Nonlinearities in the Valuation of Travel Times at the Individual Level.

Matthieu de Lapparent

1 Equipe Universitaire de Recherche en Economie Quantitative (EUREQua)

Matthieu de Lapparent,
Doctorant, Equipe Universitaire de Recherche en Economie Quantitative
Maison des sciences économiques
Université Paris 1 Panthéon-Sorbonne
106-112, bvd de l’Hôpital
75647 Paris Cedex 13, France

Courriel : Matthieu.Delapparent@malix.univ-paris1.fr

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Abstract: The valuations of prices and values of travel times are key parameters in the calibration of the demand side of traffic simulation systems. However, it seems that practical requirements are fogging the real nature of these concepts. We propose a discrete choice theoretical framework for the regular journey-to-work, within the individual is allowed to twist the effective levels of travel attributes, defining different prices of times strictly related to the traveller’s tastes and behaviors, but also depending on the travel alternatives and their corresponding effective levels of supplied attributes. We use a Box-Cox Logit specification for the calibration of a binomial mode choice model using data from the 1998 French Parisian Travel Survey. We find heterogenous results, but some robust considerations concerning the individual behavior and willingnesses to pay for travel times are emerging through our estimations.

JEL: C35, D11, J20
Keywords: Box-Cox transformations, value of time, twisting effects, discrete choice.


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Mots clé: Transformations de Box-Cox, valeur du temps, effets de distorsion, choix discret.

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1 Introduction.

The valuations of prices and values of travel times are today appearing as key parameters in the calibration of the demand side of any traffic simulation system. However, it seems that practical requirements are fogging the real nature of these concepts, which should be strictly related to the individual tastes and behaviors, but are also depending on the travel alternatives and their corresponding effective levels of supplied attributes, thus we should find individual specific prices and values of travel times, changing with the attributes levels and with transportation mode specific tastes and behavioral considerations. We argue it appears more feasible from a theoretical viewpoint to consider the individual is not only a passive unit (see [Lancaster, K.J., 1966]) to move from point to another on a network.

First theoretical approaches on the topic have considered time is a scarce resource, thus it is crucial to avoid its irrational and senseless spendings ([Becker, G.S. 1965]), but also to search for a minimal time allocation, all other things being equal, in boring but necessary activities, particularly in travel, hence it is possible time has an implicit price, at least a resource price. [de Serpa, A.C., 1971] noticed it is not exactly the case when considering the consumer is not always the producer of his/her used activities, thus time resources are not allocated the same way. Particularly in travel activity, the individual is a time-taker and rationally knows that he/she has to account for a minimal amount to be spent. Then, time resources are different from travel time, and this author define the value of travel time as the implicit transfer value from travel time to resource times. It is an extent of the [Becker, G.S. 1965] framework, within the value of travel time is defined as the difference between two implicit prices. [Truong, T.K. and D.A. Hensher, 1985] have generalized this approach, by splitting the travel time in different components: access/egress times, waiting times, commuting times, riding/on-board times..., and have found significant values of travel times savings. It has become a usual approach in practical modelling ([Bhat, R.C. 1998], [Bhat, R.C. 1998], [Horowitz, J.L., Koppelman, F.S. and S.R. Lerman], [Quinet, E. 1998]).
Another general important standpoint is to differentiate the several reasons why the travel time has to be allocated. The basic idea is the activity to be realized at destination does not improve the same way the well-being of the individual, thus the corresponding needed travel time to access it does not take the same importance in the mind of the individual. This imply different preferences for different situations, and has led to activity-based travel demand systems ([Domenich, T. and D. McFadden, 1975], [Train, K.E. and D. McFadden, 1978], [Small, K., 1992], [BenAkiva, M., dePalma, A. 1996]). We focus in this paper on the regular Journey-To-Work (JTW) framework, particularly the transportation mode choice issue.

The [MVA, 1987] report, but also [McFadden, D., 2000] have noticed in practical discrete choice modelling several possible econometric specifications corresponding to differentiated theoretical approaches (see [McFadden, D. 1973], [Hensher, D.A. 1976], [Ben-Akiva, M. and S.R. Lerman, 1985], [Maddala, G.S., 1983], [Gourieroux, C., 1989] for econometric developments). We have noticed most of the empirical applications have used functional forms with prespecified properties for the implicit valuations of travel portfolios attributes, such as income effects (see for instance [dePalma, A, Kilani, K. 1999], [de Palma, A. and C. Fontan, 2001]). The [MVA, 1987] report and [McFadden, D., 2000] have also highlighted the fact the travel time may be unreasoningly quantified. It is possible to account for these effects using simple transformations of the corresponding variables, but we argue it is a misleading approach, since it is based on a priori behavioral considerations, that are simply leading to prespecified results.

In order to give clear definition of such prices and values of travel times concepts and their properties, we develop in a first section a regular journey-to-work transportation mode discrete choice model, within the traveller is faced to mutually exclusive alternatives, each supplying distinct travel portfolios. We describe the decision process of a rational traveller. In a second section, we derive definitions for the concepts of prices and values of travel times. We analyze precisely their response functions under general assumptions. We show there is different situations emerging according to the capacity of endogenizing leisure patterns during the OD trip, defining behavioral
profiles for the traveller, such that the effective travel time is twisted by the individual which in fact does not strictly quantify it. We also allow for an income effect entering the willingness to pay for saving the time attributes. Each prices of times is specific to the individual and to the alternative. [MVA, 1987] has already noticed such results could be possible, when dealing with the limits of the standard approach. Finally, we develop a binomial Box-Cox Logit model ([Gaudry M.J.I, 1978],[Gaudry M.J.I, 1981],[Mandel B, Gaudry M.J.I, Rothengatter, 1996]), particularly adapted for joint estimation of tastes and behavior parameters as it will be motivated, for different discrete choice specifications, and we propose estimations and comments of the parameters of our theoretical prices of travel times functions in French Parisian region, using an updated 1997 subsample of the global travel survey, a.k.a. "Enquête Globale de Transport (EGT)".

