Nonlinearities in the Valuation of Travel Attributes.

Matthieu de Lapparent¹, André de Palma², Cédric Fontan²

¹ Equipe Universitaire de Recherche en Economie Quantitative (EUREQua)
  Université Paris 1 Panthéon-Sorbonne, France <http://eurequa.univ-paris1.fr>

² THEMA, Transport et Réseaux, Université de Cergy-Pontoise, France

Matthieu de Lapparent,
Doctorant, Equipe Universitaire de Recherche en Economie Quantitative
Maison des sciences économiques
Université Paris 1 Panthéon-Sorbonne
106-112, bvd de l’Hôpital
75647 Paris Cedex 13, France
Courriel : Matthieu.Delapparent@malix.univ-paris1.fr

Publication AJD-69

October, 2002
**Abstract:** The valuations of prices and values of travel times are key parameters in the calibration of the demand side of traffic simulation systems. However, it seems that practical requirements are fogging the real nature of these concepts. We propose a discrete choice theoretical framework for the regular journey-to-work, within the individual is allowed to twist the effective levels of travel attributes, defining different prices of times strictly related to the traveller’s tastes and behaviors, but also depending on the travel alternatives and their corresponding effective levels of supplied attributes. We use a Box-Cox Logit specification for the calibration of a binomial mode choice model using data from the 1998 French Parisian Travel Survey. We find heterogenous results, but some robust considerations concerning the individual behavior and willingnesses to pay for travel times are emerging through our estimations.

**JEL:** C35, D11, J20

**Keywords:** Box-Cox transformations, value of time, twisting effects, discrete choice.

---

**Résumé:** Les valorisations des prix et valeurs des temps de trajet sont des paramètres clés dans le calibrage des modèles d’assignations et simulations des flux de trafics. Cependant, il semble que la pratique a légèrement obscurci le concept même de prix du temps. Nous proposons un modèle de choix discrets pour les déplacements réguliers domicile-travail, dans lequel l’individu peut déformer les niveaux effectifs des attributs des contrats de transports offerts par différents modes, définissant ainsi différents prix et valeurs des temps dépendant du motif de déplacement, des modes de transport, mais aussi du niveau des attributs prix et temps de chacun des contrats offerts ainsi que le niveau de revenu de l’individu, ayant des influences différentes selon ses comportements de voyageur. Nous utilisons une spécification Box-Cox Logit pour l’évaluation d’un modèle binomial calibré à partir de l’Enquête Globale de Transport 1998. Nous trouvons des résultats variés, mais certains apparaissent de façon robuste au travers de nos différentes estimations.

**JEL:** C35, D11, J20

**Mots clé:** Transformations de Box-Cox, valeur du temps, effets de distorsion, choix discret.

---

**Table of Contents:**
1. Introduction. p.1
2. The journey-to-work discrete choice framework. p.2
3. Values of travel times. p.4
   3.1. Definitions. p.4
   3.2. Properties. p.5
4. Box-Cox logit models. p.6
6. Conclusion p.16
INTRODUCTION

The valuations of prices and costs of travel times appear to be key parameters in the calibration of any demand traffic simulation systems. For example in the UK, Wardman (2000) propose a complete review of the empirical estimates. However, it seems that practical requirements are fogging the real nature of these concepts, which should be strictly related to the individual tastes and behaviors, but are also depending on the travel alternatives and their corresponding effective levels of supplied attributes. Thus, we should find individual specific values of travel times, changing with the attributes levels and with transportation mode specific tastes and behavioural considerations. First, theoretical approaches on the topic have considered the time component as a scarce resource. Thus, it is crucial to avoid its irrational and senseless spending (Becker 1965), but also to search for a minimal time allocation, “all other things being equal”, in boring but necessary activities, particularly in travel, hence it is possible time has an implicit price, at least a resource price. De Serpa (1971) noticed that it is not exactly the case when we consider that the consumer is not always the producer of his/her activities, particularly in travel activity. In this case, the user is a time-taker and rationally knows that he/she has to account for a minimal amount to be spent. Then, time resources are different from travel time, and de Serpa defines the value of travel time as the implicit transfer value from travel time to resource times. Truong and Hensher (1985) have generalised this approach, by splitting the travel time in different components: access/egress times, waiting times, commuting times and riding/on-board times and have found significant values of travel times savings. It has become a usual approach in practical modelling (Bhat 1998-a and 1998-b, Horowitz and al 1980, Quinet 1998). Another general important standpoint is to differentiate the several reasons why the travel time has to be allocated. The basic idea is that the activity to be realised at destination does not improve the same way the well-being of the individual. Thus, the corresponding needed travel time to access it, does not take the same importance in the mind of the individual. This implies different preferences for different situations, and has led to activity-based travel demand systems (Domenich and McFadden 1975, Train and McFadden 1978, Small 1992, BenAkiva and dePalma 1996). We focus in this paper on the regular Journey-To-Work (JTW) framework, particularly the transportation mode choice issue. MVA (1987) and McFadden (2000) have noticed in practical discrete choice modelling, that several possible econometric specifications correspond to different theoretical approaches (McFadden 1973, Ben-Akiva and Lerman 1985). We have also noticed that most of the empirical applications have used functional forms with
pre-specified properties for the implicit valuations of travel portfolios attributes, such as income effects (see for instance dePalma and Kilani 1999, de Palma and Fontan 2001). MVA (1987) and Mcfadden (2000) have also highlighted the fact that the travel time may be unreasoningly quantified. It is possible to account for these effects using simple transformations of the corresponding variables, but we argue that it is a misleading approach, since it is based on a priori behavioural considerations, that are simply leading to pre-specified results. Thus, we develop in a first section a regular journey-to-work transportation mode choice model, within the traveller is faced to mutually exclusive alternatives, each supplying distinct travel portfolios. We describe the decision process of a rational traveller. Hence, we derive definitions for the concepts of prices and values of travel times. We analyse their response functions under general assumptions. We show that there are different situations emerging according to the capacity of endogenising leisure patterns during the OD trip, defining behavioural profiles for the traveller, so that the effective travel time may be twisted by the individual which often does not strictly quantify it. We also allow for an income effect entering the willingness to pay for saving the time attributes. Each price of times is specific to the individual and to the alternative. We develop a binomial Box-Cox Logit model (Gaudry 1978, Gaudry 1981, Gaudry and al. 1996), particularly adapted for joint estimation of tastes and behaviour parameters, as it will be motivated. We propose some estimates of the parameters of the theoretical VOT functions in French Parisian region using an updated 1998 sub-sample of the large regional travel survey.

