1}{2}}$. Solving for $\widetilde{x}$:

$$\widetilde{x} = \frac{(\alpha - \beta)\, s + (\beta p_B - \alpha p_A) + t\beta}{t\,(\alpha + \beta)}$$

Demands are found using that equation,

$$D_A(p) = \widetilde{x} = \frac{(\alpha - \beta)\, s + (\beta p_B - \alpha p_A) + t\beta}{t\,(\alpha + \beta)} \tag{19}$$

$$D_B(p) = (1 - \widetilde{x}) = \frac{(\beta - \alpha)\, s + (\alpha p_A - \beta p_B) + \alpha t}{t\,(\alpha + \beta)} \tag{20}$$

4.1 Profit maximisation problem

Each firm maximises its profit given the demand that it faces.

Firm i's problem is:

$$\Pi_i\,(p) = \max_{p_i}\,(p_i - c_i)\, D_i(p)$$

Using Equations 19 and 20 in this problem, the reaction functions are,

$$p_A = \frac{(\alpha - \beta)\, s + \beta p_B + \beta t + \alpha c_A}{2\alpha}$$

$$p_B = \frac{(\beta - \alpha)\, s + \alpha p_A + \alpha t + \beta c_B}{2\beta}$$

The solutions for this two-equation system are,

$$p_A = \frac{1}{3\alpha}\left((\alpha - \beta)\, s + 2\alpha c_A + \beta c_B + (\alpha + 2\beta)\, t\right)$$

$$p_B = \frac{1}{3\beta}\left((\beta - \alpha)\, s + \alpha c_A + 2\beta c_B + (2\alpha + \beta)\, t\right)$$

Given the optimal prices, demands (Equations 19 and 20) are,

$$D_A(p^*) = \frac{1}{3\,(\alpha + \beta)\, t}\left[(\alpha - \beta)\, s - \alpha c_A + \beta c_B + (\alpha + 2\beta)\, t\right]$$

$$D_B(p^*) = \frac{1}{3\,(\alpha + \beta)\, t}\left[(\beta - \alpha)\, s + \alpha c_A - \beta c_B + (2\alpha + \beta)\, t\right]$$

4.2 Travelled distances

Now that we know the prices, travelled distances can be computed.

For the consumer purchasing car A, using the budget constraint and the optimal price, the optimal distance is

$$d_A^* = \alpha^2 \omega_T - \left(\frac{\alpha}{3}\left((\alpha - \beta)\, s + 2\alpha c_A + \beta c_B + (\alpha + 2\beta)\, t\right)\right) \tag{21}$$

Similarly, the distance travelled by consumer B is

$$d_B^* = \beta^2 \omega_T - \left(\frac{\beta}{3} \left((\beta - \alpha) s + \alpha c_A + 2\beta c_B + (2\alpha + \beta) t \right) \right) \tag{22}$$

The difference in distances between cars B and A:

$$\begin{aligned} \Delta d &= d_B - d_A \\ &= \left(\beta^2 - \alpha^2\right) \left(\omega_T - \frac{1}{3}(s+t) \right) + \frac{\beta\alpha}{3}(c_B - c_A) + \frac{2}{3}\left(\alpha^2 c_A - \beta^2 c_B\right) \lesseqgtr 0 \end{aligned}$$

Knowing that $\alpha > \beta$, we cannot sign the last equation. By assuming that the costs are the same for both firms, the expression is simplified, but we still cannot sign it,

$$\Delta d = \left(\beta^2 - \alpha^2\right) \left(\omega_T - \frac{1}{3}(s + t + 2c) \right) \lesseqgtr 0$$

We may have thought that a consumer purchasing a sedan (A) will be left with more money and then travels a higher distance, but it is not necessarily the case since firm A can use its differentiation to increase its mark-up.

4.3 Comparative statics

I turn now to the question of the impacts of changes in the vehicle tax on car and distance demands.

Car demands - The changes in the demand for car A are as follows,

$$\frac{\partial D_A(p^*)}{\partial t_d} = \left(\frac{\partial D_A(p^*)}{\partial \alpha} \frac{\partial \alpha}{\partial t_d} + \frac{\partial D_A(p^*)}{\partial \beta} \frac{\partial \beta}{\partial t_d} \right)$$

The changes in the demand for car B is only the opposite of it since the total market is fixed.

In order to compute it, we need the differentials of α and β with respect to the tax and price,

$$\begin{aligned} \frac{\partial \alpha}{\partial t_d} &= -\frac{1}{2}\left(p_f f c + t_d\right)^{-\frac{3}{2}} = -\frac{1}{2}\alpha^3 < 0 \\ \frac{\partial \beta}{\partial t_d} &= -\frac{1}{2}(1+\gamma)\left((1+\delta) p_f f c + (1+\gamma) t_d\right)^{-\frac{3}{2}} = -\frac{1}{2}(1+\gamma)\beta^3 < 0 \end{aligned}$$

Both differentials are negative.