2 The journey-to-work discrete choices framework.

Consider the initial consumption-leisure trade-off framework, within an individual has to determine his optimal labor supply amount $t_w$ in order to consume goods, say an amount $C$, but also to keep free time resources, say $l$, for different leisure activities. Depending on his/her full income $R + w.t_w$, where $R \geq 0$ is an exogenous (non work) income and time $T$ resources, and exogenous price supply conditions (wages $w$ and consumption prices $p_c$), he/she will determine his/her optimal allocation, that is the one than reach to the highest level of satisfaction, characterized by an utility function $U(\cdot, \cdot)$ with inputs $C$ and $l$, according to exogenous market supply conditions and resources constraints. When we consider it is necessary to travel for the individual to access his/her workplace, his/her decision process is modified by the introduction of a travel market, within the origin-destination (OD) trip portfolios are determined (see also [Domenich, T. and D. McFadden, 1975], [Train, K.E. and D. McFadden, 1978]). These are defined as packages of different supply conditions, concerning fares, travel times, quality,... Because these attributes are of different types, we assume it is a quasi concave function, such that $\frac{\partial U}{\partial C} > 0$, $\frac{\partial U}{\partial l} > 0$, $\frac{\partial^2 U}{\partial C^2} \leq 0$, $\frac{\partial^2 U}{\partial l^2} \leq 0$. See [Varian, H., 1992] for more details.
they will have different impact on the resources of the individual. It seems evident the fare attributes worsen the budget resources, but we have to be more careful when considering the time attribute: it may be possible for the individual to endogenize some leisure patterns during his/her trip, thus even if travelling incurs a time resources loss, it is also possible to derive satisfaction when journeying. The underlying idea is travel time is not necessary exactly quantified as the effective time resource loss, but also may be twisted by the individual. This twist effect sign and spread is mainly depending on the traveller’s behavior.

Consider from now the (regular) motivation for trip is going to the destination workplace. This imply the amount of work hours is predetermined, say \( \bar{t}_w \), thus the full income, \( R + \bar{w}t_w \), is also known by the individual. To access the workplace, the OD trip incurs a fare \( p_T \), but also need a time allocation \( t_T \). This latter directly enters the time resources constraint, and according to the above considerations, also enters the leisure pattern \( l \) through a time evaluation function \( g(t_T) \).

Definition 1 With normalization to 1 for the goods price, thus \( C \) is the budget amount allocated to goods consumption, the rational individual will run out his/her budget resources, such that

\[
C + p_T = R + \bar{w}t_w.
\]

Definition 2 The utility leisure time input is defined as the sum between the effective free time and the perceived leisure time during trip:

\[
l = \left( T - \bar{t}_w - t_T \right) + g(t_T).
\]

Definition 3 The satisfaction the individual retrieves when realizing his/her journey-to-work trip, hence with a specific travel portfolio \( \{p_T^*, t_T^*\} \), is characterized by

\[
\begin{align*}
U(C, l) = R + \bar{w}t_w - p_T^* \\
l = \left( T - \bar{t}_w - t_T^* \right) + g(t_T^*)
\end{align*}
\]

Our frameworks becomes more realistic and sensible when considering the OD trip market has not a monopolistic supplier. In fact, we often observe at least a duopolistic situation. This follows from today’s possibilities of using different types of transportation network to realize an OD trip (road and rails), but also from different competitions operating between firms supplying the same transportation mode (airways carriers for instance), or different modes but operating

\footnote{Assume it is an \( \mathbb{R}^+ \rightarrow \mathbb{R} \) class application.}
on the same type of network (bus companies for instance). Such diversity is leading to a discrete choice framework for the traveller. The transportation mode choice issue is defined by mutually exclusive alternatives, say $K$, supplying different portfolios on the same OD trip. A rational individual will compare the different possibilities by valuating the corresponding satisfaction, then choose the one that reach to the highest level. Define $\forall k = 1, \ldots, K$, $\{p_k, t_k\}$ the $K$ available portfolios. Because the $K$ transportation modes are different, and because the time twisting effect is a behavioral consideration, we allow for the traveller to twist differently the travel times of the different competitors. Then, the corresponding levels of satisfaction are

$$\forall k = 1, \ldots, K, \begin{cases} U_k(C_k, l_k) \\ C_k = R + w_t - p_k \\ l_k = (T - t_w - t_k) + g_k(t_k) \end{cases}.$$

Hence, the individual will compare between them all the alternatives and choose

$$k^* = \arg \max_{\{1, \ldots, K\}} [U_k(C_k, l_k)],$$

that is the transportation alternative that reach to the maximum level of satisfaction (see [Ben-Akiva, M. and S.R. Lerman, 1985]). It is important to note the demand for travelling is normalized to one trip. The comparison process between modes of transportation is repeated for each OD trip. This framework is useful for describing the equilibrium of the traveller, in terms of implicit valuation of travel attributes. It is an useful tool in order to valuate the modification of the well-being of the individual following a modification of supplied travel attributes. For a planner, but also for strategical purposes, it is important to know how to modify the supply conditions without worsening the level of satisfaction of the traveller. More precisely, it is also interesting for a transportation operator to know how to substitute fare and time OD supply condition in order to keep the same market share, that is the level of satisfaction of the traveller remains unchanged. This need of implicit valuations of the travel portfolios leads to the concepts of prices and values of travel times.
3 Prices and values of travel times.