1. THE JOURNEY to WORK DISCRETE CHOICES FRAMEWORK

Consider the initial consumption-leisure trade-off framework, within an individual has to determine his optimal labour supply amount \( t_w \) in order to consume goods, say an amount \( C \), but also to keep free time resources, say \( l \), for different leisure activities. Let us define the full income: \( R+w.l+w \), where \( R \geq 0 \) is an exogenous (non-work) income and time \( T \) resources. Depending on this income and exogenous price supply conditions (wages \( w \) and consumption prices \( p_c \)), each user will determine his/her optimal allocation, that is the one than reach to the highest level of satisfaction, characterised by an utility function \( U(.,.)^1 \) with inputs \( C \) and \( l \), according to exogenous market supply conditions and resources constraints. When we consider it is necessary to travel for the individual to access his/her workplace, his/her decision process is modified by the introduction of a travel market, within the origin-destination (OD) trip portfolios are determined (see also Domenich and McFadden 1975, Train and McFadden 1978). These are defined as packages of different supply conditions, concerning fares, travel times, quality,..... Because these attributes are of different types, they will have different impact on the resources of the individual. It seems evident that the fare attributes worsen the budget resources. But we have to be more careful when considering the time attribute: it may be possible for the individual to endogenise some leisure patterns during his/her trip. Thus even if travelling incurs a time resources loss, it is also possible to derive satisfaction when journeying. The underlying
idea is travel time is not necessary exactly quantified as the effective time resource loss, and may be twisted by the individual. This twist effect sign and spread is mainly depending on the traveller's behaviour.

We assume from now the (regular) motivation for trip is going to the destination workplace. This implies the amount of work hours is predetermined, say $\bar{t}_w$, thus the full income, $R = R + wt_{w}$, is also known by the individual. To access the workplace, the OD trip incurs a fare $p_T$, but also need a time allocation $t_T$. This latter directly enters the time resources constraint, and according to the above considerations, also enters the leisure pattern $l$ through a time evaluation function $g(t_T)$.

**Definition 1** With normalisation to 1 for the goods price, thus $C$ is the budget amount allocated to goods consumption, the rational individual will run out his/her budget resources, such that

$$C + p_T = R + wt_{w}$$

**Definition 2** The utility leisure time input is defined as the sum between the effective free time and the perceived leisure time during trip:

$$l = (T - \bar{t}_w - t_T) + g(t_T)$$

**Definition 3** The satisfaction the individual retrieves when realising his/her journey-to-work trip, hence with a specific travel portfolio $\{p_T^*, t_T^*\}$, is characterised by

$$U(C, l)$$

$$C = R + wt_{w} - p_T^*$$

$$l = (T - \bar{t}_w - t_T^*) + g(t_T^*)$$

Our framework becomes more realistic and sensible when we consider that the OD trip market has not a monopolistic supplier. In fact, we often observe at least a duopolistic situation. This follows from today's possibilities of using different types of transportation network to realise an OD trip (road and rails), but also from different competitions operating between firms supplying the same transportation mode (airways carriers for instance), or different modes but operating on the same type of network (bus companies for instance). Such diversity is leading to a discrete choice framework for the traveller. The transportation mode choice issue is defined by mutually exclusive alternatives, say $K$, supplying different portfolios on the same OD trip. A rational individual will compare the different possibilities by evaluating the corresponding satisfaction, then choose the one that reach to the highest level. Define $\forall k=1,\ldots,K, \{p_{Tk}, t_{Tk}\}$ the $K$ available portfolios. Because the $K$ transportation modes are different, and because the time twisting effect is a behavioural consideration, we allow for the traveller to twist differently the travel times of the different competitors. Then, the corresponding levels of satisfaction are
Hence, the individual will compare between them all the alternatives and choose

\[ k^* = \arg \max_{k \in [1, \ldots, K]} \left[ U_k(C_k, l_k) \right] \]

that is the transportation alternative that reaches to the maximum level of satisfaction (see Ben-Akiva and Lerman 1985). It is important to note the demand for travelling is normalised to one trip. The comparison process between modes of transportation is repeated for each OD trip. This framework is useful for describing the equilibrium of the traveller, in terms of implicit valuation of travel attributes. It is a useful tool in order to evaluate the modification of the well being of the individual following a modification of supplied travel attributes. For a planner, but also for strategic purposes, it is important to know how to modify the supply conditions without worsening the level of satisfaction of the traveller. This need of implicit valuations of the travel portfolios leads to the concepts of prices and values of travel times.