The differentials of the demand with respect to the parameters are,

$$\frac{\partial D_A(p^*)}{\partial \alpha} = -\frac{1}{3(\alpha+\beta)^2 t}\left[(\alpha-\beta)s - \alpha c_A + \beta c_B + (\alpha+2\beta)t\right] + \frac{1}{3(\alpha+\beta)t}\left[s - c_A + t\right]$$
$$\frac{\partial D_A(p^*)}{\partial \beta} = -\frac{1}{3(\alpha+\beta)^2 t}\left[(\alpha-\beta)s - \alpha c_A + \beta c_B + (\alpha+2\beta)t\right] + \frac{1}{3(\alpha+\beta)t}\left[-s + c_B + 2t\right]$$

The changes in the demand can now be computed,

$$\frac{\partial D_A(p^*)}{\partial \alpha}\frac{\partial \alpha}{\partial t_d} = -\frac{1}{6}\frac{\beta\alpha^3}{(\alpha+\beta)^2 t}\left(2s - (c_A + c_B) - t\right) < 0 \quad \text{(By assumption of covered market)}$$
$$\frac{\partial D_A(p^*)}{\partial \beta}\frac{\partial \beta}{\partial t_d} = \frac{(1+\gamma)}{6}\frac{\alpha\beta^3}{(\alpha+\beta)^2 t}\left(2s - (c_A + c_B) + t\right) > 0$$

Using the last two equations,

$$\begin{aligned}\frac{\partial D_A(p^*)}{\partial t_d} &= \left(\frac{\partial D_A(p^*)}{\partial \alpha}\frac{\partial \alpha}{\partial t_d} + \frac{\partial D_A(p^*)}{\partial \beta}\frac{\partial \beta}{\partial t_d}\right) \\ &= -\frac{1}{6}\frac{\alpha\beta}{(\alpha+\beta)^2 t}\left(\alpha^2 - (1+\gamma)\beta^2\right)(2s - c_B - c_A - t) \qquad (23)\end{aligned}$$

From Equation 23, the expression $-\frac{1}{6}\frac{\alpha\beta}{(\alpha+\beta)^2 t}$ is negative. By the assumption of covered market, $(2s - c_B - c_A - t)$ is positive. The sign of Equation 23 depends on the expression $\left(\alpha^2 - (1+\gamma)\beta^2\right)$. This expression is positive if $\delta > \gamma$. The change in car A demand with respect to the per-distance tax depends on the difference between the parameters on fuel consumption and emission rate. We have,

$$\begin{aligned}\frac{\partial D_A(p^*)}{\partial t_d} &> 0 \quad \text{if} \quad \delta < \gamma \\ \frac{\partial D_A(p^*)}{\partial t_d} &= 0 \quad \text{if} \quad \delta = \gamma \\ \frac{\partial D_A(p^*)}{\partial t_d} &< 0 \quad \text{if} \quad \delta > \gamma\end{aligned}$$

Then an increase in the per-distance tax would increase the demand for car A (and decrease the demand for car B) only if the differential in the emission rate between cars A and B (γ) is higher than the differential in fuel consumption (δ).

Distance demands - The differentials of the distance with respect to the vehicle tax are

$$\frac{\partial d_A}{\partial t_d} = \left(\frac{\partial d_A}{\partial \alpha}\frac{\partial \alpha}{\partial t_d} + \frac{\partial d_A}{\partial \beta}\frac{\partial \beta}{\partial t_d}\right) \qquad (24)$$
$$\frac{\partial d_B}{\partial t_d} = \left(\frac{\partial d_B}{\partial \alpha}\frac{\partial \alpha}{\partial t_d} + \frac{\partial d_B}{\partial \beta}\frac{\partial \beta}{\partial t_d}\right) \qquad (25)$$

where $\frac{\partial \alpha}{\partial t_d}$ and $\frac{\partial \beta}{\partial t_d}$ are taken from last section, and using Equations 21 and 22,

$$\begin{aligned}\frac{\partial d_A}{\partial \alpha} &= 2\alpha\left(\omega_T - \frac{1}{3}(s+2c_A+t)\right) + \frac{\beta}{3}(s-c_B-2t)\\ \frac{\partial d_A}{\partial \beta} &= \frac{\alpha}{3}(s-c_B-2t)\end{aligned}$$

Using these equations,

$$\begin{aligned}\frac{\partial d_A}{\partial \alpha}\frac{\partial \alpha}{\partial t_d} &= -\alpha^4\left(\omega_T - \frac{1}{3}(s+2c_A+t)\right) - \frac{\alpha^3\beta}{6}(s-c_B-2t)\\ \frac{\partial d_A}{\partial \beta}\frac{\partial \beta}{\partial t_d} &= -\frac{(1+\gamma)\alpha\beta^3}{6}(s-c_B-2t)\end{aligned}$$

Equation 24 becomes,

$$\frac{\partial d_A}{\partial t_d} = -\alpha^4\left(\omega_T - \frac{1}{3}(s+2c_A+t)\right) - \left(\frac{\alpha^3\beta}{6} + \frac{(1+\gamma)\alpha\beta^3}{6}\right)(s-c_B-2t)$$

From that equation, the expressions $\left(\omega_T - \frac{1}{3}(s+2c_A+t)\right)$ and $(s-c_B-2t)$ cannot be signed. Hence, the change in the distance cannot be known without knowing the values.