The values and/or prices of travel times are key parameters in traffic assignment, simulation and forecasting ([?], [?], [?]). These are used as calibration parameters to generate a Wardrop user optimum equilibrium and implement microsimulation experiments. All traffic (dynamical) simulation systems require the knowledge of these prices to implement assignation and forecast (dynamical) OD flows. However, the way it is made is based on a representative, or guardianship, price of time, unchanging whatever are levels of travel attributes. From a theoretical viewpoint, it appears as a particular framework, within the individual is considered to be a passive person to move from one point to another on a network. At the contrary, we think the traveller is sensitive to the level of supplied attributes when valuating their implicit price. It is because an individual has different response according to the travel conditions we argue there is not one representative price of travel time, but individual OD trip transportation mode prices of times.

3.1 Definitions and concepts.

The basic idea is time is money, particularly travel time since it is partly a constraining activity, thus the individual is willing to pay for saving time resources, that is to allocate less time to travel. His/her equilibrium is such that the price he/she is willing to pay in order to save an amount of travel time leaves unchanged his/her level of satisfaction. It is marginal substitution rate, as initially proposed in [Becker, G.S. 1965], in [Tipping, D.G., 1968], and in [de Serpa, A.C., 1971].

Definition 4 The price of travel time is the monetary amount the individual is willing to pay in order to save a marginal time unit and keep the same level of satisfaction.

\[ POT_k = \frac{\partial U_k}{\partial C_k} \frac{dl_k}{dp_{kT}} = \frac{\partial U_k}{\partial C_k} \frac{dl_k}{dp_{kT}} \left[ 1 - \frac{\partial g_k}{\partial t_{kT}} \right]. \]

By definition of a price, it must have a positive value, thus

\[ \frac{\partial g_k}{\partial t_{kT}} \leq 1. \]

The price of travel time (for alternative \( k \)) is a weighted marginal substitution rate between leisure and consumption. This weight represents the part of time loss during one time unit, since
\( \frac{\partial g_k}{\partial t_k T} \) is the marginal amount of leisure time during trip when this latter increase from one time unit. In other words, it is equivalent to the rate of endogenization of leisure patterns when travelling (see [McFadden, D., 2000]). This characterize the possibility for the individual to feel some leisure times during his/her journey. It implies the effective travel time amount is also used to other satisfying experiences, thus is not perceived as a pure loss of time resources. Only a part of the effective travel time is pure loss. The corresponding regularity condition for this rate is to be less than 1: for one more travel time unit, there is less than one leisure time unit that can be endogenized. Because \( \left. \frac{\partial U_m}{\partial l_m} \right|_{l_k = (T - t_m - t_k T)} + g_k (t_k T) \) is the price of time resources, that is the price the individual is willing to pay to have one more leisure time unit, we deduce the price of travel time using mode \( k \) is a weighted price of time resources. We can extend this result to a more general framework, considering it is possible to transfer time from one activity to another, particularly from travel to another activity. [Truong, T.K. and D.A. Hensher, 1985] have worked on this topic and found significant transfer values between different time components. Then, there will be trade-offs between the different implicit prices of activity times, which will define associated values of times.

**Proposition 5** The value of travel time saved is a transfer value. It represents the possible benefit or loss that rely on time transfer from one activity to another. It is the difference between two prices of time: one is the monetary amount the individual is willing to receive to allocate his/her marginal time unit to a specific activity, and the other is the amount he/she is willing to pay for saving the same time amount in another activity.

From this proposition we can derive several types of values of times: it depends on the two activities we consider. The table below gives for instance three types of values, all relative to the travel activity. They have each different interprettings:

<table>
<thead>
<tr>
<th>Values of travel time</th>
<th>VOT(_{mk})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer from one mode ( k ) to another mode ( m )</td>
<td>( \frac{\partial U_m}{\partial l_m} ) ( \left[ 1 - \frac{\partial u_m}{\partial l_m T} \right] - \frac{\partial U_k}{\partial l_k} ) ( \frac{\partial C_k}{\partial l_k} ) ( \left[ 1 - \frac{\partial u_k}{\partial l_k T} \right] )</td>
</tr>
<tr>
<td>Transfer from travel to work</td>
<td>( w ) ( \frac{\partial U_k}{\partial C_k} ) ( \left[ 1 - \frac{\partial u_k}{\partial l_k T} \right] )</td>
</tr>
</tbody>
</table>
They both express the implicit benefits and/or losses when a transfer of a time amount occurs between two activities. However, it also possible to use such concepts to valuate this implicit variation when the individual switches from one alternative to another. From a general viewpoint, these values and prices are measuring the willingness-to-pay (see [Train, K.E. and D. McFadden, 1978]) for realocating resources between activities. Our particular case is the travel activity, and we desire to analyze how such willingnesses are responding to changes in the supply side of the OD travel market.

3.2 Properties of the prices of travel times.

Since we have defined the concept of the price of travel time, we have to develop its response functions to variations of the travel attributes. The underlying basement is it appears quite unfeasible to assume these prices are unchanging with levels of attributes. More precisely, we argue there is a price effect entering the valuation of travel time, but also a time twisting effect, due to the possibility of endogenizing leisure during trip.