### 2.1 Values of travel times

All traffic (dynamical) simulation systems require the knowledge of these prices to implement assignation and forecast (dynamical) OD flows. However, the way it is made is based on a representative, or tutelary, value of time, unchanging whatever are the levels of travel attributes. From a theoretical viewpoint, it appears as a particular framework, within the individual is considered to be a passive person to move from one point to another on a network. At the contrary, we think the traveller is sensitive to the level of supplied attributes when evaluating the implicit price. It is because an individual has different responses according to the travel conditions that we argue that there is not only one representative value of travel time. The values of travel times are individual depending on OD and mode.

**Definitions and concepts.**

The basic idea is time is money, particularly travel time since it is partly a constraining activity, thus the individual is willing to pay for saving time resources, that is to allocate less time to travel. His/her equilibrium is such that the price he/she is willing to pay in order to save an amount of travel time leaves unchanged his/her level of satisfaction.

**Definition 4** The value of travel time (VOT) is the monetary amount the individual is willing to pay in order to save a marginal time unit and keep the same level of satisfaction.
\[
POT_k = \frac{\partial U_k}{\partial l_k} \frac{\partial l_k}{\partial C_k} = \frac{\partial U_k}{\partial C_k} \partial C_k \partial p_{kT} = \frac{\partial U_k}{\partial C_k} \partial T_{kT} \left[ 1 - \frac{\partial g_k}{\partial T_{kT}} \right]
\]

By definition of a price, it must have a positive value, thus

\[
\frac{\partial g_k}{\partial T_{kT}} \leq 1
\]

The VOT (for alternative \(k\)) is a weighted marginal substitution rate between leisure and consumption. This weight represents the part of time loss during one time unit, since \(\frac{\partial g_k}{\partial T_{kT}}\) is the marginal amount of leisure time during trip when this latter increase from one time unit. In other words, it is equivalent to the rate of endogenisation of leisure patterns when travelling (see Mcfadden 2000). This characterise the possibility for the individual to feel some leisure times during his/her journey. It implies the effective travel time amount is also used to other satisfying experiences, thus is not perceived as a pure loss of time resources. The corresponding regularity condition for this rate is to be less than 1: for one more travel time unit, there is less than one leisure time unit that can be endogenised. We wish to analyse how such willingness is responding to changes in the supply side of the OD travel market.

**Properties of the prices of travel times**

It appears quite unfeasible to assume these VOT are unchanging with levels of attributes. We argue that under general assumptions for the utility function, there is a price effect entering the valuation of travel time, but also a time twisting effect, due to the possibility of endogenising leisure during trip.

**Definition 5** The variation of the willingness to pay (WTP) for saving travel time using alternative \(k\) according to its price component is

\[
\frac{dPOT_k}{dp_{kT}} = \frac{\partial U_k}{\partial l_k} \frac{\partial^2 U_k}{\partial C_k^2} \left[ 1 - \frac{\partial g_k}{\partial T_{kT}} \right] \leq 0
\]

The variation of the WTP is depending on the rate of lost time by time unit\(^3\), that is weighted by a ratio of derivatives of the utility function. We observe the VOT are varying if and only if the utility function is strictly concave in its consumption argument. The sign of the variation incurred by a fare positive variation is negative: it implies the WTP for travel times are decreasing when
fares are increasing. The weight \( \frac{\partial^2 U_k}{\partial C_i^2} \left( \frac{\partial U_k}{\partial C_i} \right)^2 \) is an individual scaling one, characterising the growth rate of the marginal consumption satisfaction from the initial situation to the new situation after the variation of the fare occurred. It implies people with higher initial budget resources will be less sensitive to a fare increasing, thus the satisfaction modification rate will be lower than the one of people with low initial income, and the price of time will be less decreasing for high income travellers. This income effect (see also Mcfadden 2000 and de Palma and Fontan 2001) implies for high income individuals to have a higher WTP for saving travel time. Alternatively, we also can study the response function of the prices of times according to a marginal variation of effect travel times.

**Definition 6** The variation of the WTP for saving travel time using alternative \( k \) according to its time component is

\[
\frac{d\text{POT}_k}{dt_{kr}} = \frac{\partial^2 U_k}{\partial t_{kr}^2} \left[ \frac{\partial t_{kr}}{\partial U_k} \right]^2 \left( \frac{\partial U_k}{\partial C_i} \right) - \frac{\partial U_k}{\partial C_i} \frac{\partial^2 g_k}{\partial t_{kr}^2} \tag{2}
\]

and depend on the rate of endogenisation of leisure patterns during trip and its evolution with travel time.