Doing the similar computations for d_B,

$$\begin{aligned}\frac{\partial d_B}{\partial \alpha} &= \frac{\beta}{3}(s-c_A-2t)\\ \frac{\partial d_B}{\partial \beta} &= 2\beta\left(\omega_T - \frac{1}{3}(s+2c_B+t)\right) + \frac{\alpha}{3}(s-c_A-2t)\end{aligned}$$

Equation 25 becomes,

$$\frac{\partial d_B}{\partial t_d} = -(1+\gamma)\beta^4\left(\omega_T - \frac{1}{3}(s+2c_B+t)\right) - \left(\frac{\alpha^3\beta}{6} + \frac{(1+\gamma)\alpha\beta^3}{6}\right)(s-c_A-2t)$$

In the case of d_B, the sign stays unknown too.

However, by the maximisation problem, the consumer spends all his income. If the cost of a unit of distance increases (here t_d), we know by the substitution effect that the distance cannot increase. Following an increase in the per-distance tax, the firms may decrease their price, but the decrease cannot be such that it leaves enough money to the consumer to increase its distance. Then we have,

$$\begin{aligned}\frac{\partial d_A}{\partial t_d} &\leq 0\\ \frac{\partial d_B}{\partial t_d} &\leq 0\end{aligned}$$

4.4 Reduction in pollution

The total pollution level and externality cost created by cars can be computed as,

$$\begin{aligned} P &= D_A \cdot r \cdot d_A + D_B \cdot (1+\gamma) \cdot r \cdot d_B \\ &= (D_A d_A + (1+\gamma) D_B d_B) \cdot r \end{aligned}$$

The change in pollution following an increase in the per-distance tax is,

$$\begin{aligned} \frac{\partial P}{\partial t_d} &= \left(\frac{\partial D_A(p^*)}{\partial t_d} d_A + \frac{\partial d_A}{\partial t_d} D_A + (1+\gamma) \left(-\frac{\partial D_A(p^*)}{\partial t_d} d_B + \frac{\partial d_B}{\partial t_d} D_B \right) \right) r \\ &= \underbrace{\frac{\partial D_A(p^*)}{\partial t_d} (d_A - (1+\gamma) d_B)}_{\text{(a)}} + \underbrace{\frac{\partial d_A}{\partial t_d} D_A + (1+\gamma) \frac{\partial d_B}{\partial t_d} D_B}_{\text{(b)}} \end{aligned}$$

From the previous part, we know that the term (b) is non-positive. For the term (a), the sign of $\frac{\partial D_A(p^*)}{\partial t_d}$ depends on the parameters δ and γ. The term $(d_A - (1+\gamma) d_B)$ represents the difference in "distances in terms of pollution". By that expression, I mean that: If the two cars travel the same distance, the car B will produce more pollution since its rate emission is higher. However, the car A will produce more pollution if it travels sufficiently more than the car B. Then the term (a) can be positive if $\frac{\partial D_A(p^*)}{\partial t_d}$ and $(d_A - (1+\gamma) d_B)$ are both positive or negative, situations that can be possible. That happens if, following an increase in the per-distance tax, consumers may switch towards cleaner vehicles, but travel more with these vehicles; or may switch towards dirtier vehicles.

So, it is not clear that the pollution level will decrease following an increase in the per-distance tax. There is a (small) possibility that the term (a) is positive and outweighs the term (b).

5 Discussion and Conclusion

Sport utility vehicles (SUV) consume more fuel and emit more pollutants than sedans do. There is room for the government to intervene to reduce environmental cost. I use a horizontal differentiation model adapted to include externality. Externality production depends on firms' locations (it is increasing along the line). There are two firms each producing a model.

In a first part, the travelled distance with each car is assumed to be fixed. In an extreme locations setting, the optimal tax is equal to the marginal damage. As the per-distance tax increases, the demand for that car decreases and the demand for the sedan increases. The pollution level and total externality cost decrease in accordance to the decrease in the demand for the SUV.

When product selection is allowed, one firm always produces the less pollutant car. The other firm may locate everywhere along the line depending on the values of the parameters, but not at the same location as firm A. There is always some differentiation. When firm B does not locate at the extreme, the demand for its good may not decrease. However, pollution level decreases as the SUV is now cleaner.

In the second part, I use quasilinear preferences in an extreme locations setting. This allows each type of consumer to choose its distance. The per-distance tax does necessarily induce a decrease in the demand for the SUV. The impact of the tax depends on the difference between the fuel consumption and emission rate parameters. Finally, there is a possibility that the pollution level increases following a tax: people may purchase dirtier vehicles or people may purchase cleaner vehicles but travel more.

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