**Definition 6** The variation of the willingness to pay (WTP) for saving travel time using alternative $k$ according to its price component is

$$\frac{d\text{POT}_k}{dp_{kT}} = \frac{\partial U_k}{\partial t_k} \frac{\partial^2 U_k}{\partial C^2_k} \left[ 1 - \frac{\partial g_k}{\partial t_k} \right] \leq 0. \quad (1)$$

The variation of the WTP is depending on the rate of lost time by time unit, that is weighted by a ratios of derivatives of the utility function. We observe the prices of times are varying if and only if the utility function is strictly concave in its consumption argument. The sign of the variation incurred by a fare positive variation is negative: it implies the WTP for travel times are decreasing when fares are increasing.

**Corollary 7** The growth rate of the $k$-th price of time is equal to a weighted variation of the corresponding travel fare:

$$\frac{d\text{POT}_k}{\text{POT}_k} = \frac{\partial^2 U_k}{\partial C^2_k} dP_{kT}.$$  

\(^3\) Remind it is characterized by the formula $\left[ 1 - \frac{\partial g_k}{\partial t_k} \right]$. 

8
**Proof.** We start from the definition of the variation of the WTP travel time when a marginal fare variation occurs. When developing the denominator on the right side of (1), we fall back for a part of the right side formula the definition of the price of time, thus

\[
\frac{dPOT_k}{dp_{kT}} = \frac{\partial^2 U_k}{\partial C_k^2} \frac{\partial^2 U_k}{\partial C_k} \Rightarrow \frac{dPOT_k}{POT_k} = \frac{\partial^2 U_k}{\partial C_k} dp_{kT}
\]

The weight \(\frac{\partial^2 U_k}{\partial C_k}\) is an individual scaling one, characterizing the growth rate of the marginal consumption satisfaction from the initial situation to the new situation after the variation of the fare occurred. It implies people with higher initial budget resources will be less sensitive to a fare increasing, thus the satisfaction modification rate will be lower than the one of people with low initial income, and the price of time will be less decreasing for high income travellers. This income effect (see also [McFadden, D., 2000], [de Palma, A. and C. Fontan, 2001]) implies for high income individuals to have a higher WTP for saving travel time.

Alternatively, we also study the response function of the prices of times according to a marginal variation of effect travel times.

**Definition 8** The variation of the WTP for saving travel time using alternative \(k\) according to its time component is

\[
\frac{dPOT_k}{dt_{kT}} = -\frac{\partial^2 U_k}{\partial C_k^2} \left[ \frac{\partial g_k}{\partial t_{kT}} - 1 \right]^2 - \frac{\partial^2 g_k}{\partial t_{kT}^2} \frac{\partial^2 g_k}{\partial C_k},
\]

and depend on the rate of endogenization of leisure patterns during trip and its evolution with travel time.

The sign of the response is undetermined. It relies on the strict concavity of the utility function \(\frac{\partial^2 U_k}{\partial t_{kT}}\) in its leisure argument, and the variation \(\frac{\partial^2 g_k}{\partial t_{kT}}\) of the rate of leisure endogenization \(\frac{\partial g_k}{\partial t_{kT}}\). If the endogenization rate is constant whatever is the effective travel time, that is \(\frac{\partial^2 g_k}{\partial t_{kT}^2} = 0\), then the prices of times are increasing with the effective level of travel time if and only if the utility function is strictly concave in leisure argument. Assume it is the case, and allow for variation of the endogenization rate. Then the prices of times can be either overamplified or underestimated according to the sign of the rate variation. If the individual can endogenize more and more leisure
patterns when travel time increase, we have $\frac{\partial^2 g_k}{\partial t^2 kT} > 0$, then his/her WTP for saving travel time is decreasing. It is an intuitive result since when having increasing leisure time during his/her trip, he/she does feel less lost resources time, thus is willing to pay less for saving time. At the contrary, when effective travel durations are increasing and when the individual is endogenizing less and less leisure times, that is $\frac{\partial^2 g_k}{\partial t^2 kT} < 0$, he/she is perceiving increasing an time resources loss, thus is willing to pay more and more for saving travel time. It is the consequence of a lack of flexible scheduling leisure activities.

**Corollary 9** The growth rate of the prices of times according to their time components is a weighted variation of the corresponding travel times.

$$\frac{d \text{POT}_k}{\text{POT}_k} = - \left[ \frac{\partial^2 U_k}{\partial t^2 kT} \left[ 1 - \frac{\partial g_k}{\partial t kT} \right] + \frac{\partial^2 g_k}{\partial t^2 kT} \right] dt_{kT},$$

and depends on the modification rate of corrected marginal satisfaction and the modification rate of perceived time resources loss.

**Proof.** We start from the definition of the variation of the WTP travel time when a marginal time variation occurs in (2). Since $\left[ \frac{\partial g_k}{\partial t kT} - 1 \right]^2 = \left[ 1 - \frac{\partial g_k}{\partial t kT} \right]^2$, and dividing by the corresponding price of time

$$\frac{d \text{POT}_k}{\text{POT}_k} = - \left[ \frac{\partial^2 U_k}{\partial t^2 kT} \left[ 1 - \frac{\partial g_k}{\partial t kT} \right] - \frac{\partial^2 g_k}{\partial t^2 kT} \right] dt_{kT}.$$

When the effective travel time is increasing, the variation of the marginal leisure satisfaction is depending on the time resources level. People with higher initial leisure time will be less sensitive to a travel time increasing.

$$\frac{\partial^2 U_k}{\partial t^2 kT} \left[ 1 - \frac{\partial g_k}{\partial t kT} \right]$$

is characterizing the growth (worsening) rate of the marginal leisure utility of the traveller. Note this modification rate is corrected by the rate of leisure endogenization. We account for the leisure dimension of the travel time. When also setting

$$\frac{\partial^2 g_k}{\partial t^2 kT} = - \frac{\partial^2 g_k}{\partial t^2 kT},$$
it appears as the growth rate of perceived time resources loss. Finally, the growth rate of the prices of travel times with their travel times effective levels is the part of additional effective time corresponding to the sum of the growth rate of the corrected marginal leisure utility and the growth rate of the perceived time loss rate. It is corrected by the additional leisure amount incurred by the additional travel time but also by the fact the traveller may or not endogenize more leisure patterns. This lead him/her to twist the effective time to a perceived one.