The sign of the response is undetermined. It relies on the strict concavity of the utility function \( \frac{\partial^2 U_k}{\partial t_{kr}^2} \) in its leisure argument, and the variation \( \frac{\partial^2 g_k}{\partial t_{kr}^2} \) of the rate of leisure endogenisation \( \frac{\partial g_k}{\partial t_{kr}} \). For instance, if the endogenisation rate is constant whatever is the effective travel time, that is \( \frac{\partial^2 g_k}{\partial t_{kr}^2} = 0 \), then the prices of times are increasing with the effective level of travel time if and only if the utility function is strictly concave in leisure argument. The most intuitive case is when effective travel duration are increasing and when the individual is endogenising less and less leisure times, that is \( \frac{\partial^2 g_k}{\partial t_{kr}^2} < 0 \), then he/she is perceiving an increasing time resources loss, thus is willing to pay more and more for saving travel time. It is the consequence of a lack of flexible scheduling leisure activities. It is important to note when the individual is not a passive traveller, which is represented by the strict concavity of the utility function in its arguments, the prices of travel times are responding to supply fares and effective times levels changes. Although the income effect has a deterministic sign. The times twisting effects are mainly depending on the capacity to endogenise leisure patterns during the journey. This capacity is variable with the effective travel time. The traveller may have run out his/her endogenisation capacities and may feels the resting time as a pure loss of resources.

3 THE BOX-COX LOGIT MODELS
In standard approaches, we generally assume the variable is entering the utility function either in its original form, or in logarithmic form, or eventually in a quadratic form. For these two latter, it is a way to account for twisting effects entering the individual decisional process. But once it is specified, it is no more possible to verify if it is an "a priori" feasible assumption, except by comparing it with other functional forms, that also rely on "a priori" assumption sets. The Box-Cox transformation (Tukey 1957 and Box and Cox 1964) is giving a less constraining specification framework. It appears as a useful tool for testing the presence of twisting effects on some decisional determinants entering the utility function. It allows for more generality because the way these are perceived is not predetermined.

**Definition 7** Consider \( X \) a strictly positive continuous variable.

\[
f(X, \lambda) = \begin{cases} 
\frac{X^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\
\ln(X) & \text{if } \lambda = 0
\end{cases}
\]

is the Box-Cox transformation of the variable \( X \).

For our purposes, it is not interesting to analyse a negative \( \lambda \) situation. However, when it is a positive parameter, we have to distinguish three sub-cases. The case where \( \lambda = 1 \), that is when the individual exactly quantify the variable implies for one effective unit, one unit is perceived. When \( \lambda \) differs from 1, we observe either overamplification (\( \lambda > 1 \)), or underestimation (\( \lambda < 1 \)), that is for one effective unit, it is not necessary one unit that is perceived. \( \lambda \) can be considered as a twisting or perception factor, in the sense it account for the fact the individual may not exactly quantify the effective level of the variable \( X \) when levelling his/her satisfaction. It is a behavioural parameter that should have an impact on the relative importance of its corresponding variable in the decisional process. It is possible to use these Box-Cox transformations to define flexible empirical functional forms for the utility functions respecting our objective (see Gaudry 1981). We assume that the global utility function can be written as the sum of weighted sub-utility function\(^4\). The weights are characterising the (relative) tastes of the travellers, and the sub-utility functions are for strictly positive continuous variables (such as price and time attributes) Box-Cox transformations of the effective levels. Even if we can introduce other determinants than those included in the theoretical framework, we have basically an indirect utility function that is depending on the net from travel costs income and the travel times. Because we are just observing the effective choices of the individuals, we may not have accounted for unsignificant determinants, but also unmeasurable reasons and unobservable externalities between the different alternatives. Hence, we also assume there is a perturbation factor for each individual \( n=1,...,N \), \( \varepsilon_n \), which is randomly distributed over the population, but we assume with a stable and identical individual generation process. It is a regularity condition for a stable random utility framework. With all our notations, we define the latent data
generating process for two alternatives to be equal to a system of weighted sums of sub-utility functions. Some of these latter are just the effective observation, some others are twisted variables. Consider $\mu$ a taste for heterogeneity factor, such that $\forall n=1,\ldots,N$

$$
\begin{align*}
U_{1n} = C_1 + \beta \frac{(\tilde{R} - p_{1n,T})^{\gamma_1}}{\lambda_1} - 1 + \gamma \frac{t_{1n,T}^{\lambda_2}}{\lambda_2} + \mu \epsilon_{n1} \\
U_{2n} = C_2 + \beta \frac{(\tilde{R} - p_{2n,T})^{\gamma_1}}{\lambda_1} - 1 + \gamma \frac{t_{2n,T}^{\lambda_2}}{\lambda_2} + \mu \epsilon_{n2}
\end{align*}
$$

We have stated above an individual will choose the alternative that reach to the highest level of utility. For a sample $n=1,\ldots,N$ of individuals faced to the same discrete choice issue, we will only observe the effective decisions they made. The decision rule can be written

$$
\forall n = 1,\ldots,N, \ y_n = \begin{cases} 
1 & \text{if } U_{1n} \geq U_{2n} \\
0 & \text{if } U_{1n} < U_{2n}
\end{cases}
$$