It is important to notice when the individual is not a passive traveller, which is represented by the strict concavity of the utility functions in their arguments, the prices of travel times are responding to supply fares and effective times levels changes. Although the income effect has a deterministic sign, the times twisting effects are mainly depending on the capacity to endogenize leisure patterns during the journey, this capacity being variable with the effective travel time, that it is possible to have situations where the traveller has run out his/her endogenization capacities, and feels the resting time as a pure loss of resources, thus can lead to impatience, that is characterized by an increasing with effective travel times prices of times. It is possible to use Box-Cox transformations to define flexible functional empirical forms for the utility functions respecting our objective (see [Gaudry M.J.I, 1981]). If we assume the global utility function can be written as the sum of weighted sub-utility function, where the weights are characterizing the (relative) tastes of the travellers, and where the sub utility functions are for strictly positive continous variables (such as price and time attributes) Box-Cox transformations of the effective levels, then the Box-Cox parameters are characterizing the way the attributes are twisted, that is if the traveller overamplify and/or underestimate the effective levels of attributes. In a random utility framework (see [McFadden, D., 2000]), if the unobservables are independently and identically Gumbel distributed, we derive the Box-Cox Logit model class (Gaudry M.J.I, 1978], [Mandel B, Gaudry M.J.I, Rothengatter, 1996]), which lead to empirical valuation of travel prices

\footnote{Note it is the standard case for most of the empirical application.}

\footnote{Note the normality assumption has never been developped for discrete choices frameworks with Box-Cox transformations.}
of times. Moreover, it appears an econometric specification robust to the independence of irrelevant alternatives axiom (see [Hausman, J. and D. McFadden, 1984] for specification tests of this axiom in the standard logit model class).

4 Econometric specification of the discrete choices framework.

TALK FROM OTHER DCM SUCH AS PROBIT MNP RPL...

4.1 On the use of the Box-Cox transformation.

This transformation ([Tukey, J.W., 1957], [Box, G.P. and D.R. Cox, 1964]) is a power transformation class, that allow for more flexible functional forms when specifying the individual utility functions. In this section, consider the satisfaction $f$ of the traveller is a function of a determinants $X$. In standard approaches, we generally assume the variable is entering the utility function either in its original form, or in logarithmic form, or eventually in a quadratic form. For these two latters, it is a way to account for twisting effects entering the individual decisional process. But once it is specified, it is no more possible to verify if it is an "a priori" feasible assumption, except by comparing it with other functional forms, that also rely on "a priori" assumption sets.

The Box-Cox transformation is giving a less constraining specification framework. It appears as an useful tool for testing the presence of twisting effects on some decisional determinants entering the utility function. It allows for more generality because the way these are perceived is not predetermined.

Definition 10 Consider $X$ a strictly positive continuous variable.

$$ f(X, \lambda) = \begin{cases} \frac{X^\lambda - 1}{\ln X} & \text{if } \lambda \neq 0 \\ \ln X & \text{if } \lambda = 0 \end{cases} $$

is the Box-Cox transformation of the variable $X$.

Definition 11 The first order derivative with respect to $\lambda$ of the Box-Cox transformation are $\forall X > 0$ at a point $X$:

$$ \frac{\partial f(X, \lambda)}{\partial \lambda} \bigg|_{\lambda=X} = \begin{cases} \frac{1}{X} \left[ \ln X \cdot X^X - f(X, X) \right] & \text{if } X \neq 0 \\ \ln X^2 & \text{if } X = 0 \end{cases} $$

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Definition 12 The second order derivative with respect to $\lambda$ of the Box-Cox transformation are $\forall X > 0$ at a point $\bar{X}$:

$$\frac{\partial^2 f(X, \lambda)}{\partial \lambda^2} \bigg|_{\lambda=\bar{X}} = \begin{cases} \frac{1}{\bar{X}^2} \ln^2 (X) \bar{X}^\lambda - 2 \frac{\partial f(X, \lambda)}{\partial \lambda} \bigg|_{\lambda=\bar{X}} & \text{if } \bar{X} \neq 0, \\ \frac{1}{3} \ln^3 (X) & \text{if } \bar{X} = 0. \end{cases}$$

For our purposes, it is not interesting to analyze a negative $\lambda$ situation. However, when it is a positive parameter, we have to distinguish three subcases, which can be compared as shown in figures below. The right line with slope equal to 1 is the case where $\lambda = 1$, that is when the individual exactly quantify the variable. For one effective unit, one unit is perceived. When $\lambda$ differs from 1, we observe either overamplification (left figure), or underestimation (right figure), that is for one effective unit, it is not one unit that is perceived, but either more when $\lambda > 1$, or less when $\lambda < 1$. We also notice the use of such transformations is a wider approach, since it admits for specific cases the linear specification ($\lambda = 1$) and the log-linear specification $\lambda = 0$.

![Twisting effect](image1.png)

$\lambda$ can be considered as a twisting or perception factor, in the sense it account for the fact the individual may not exactly quantify the effective level of the variable $X$ when levelling his/her satisfaction. It is a behavioral parameter, that should have on impact on the relative importance of its corresponding variable in the decisonal process.
4.2 Notations and the latent utility system.

Our following development is focused on a binomial choice set. We have stated above an individual will choose the alternative that reach to the highest level of utility. For a sample $n = 1, \ldots, N$ of individuals faced to the same discrete choice issue, we will only observe the effective decisions they made. The decision rule can be written

$$
\forall n = 1, \ldots, N, y_n = \begin{cases} 
1 & \text{if } U_{1n} \geq U_{2n} \\
0 & \text{if } U_{1n} < U_{2n}
\end{cases}.
$$

As in classical latent dependent variable (LDV) models, we will turn our problem in probabilistic terms. Conditionally to all the determinants of the decision process, define the choice probabilities

$$
\Pr[1]_n \equiv \Pr[U_{1n} \geq U_{2n}], \Pr[2]_n \equiv 1 - \Pr[1]_n.
$$

In order to give clear results for the estimators we propose, we define some useful notations for the derivation of analytical formulas. The constant terms will be denoted by

$$
C_1, C_2,
$$

and are expressing the reservation utility levels corresponding respectively to alternative 1 and alternative 2. Another type of explanatory variables we will consider is individual specific variables. They are characterizing socioeconomical qualitative and quantitative levels such as age, sex, number of children, ... Because they are the individual’s own, they will appear in the two alternatives, from which they do not depend on. Define them as the $(J \times 1)$ array

$$
Z_n \equiv (Z_{n1}, \ldots, Z_{nJ})'.
$$

However it may be credible to argue the relative importances (weights) of such variables are not the same for each modality. Then, we define alternative indexed weights corresponding to the individual specific variables, such that they are two $(J \times 1)$ arrays

$$
\forall k = 1, 2, \beta_k \equiv (\beta_{k1}, \ldots, \beta_{kJ})'.
$$
We also distinguish ordinal and qualitative alternative specific attributes from the strictly positive continuous quantitative attributes. These formers are stacked for each individual and each alternative \( k \) in a \((L \times 1)\) array

\[
W_{nk} \equiv (W_{nk1}, \ldots, W_{nkL})', \gamma_k \equiv (\gamma_{k1}, \ldots, \gamma_{kL})'
\]

are the corresponding weights arrays, we also assume they differ between choice possibilities. Precisely, we consider alternative specific explanatory variables, strictly related to their corresponding choice alternative, within we cluster them as mentioned above.

For the last type of explanatory variables, say \( P \) for each modality, we define them by

\[
\forall p = 1, \ldots, P, X_{n1p} \text{ and } X_{n2p}.
\]

The particularity is for these variables is not to be solely weighted, but also to be twisted through the perception factor defining the Box-Cox transformation we will apply to each of these. Therefore, for each variable \( p \) of each alternative 1, 2, we have the corresponding parameters

\[
\theta_{1p}, \lambda_{1p} \text{ and } \theta_{2p}, \lambda_{2p}.
\]

Finally, because we are just observing the effective choices of the individuals, we may not have accounted for unsignificant determinants, but also unmeasurable reasons and unobservables externalities between the different alternatives. Hence, we also assume there is a perturbation factor for each individual

\[
\varepsilon_n = (\varepsilon_{n1}, \varepsilon_{n2})',
\]

which is randomly distributed over the population, but we assume with a stable and identical individual generation process. It is a regularity conditions for a stable random utility framework.

With all our notations, we define the latent data generating process to be equal to a system of weighted sums of sub-utility functions. Some of these latters are just the effective observation, some others are twisted variables. Consider \( \mu \) an heterogenetity factor, such that \( \forall n = 1, \ldots, N \)

15
\[
\begin{align*}
U_{1n} &= C_1 + Z'_n \beta_1 + W'_n \gamma_1 + \sum_{p=1}^{P} X_{n1p} (\lambda_{1p}) \theta_{1p} + \mu \varepsilon_{n1} \\
U_{2n} &= C_2 + Z'_n \beta_2 + W'_n \gamma_2 + \sum_{p=1}^{P} X_{n2p} (\lambda_{2p}) \theta_{2p} + \mu \varepsilon_{n2}.
\end{align*}
\]

4.3 The Logit maximum likelihood estimator.

The Logit models class is obtained when assuming \(\forall n = 1, \ldots, N,\)

\[\varepsilon_{n1}, \varepsilon_{n2} \overset{iid}{\sim} \text{Gumbel}, F_{\varepsilon_{n1}} (\omega) = F_{\varepsilon_{n2}} (\omega) = \exp \left(-\exp \left(-\sqrt{\frac{\pi^2}{6}} \omega \right) \right).\]

We derive the choice probability for the alternative 1 as

\[\Pr[1]_{n} = \frac{1}{1 + \exp \left(\frac{C_2 - C_1}{\mu} + Z'_n \left(\frac{\beta_2 - \beta_1}{\mu} \right) + W'_n \left(\frac{\gamma_2 - \gamma_1}{\mu} \right) - \sum_{p=1}^{P} X_{n1p} (\lambda_{1p}) \theta_{1p} - \sum_{p=1}^{P} X_{n2p} (\lambda_{2p}) \theta_{2p} \right)}.
\]