As in classical latent dependent variable (LDV) models, we will turn our problem in probabilistic terms. Conditionally to all the determinants of the decision process, define the choice probabilities

$$
\Pr[1]_n \equiv \Pr[U_{1n} \geq U_{2n}], \quad \Pr[2]_n \equiv 1 - \Pr[1]_n
$$

In a random utility framework (see McFadden 2000), if the unobservable are independently and identically Gumbel distributed, we derive the Box-Cox Logit model class$^5$ (see Gaudry 1978 and Gaudry et al. 1996, for a discussion of its properties and applications).

$$
\forall n=1,\ldots,N, \epsilon_{1n}, \epsilon_{2n} \overset{iid}{\rightarrow} \text{Gumbel}
$$

$$
F_{\epsilon_{1n}}(w) = F_{\epsilon_{2n}}(w) = \exp \left( - \exp \left( - \sqrt{\frac{\pi^2}{6}} w \right) \right)
$$

We derive the choice probability for the alternative 1 as

$$
\Pr[1] = \frac{\exp \left[ \frac{C_1}{\mu} + \beta \frac{(\tilde{R} - p_{1n,T})^{\gamma_1}}{\lambda_1} - 1 + \gamma \frac{t_{1n,T}^{\lambda_2}}{\lambda_2} \right]}{1 + \exp \left[ \frac{C_2}{\mu} + \beta \frac{(\tilde{R} - p_{2n,T})^{\gamma_1}}{\lambda_1} - 1 + \gamma \frac{t_{2n,T}^{\lambda_2}}{\lambda_2} \right]}
$$
We clearly see that parameters identification issues are emerging, which correspond to specific types of explanatory variables, but also which depend on a scale parameter. Concerning the constant terms and the individual specific variable, it is only the scaled difference between the relative weights that can be estimated. Because of the unknown scale, we often assume $\mu=1$, but with the implicit consequence that estimated numeric values have not a meaningful sense. It is only the sign of the difference and its significance that can be interpreted. Note the scale measure oblige to be also careful about the numerical values of all weights parameters. The Box-Cox parameters are directly estimable since they enter the specification through a power transformation. Consequently, the estimable taste parameters are

$$C_i \equiv \frac{C_2 - C_1}{\mu}, \quad \gamma_\mu \equiv \frac{\gamma}{\mu}$$

whereas there are no identification issue for the Box-Cox parameters, thus they all can be estimated. The log-likelihood function is derived using the observed decision rule and the choice probabilities:

$$\ln \ell_N = \sum_{n=1}^{N} [y_n \ln(\Pr[1|n]) + (1 - y_n)\ln(1 - \Pr[1|n])]$$

**4. ESTIMATES FOR PARIS AREA TRAVEL SURVEY**

We will use in our empirical application data at the individual level, extracted from the EGT primary datafile, which is a revealed preferences (RP) survey. It appears the travel portfolio is itself absorbing the major effects in the transportation mode discrete choice process (de Palma and Fontan 2001 used the same initial dataset to estimate transportation mode discrete choice models). This survey is prepared and paid by the city of Paris (France), the STIF (transit syndicate), the SNCF (the rail operator), the RATP (major subways and bus networks) and COFIROUTE (highways). Because our topic is to focus on the valuation of travel times, we will limit the range of individual specific variables. The RP survey is matched with the results of the traffic assignment model of the IAURIF, the urban planning institute of the Parisian region, in order to obtain a pseudo stated preferences (SP) datasets (see Hensher 1994 for an extensive review of such matching). We advise the reader to refer to the recent work about traffic simulation tools of TTR (2000) for a better understanding of these. The main characteristics of the IAURIF model follow a classical four steps approach (see Quinet1998), and cluster the geographical region in 488 zones. The road network is built using the land use planning and is defined with 6800 arcs and 4500 nodes. The transit network is including rail, bus and subways, and is constituted with 2900 arcs and 1000 nodes. Once calibrated, our sample has a size $N=865$. It contains individual trip that occurs in morning 7.00-10.00 PM weekdays. All sampled individuals are licensed drivers with at least a vehicle for his/her corresponding household. They also have access both to transit and road networks, and they all have carried out a regular journey-to-work.
4.1 Values of times surface functions

The data generating process for the latent system is assumed to be the one developed above. We expect that for the net incomes weight parameters to have a positive effect on their corresponding choice probabilities, and for times weights to have negative effects. Concerning Box-Cox parameters, they are applied to attribute specific variables, and we expect all of them to be positive. Hence, the prices of times function, which are depending on the travel mode $k$, the time component type $l$, the amplitude of the twisting effects $\lambda_{k,m}$ and $\lambda_{k,inc}$, the attribute specific levels $p_{n,k,l}$ and $t_{n,k,l}$, the individual $n$ income and the tastes parameters, are equal to

$$\forall l, \forall n, \forall k, \pi_{n,k,l} = -\frac{\gamma}{\beta} t_{n,k,l}^{\lambda_{inc} - 1} \left[R_n - p_{n,k,l}\right]^{\lambda_{inc}}$$

According to different possible values for the Box-Cox parameters, these VOT of times surfaces are giving different results. Each of them is corresponding to a specific traveller's behaviour. For instance, an intuitive case is considering $\beta>0$, $0 \leq \lambda_{inc}<1$, $\gamma<0$ and $\lambda_{t}>1$. It is characterizing an increasing with effective time and net income price of travel time. It is a situation where higher income people have a higher WTP, but whatever is their income, all the travellers are willing to pay more and more as travel time is increasing. Such responses are denoting an impatient behaviour for the individuals: they are more and more willing to get rid partly of their travel times, thus a higher WTP, because they are perceiving more and more time resources losses in when travelling. Graphically,

![Figure 1. POT surface function](image)

Another case is also when people can endogenise more and more leisure patterns as their travel time are increasing, so that $\beta>0$, $0 \leq \lambda_{inc}<1$, $\gamma<0$ and $\lambda_{t}<1$ thus they are perceiving fewer time losses in some of its dimensions. Because they can improve their satisfaction by doing more leisure during their trips, they desire to pay less and less for saving their travel times as those are
during more and more. When a Box-Cox parameters are equal to one, then the corresponding attribute impact on the associated WTP is null: the corresponding marginal utility (or cost) is constant whatever is the initial level. Note several other situations are possible, each defining a particular price of time surface.

4.2 Estimates from the Box-Cox Logit models

Clearly, the general econometric specification developed above allows for several joint tastes and behaviours econometric results. We add the following variables to the theoretical model for estimation purpose: Constant, Age, Sex for individual specific (IS) variables, which are associated with the private vehicle transportation mode for identification purpose. Constant and age parameters denote the difference of weights between modes. The IS variable sex is built to introduce a difference between male and female. The corresponding parameter characterises the impact of being a male on the transportation mode choice and in which mode it is the most present. We also have attribute specific (AS) variables: travel times are split in times "in" and times "out". This former is for the transit mode the "on-board" travel time, and for the private vehicle (PV) mode the net from local access/digress time. Times "out" are access/digress, wait and walk times for the transit mode, and local access/egress times and wait and walk times for the PV mode. The transit fares are computed with the actual transit card fare grid and aggregate statistics given by the STIF. It is crossed with the survey variable denoting the possessing or not of a transit card to define the fare variable. Concerning the PV mode, the STIF gives a using kilometre cost, accounting for gas and non-gas costs. Finally, we define the net from travel cost income for transit and PV. The following table presents the three different models that we discuss in the following
Table 1. Estimates of the box-cox logit models

<table>
<thead>
<tr>
<th></th>
<th>Model M1</th>
<th>Model M2</th>
<th>Model M3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BOX COX LOGIT</strong></td>
<td>Estimates</td>
<td>Estimates</td>
<td>Estimates</td>
</tr>
<tr>
<td><strong>Constant (PV)</strong></td>
<td>-0.5231 [-1,039]</td>
<td>-0.4937 [-0,999]</td>
<td>-0.0578 [-0,929]</td>
</tr>
<tr>
<td><strong>Age (PV)</strong></td>
<td>0.0267 [2,984]</td>
<td>0.0254 [2,836]</td>
<td>0.0256 [2,857]</td>
</tr>
<tr>
<td><strong>Gender Male (PV)</strong></td>
<td>-0.1390 [-0,765]</td>
<td>-0.1279 [-0,702]</td>
<td></td>
</tr>
<tr>
<td><strong>Travel time in (difference)</strong></td>
<td>-0.0829 [-12,41]</td>
<td>-0.0545 [-12,85]</td>
<td>-0.0431 [-13,43]</td>
</tr>
<tr>
<td><strong>Travel time out (difference)</strong></td>
<td>-0.0714 [-7,687]</td>
<td>-0.0507 [-7,927]</td>
<td>-0.0384 [-6,488]*</td>
</tr>
<tr>
<td><strong>Box Cox Net Income</strong></td>
<td>0.0668 [0,144]</td>
<td>-0.0264 [0,052]</td>
<td></td>
</tr>
<tr>
<td><strong>Box-Cox Travel time</strong></td>
<td>1.1122 [9,274]</td>
<td>1.1653 [9,559]</td>
<td></td>
</tr>
<tr>
<td><strong>Log-Likelihood</strong></td>
<td>-372.31</td>
<td>-369.64</td>
<td>-368.60</td>
</tr>
<tr>
<td><strong>LR stat (df)</strong></td>
<td>453.78 (5)</td>
<td>459.12 (7)</td>
<td>461.22 (6)</td>
</tr>
<tr>
<td><strong>% Right</strong></td>
<td>79.54</td>
<td>80.00</td>
<td>79.77</td>
</tr>
<tr>
<td><strong>Rho-square</strong></td>
<td>0.378</td>
<td>0.382</td>
<td>0.384</td>
</tr>
<tr>
<td><strong>Rho-square bar Akaike</strong></td>
<td>0.369</td>
<td>0.371</td>
<td>0.373</td>
</tr>
<tr>
<td><strong>Sum of Sq. Residuals</strong></td>
<td>243.15</td>
<td>241.06</td>
<td>240.05</td>
</tr>
<tr>
<td><strong>Sample Size</strong></td>
<td>865</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The student for box-cox parameters correspond to the null hypothesis $\lambda =0$ for the left brackets and $\lambda =1$ for the right ones

The first model M1 has no Box-Cox transformations on any variable, we fall back on a conditional logit analysis framework (see McFadden 1973), where the decision process is depending only on the differences between the attribute specific variables. It is important to note it is a widely used logit specification in transportation analysis, used to obtain the tutelary VOT. We deviates slightly by introducing IS variables, because it captures more detailed results. We observe there is no significant difference between the reservation levels of utility corresponding to the two alternatives. Only the age seems to have a significant effect on the individual choice probabilities concerning the IS variables, and leads to the quite natural fact that older people have preferences for private vehicle trip. Using our general formula for the prices of times, we obtain for time "in" 95.11 FF/Hr and for times "out" 81.91FF/Hr. These are considered as standard results for the French Parisian region (see for instance Quinet 1998, and more recently Boiteux 2001 or de Palma and Fontan 2001 for empirical results). Thus we are satisfied to refind a stylised fact as a starting point for the extension of these primal model.