We clearly see that parameters identification issues are emerging, which correspond to specific types of explanatory variables, but also which depend on a scale parameter. Concerning the constant terms and the individual specific variable, it is only the scaled difference between the relative weights that can be estimated. Because of the unknown scale, we often assume \(\mu = 1,\)

but with the implicit consequence that estimated numeric values have not a meaningful sense. It is only the sign of the difference and its significance that can be interpreted. Note the scale measure oblige to be also careful about the numerical values of all weights parameters. the Box-Cox parameters are directly estimable since they enter the specification through a power transformation. Consequently, the estimable tastes parameters are

\[C_* \equiv \frac{C_2 - C_1}{\mu}, \beta_* \equiv \left(\frac{\beta_2 - \beta_1}{\mu} \right), \gamma_{s2} \equiv \frac{\gamma_2}{\mu}, \gamma_{s1} \equiv \frac{\gamma_1}{\mu}, \theta_{s2p} \equiv \frac{\theta_{2p}}{\mu}, \theta_{s1p} \equiv \frac{\theta_{1p}}{\mu},\]

whereas there no identification issue for the Box-Cox parameters, thus they all can be estimated.

The log-likelihood function is derived using the observed decision rule and the choice probabilities:

\[\ln \ell_N = \sum_{n=1}^{N} [y_n \ln (\Pr[1]_{n}) + (1 - y_n) \ln (1 - \Pr[1]_{n})].\]
It is important to note it is an estimator belonging to the $M$-estimator class, thus it must respect some regularity conditions. These are extensively developed in Gourieroux and Montfort, Newey and McFadden. We define for clear computations of the scores and the hessian of our following estimator compact notations for the parameters and also the choice probabilities derivatives with respect to the parameters characterizing tastes:

$$\theta \equiv (C_*, \beta_*, \gamma_1, \gamma_2, \theta_{12}, \ldots, \theta_{11}, \ldots, \theta_{11})',$$

and Box-Cox parameters designing behavioral twisting effects:

$$\lambda = (\lambda_{21}, \ldots, \lambda_{2P}, \lambda_{11}, \ldots, \lambda_{1P})',$$

thus

$$\ell_N \equiv \ell_N \left( y \mid 1, Z, W_2, W_1, X_2, X_1; \theta, \lambda \right).$$

Lemma 13 Let

1. $\pi \equiv (\theta', \lambda')' \in \Pi$, a compact subset of $\mathbb{R}^K$.
2. $(y, I)$ independently and identically distributed.
3. $\ell_N$ is continuous for $\pi$, integrable with respect to the true distribution of $(y, I)$ for all $\pi \in \Pi$.
4. $\frac{1}{n}\ell_N \xrightarrow{p} \ell_\infty \equiv \mathbb{E}_I \mathbb{E}_{y(I)} [\ell_n (y_n, I; \theta, \lambda)]$,
   the associated limit program.
5. $\ell_\infty$ is continuous for $\pi$.
6. $\pi_0 = \arg \max_\pi \ell_\infty$ exists and is unique.
   Then $\bar{\pi} = \arg \max_\pi \ell_N$ is such that $\bar{\pi} \xrightarrow{p} \pi_0$.

If the subset $\Pi$ is not compact, but open, then if $\ell_N$ is derivable, there is an asymptotic solution for the first order conditions, which almost surely converge to the true value of the parameters. It is important to note this solution is corresponding to a local maximum. We assume from now these properties verified for our estimator. Following our above notations, we
obtain a stacked form for the individual probability derivatives:

\[
\nabla_\theta \Pr [1]_{1n} \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} = \begin{bmatrix}
-1 \\
-Z_n \\
-W_{n1} & -W_{n2}
\end{bmatrix} [\Pr [1]_{1n} (1 - \Pr [1]_{1n})]_{\theta = \tilde{\theta}; \lambda = \lambda}, \text{ and}
\]

\[
\nabla_\lambda \Pr [1]_{1n} \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} = \begin{bmatrix}
-\theta_{121} & \theta_{121} & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-\theta_{21p} & \theta_{21p} & \cdots & \cdots & \cdots \\
\theta_{111} & \theta_{111} & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\theta_{11p} & \theta_{11p} & \cdots & \cdots & \cdots \\
\end{bmatrix} \tilde{W}_{n} (\theta, \lambda) \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda}
\]

They are useful to derive the system of first order analytical conditions for the log-likelihood function, given by:

\[
\left\{ \begin{array}{l}
\frac{\partial \ln \ell_{\theta \lambda}}{\partial \theta} \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} = \sum_{n=1}^{N} \left[ \tilde{X}_{n} (\lambda) (y_{n} - \Pr [1]_{1n}) \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} \right] \\
\frac{\partial \ln \ell_{\theta \lambda}}{\partial \lambda} \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} = \sum_{n=1}^{N} \left[ \tilde{W}_{n} (\theta, \lambda) (y_{n} - \Pr [1]_{1n}) \bigg|_{\theta = \tilde{\theta}; \lambda = \lambda} \right].
\end{array} \right.
\]

These results are quite similar to the classical MNL model in their analytical forms, but with the additional parameters characterizing the Box-Cox transformations, we obtain however different first order conditions, because the parameters are also appearing in the explanatory matrices. We also have to study the second order derivatives. Define for these latters the notation:

which can be stacked in a matrix $J(\theta, \lambda) = \sum_{n=1}^{N} J_n(\theta, \lambda)$ such that

$$
J_n(\theta, \lambda) \bigg| \begin{array}{c} \theta = \tilde{\theta} \\ \lambda = \tilde{\lambda} \end{array} = \left[ \begin{array}{cc} 0 & 0 \\ 0 & \partial \tilde{W}_n(\theta, \lambda) / \partial \lambda \end{array} \right] [y_n - \Pr[1]_n]
$$

$$
= \left[ \begin{array}{ccc} \tilde{X}_n(\lambda) \tilde{X}_n(\lambda)' & \tilde{X}_n(\lambda) \bar{W}_n(\theta, \lambda)' \\ \bar{W}_n(\theta, \lambda) \tilde{X}_n(\lambda) & \bar{W}_n(\theta, \lambda) \bar{W}_n(\theta, \lambda)' \end{array} \right] \left[ \begin{array}{c} \Pr[1]_n (1 - \Pr[1]_n) \end{array} \right]
$$

This second order derivatives and their properties, notably concerning the hessian matrix, is more generally explained in [Gaudry, M.J.I, Duclos, L.P, Dufort, F. and Tran Liem, 1994].