The second model M2 we estimate is releasing the assumption on the Box-Cox power values, but keep the same explanatory variables. Since it is not an intuitive assumption to allow for alternative differentiated Box-Cox transformations, we assume there is one unique transformation for each type of travel component, whatever is the alternative. We also introduce one on the net incomes. The major shifting occurs in the attribute specific weights,
because of such transformation. The major difference with the first specification is there is now one set of VOTs (one by mode) for each individual of the sample. Because there is a huge improvement in the output results of the model, it becomes more difficult to find tutelary values. From a general standpoint, the results gives regular signs for the AS variables. It appears the income effect is significantly present as long as the corresponding Box-Cox parameter is set to be not significantly different from zero, but is numerically a positive number. Concerning the travel times, two remarks have to be noticed: they are significant variables, and the Box-Cox parameter is clearly different from zero, but is tested to be not significantly different from one, even if the numerical value is higher than 1. This implies the WTP is neutral to effective travel time variation. The perceived cost in travel time is not convex, but linear for the JTW framework with this specification, where we group the different types of travel times such that they are twisted the same way. We use below graphical representations of the different unconditional distributions of the several VOT to characterise explicitly the sample willingness to pay.

The shapes of the distributions are closed to log-normal. Such results are aware with the analysis of Arduin et al. (1994). Note that the assumption of log-normal distribution is often made when using the modal split price-time model (Abraham and Blanchet 1973).

From an aggregated viewpoint, it appears that times "in" have higher values than times "out". It is denoted in average, median and modal distributional statistics. Graphically, the response functions show times "out" are lower than times "in" for a strict majority of the individuals. However, times "in" are more volatile, since their support is longer and their standard deviations higher. It is not a surprising result since out-of-vehicle times (or access/digress times for PV) can be easier used to complement the trip with leisure activities. Another remark is that the PV prices of times are higher than transit implicit prices. This is due to the fact the PV mode is a more constraining riding framework for endogenising leisure patterns. Moreover, this model gives higher WTP for time attributes than the simple conditional logit model: this is due to the
convexity of the indirect utility in travel cost, which imply the utility function is concave in consumption argument, thus an income effect improving the WTP. These empirical response functions to an income increasing also show the individual with higher budget income has an implicit time purchase power that is higher than the individual with a low income, thus has a higher WTP, for all times types ("in" and "out"). Precisely, we notice there is different regimes according to specific income range. The sensitivity of the response function is increasing with the income class. This strange effect is maybe due to the fact we have available only very few observations for high income classes, thus it may be biasing the form of the response function. In middle classes, we observe a regular concave function for all prices of times.

Figure 3. VOT and income
Table 2. POT Statistics

<table>
<thead>
<tr>
<th>POT FF/HR</th>
<th>PV in</th>
<th>PV out</th>
<th>Transit in</th>
<th>Transit out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>149.34</td>
<td>138.93</td>
<td>139.57</td>
<td>129.84</td>
</tr>
<tr>
<td>Median</td>
<td>129.87</td>
<td>120.82</td>
<td>118.71</td>
<td>110.43</td>
</tr>
<tr>
<td>Mode</td>
<td>158.46</td>
<td>147.41</td>
<td>213.87</td>
<td>198.96</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>97.13</td>
<td>90.35</td>
<td>97.14</td>
<td>90.37</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.76</td>
<td>5.76</td>
<td>5.79</td>
<td>5.79</td>
</tr>
<tr>
<td>Asymmetry coef.</td>
<td>1.90</td>
<td>1.90</td>
<td>1.91</td>
<td>1.91</td>
</tr>
<tr>
<td>Min</td>
<td>7.99</td>
<td>7.44</td>
<td>7.13</td>
<td>7.56</td>
</tr>
<tr>
<td>Max</td>
<td>728.85</td>
<td>678.03</td>
<td>716.36</td>
<td>666.41</td>
</tr>
</tbody>
</table>

The third model M3 is considering no time split in the PV mode: there is only times "in" when using roads. The estimates gives the same interpreting, that is there is an income effect, but no time twisting effects, so that one minute is perceived as one minute. Here again is noticed the fact the VOT is "higher" than the tutelary value. In the same way, we observe a log-normal distribution for the unconditional VOT distribution, and the income effect is represented the same way. For instance, for PV mode, the graphical representation of the estimated distribution has the following form:

Figure 4. VOT distribution of the private modes users

The statistics corresponding to the three distributions are:
A priori, whatever is the specification of the Box-Cox model, we fall back on a leisure linear specification as optimal functional form for the utility function.

**CONCLUDING REMARKS**

Our findings have refund some important results concerning the concept of the price of travel time. The standard conditional logit specification lead to VOT that correspond to the tutelary values reported in Boiteux (2001). When we allow for Box-Cox transformations, it appears an income effect, which improve the significance of the net income weight parameter in the decisional process. This result conforms with Mcfadden (2000): the WTP for saving travel time increase with the budget resources, since an implicit higher time purchase power. It is not evidently stated there are increasing prices of times with the trip duration, because of Box-Cox transformations with values unsignificantly different from 1. An intuitive age effect with a stable and significant weight across the different models is also found to be significant, respecting the fact that older active people have a higher preference for private vehicle trip, since their (assumed) better socio-economical conditions.

In addition, Times "in" are higher evaluated than times "out", because of the higher hardness they imply: there are less possibilities to endogenise leisure patterns. The issue concerning the preordering of the reservation utility levels is not solved: there is trade-offs between comfort and more flexible leisure patterns to be realised during trips. The main limits of our approach is to have reduced the trip scheduling to the mode choice issue, by assuming departure hour, destination and route already chosen from the origin point. Thus we can not account for substitution patterns that could emerge in joint decisional processes concerning the trip schedule (see Bhat 1998b). We may obtain excessive WTP when the individual is on his/her optimal schedule and/or when only one component of the global decision process varies. As the database we used seems to be rigorous to estimate values of time, our following research will be based on nested choices and computes of compensatory values (following de Plama and Kilani 1999) that is crucial to study, for example, the impact of a change in public fare.

**APPENDICE**
### Statistics of quality of fit for Box-cox models

#### MODEL M1

<table>
<thead>
<tr>
<th></th>
<th>Predicted PV</th>
<th>Predicted Transit</th>
<th>Obs Counts</th>
<th>Obs Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual PV</td>
<td>323.36</td>
<td>121.63</td>
<td>445</td>
<td>0.514</td>
</tr>
<tr>
<td>Actual Transit</td>
<td>121.63</td>
<td>298.36</td>
<td>420</td>
<td>0.485</td>
</tr>
<tr>
<td>Predicted counts</td>
<td>445</td>
<td>420</td>
<td>865</td>
<td>1</td>
</tr>
<tr>
<td>Predicted shares</td>
<td>0.514</td>
<td>0.485</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Proportion successfully predicted</td>
<td>0.726</td>
<td>0.710</td>
<td>0.718</td>
<td></td>
</tr>
<tr>
<td>Prediction success index</td>
<td>0.212</td>
<td>0.224</td>
<td>0.218</td>
<td></td>
</tr>
<tr>
<td>Proportional error in predicted share</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### MODEL M2

<table>
<thead>
<tr>
<th></th>
<th>Predicted PV</th>
<th>Predicted Transit</th>
<th>Obs Counts</th>
<th>Obs Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual PV</td>
<td>324.27</td>
<td>120.72</td>
<td>445</td>
<td>0.514</td>
</tr>
<tr>
<td>Actual Transit</td>
<td>120.72</td>
<td>299.27</td>
<td>420</td>
<td>0.485</td>
</tr>
<tr>
<td>Predicted counts</td>
<td>445</td>
<td>420</td>
<td>865</td>
<td>1</td>
</tr>
<tr>
<td>Predicted shares</td>
<td>0.514</td>
<td>0.485</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Proportion successfully predicted</td>
<td>0.728</td>
<td>0.712</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td>Prediction success index</td>
<td>0.214</td>
<td>0.227</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>Proportional error in predicted share</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### MODEL M3

<table>
<thead>
<tr>
<th></th>
<th>Predicted PV</th>
<th>Predicted Transit</th>
<th>Obs Counts</th>
<th>Obs Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual PV</td>
<td>324.81</td>
<td>120.18</td>
<td>445</td>
<td>0.514</td>
</tr>
<tr>
<td>Actual Transit</td>
<td>120.18</td>
<td>299.81</td>
<td>420</td>
<td>0.485</td>
</tr>
<tr>
<td>Predicted counts</td>
<td>445</td>
<td>420</td>
<td>865</td>
<td>1</td>
</tr>
<tr>
<td>Predicted shares</td>
<td>0.514</td>
<td>0.485</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Proportion successfully predicted</td>
<td>0.730</td>
<td>0.713</td>
<td>0.722</td>
<td></td>
</tr>
<tr>
<td>Prediction success index</td>
<td>0.215</td>
<td>0.228</td>
<td>0.221</td>
<td></td>
</tr>
<tr>
<td>Proportional error in predicted share</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### REFERENCES


and Training Program, U.S. Federal Transit Administration, National Transportation Library.


NOTES

1- We assume that it is a quasi concave function, such that \( \frac{dU}{dC} > 0, \frac{dU}{dl} > 0, \frac{d^2U}{dC^2} \leq 0 \) and \( \frac{d^2U}{dl^2} \leq 0 \) (see Debreu 1954, for more details).
2- Assume it is an R* → R C² class application.
3- Remind it is characterized by the formula \([1 - \hat{c}_{g_i} / \hat{c}_{r_i}].\)
4- Note it is the standard case for most of the empirical application.
5- The normality assumption has never been developed for discrete choices frameworks with Box-Cox transformations.
6- Both for comfort and easier budget accessibility