Moreover, a discussion with M.Gaudry has helped us to understand the possibility of a non unique solution because it is a non trivial optimization program.
Lemma 14 Under the regularity properties of the estimator, and if parameters, we can obtain the asymptotic distribution of our estimator.

Since we assume the other lemma verifies, then we give their analytical expressions:

\[
\sum_{n=1}^{N} \frac{\partial \ln \ell_n}{\partial \theta} \frac{\partial \ln \ell_n}{\partial \lambda} \bigg|_{\theta = \hat{\theta}, \lambda = \hat{\lambda}} = \sum_{n=1}^{N} \left[ \bar{X}_n(\lambda) \bar{X}_n(\lambda)' [y_n - \Pr[1]]^2 \right]_{\theta = \hat{\theta}, \lambda = \hat{\lambda}}
\]

\[
\sum_{n=1}^{N} \frac{\partial \ln \ell_n}{\partial \lambda} \frac{\partial \ln \ell_n}{\partial \lambda} \bigg|_{\theta = \hat{\theta}, \lambda = \hat{\lambda}} = \sum_{n=1}^{N} \left[ \bar{W}_n(\theta, \lambda) \bar{W}_n(\theta, \lambda)' [y_n - \Pr[1]]^2 \right]_{\theta = \hat{\theta}, \lambda = \hat{\lambda}}
\]

\[
\sum_{n=1}^{N} \frac{\partial \ln \ell_n}{\partial \theta} \frac{\partial \ln \ell_n}{\partial \lambda} \bigg|_{\theta = \hat{\theta}, \lambda = \hat{\lambda}} = \sum_{n=1}^{N} \left[ \bar{W}_n(\theta, \lambda) \bar{X}_n(\lambda)' [y_n - \Pr[1]]^2 \right]_{\theta = \hat{\theta}, \lambda = \hat{\lambda}}
\]

which can be stacked in a matrix \( \mathbf{I}(\theta, \lambda) = \sum_{n=1}^{N} \mathbf{I}_n(\theta, \lambda) \) such that

\[
\mathbf{I}_n(\theta, \lambda) \bigg|_{\theta = \hat{\theta}, \lambda = \hat{\lambda}} \equiv \begin{bmatrix}
\bar{X}_n(\lambda) & \bar{X}_n(\lambda)' \\
\bar{W}_n(\theta, \lambda) & \bar{W}_n(\theta, \lambda)'
\end{bmatrix}
\begin{bmatrix}
\bar{X}_n(\lambda)' & \bar{X}_n(\lambda) \\
\bar{W}_n(\theta, \lambda)' & \bar{W}_n(\theta, \lambda)
\end{bmatrix}
\left[ y_n - \Pr[1] \right]^2
\]

Using a Taylor expansion of the system of first order conditions around the true values of the parameters, we can obtain the asymptotic distribution of our estimator.

**Lemma 14** Under the regularity properties of the estimator, and if

1. \( \ell_n \) is twice continuously differentiable in \( \pi \equiv (\theta', \lambda')' \).

2. The matrix

\[
\mathbf{E}_\pi \mathbf{E}_n \left[-\frac{\partial^2 \ln \ell}{\partial \pi \partial \pi'} \right]_{\pi = \pi_0}
\]

exists and is invertible.

Then

\[
\sqrt{N} \begin{bmatrix}
\hat{\theta} - \theta_0 \\
\hat{\lambda} - \lambda_0
\end{bmatrix} \xrightarrow{\mathcal{L}} \mathcal{N} \left( \begin{bmatrix}
0 \\
0
\end{bmatrix}, \mathbf{E} \left[ \mathbf{J}_N(\theta_0, \lambda_0)^{-1} \mathbf{E} \mathbf{I}_n(\theta_0, \lambda_0) \mathbf{E} \mathbf{J}_N(\theta_0, \lambda_0)^{-1} \right] \right),
\]

where the expectations at true values are estimated by

\[
\frac{1}{N} \begin{bmatrix}
\mathbf{I} \left( \hat{\theta}, \hat{\lambda} \right) \\
\mathbf{J} \left( \hat{\theta}, \hat{\lambda} \right)
\end{bmatrix},
\]

since we assume the other lemma verified.
5 Empirical application to the french parisian travel survey.

We will use in our empirical application data at the individual level, extracted from the EGT primary datafile, which is a revealed preferences (RP) survey. It appears the travel portfolio is itself absorbing the major effects in the transportation mode discrete choice process ([de Palma, A. and C. Fontan, 2001] used the same initial dataset to estimate transportation mode discrete choices models). This survey is prepared and paid by the city of Paris (France), the STP (transit syndicate), the SNCF (the rail operator), the RATP (major subways and bus networks) and COFIROUTE (highways). Because our topic is to focus on the topic of the valuation of travel times, we will limit the range of individual specific variables. The RP survey is matched with the results of the traffic assignment model of the IAURIF, the urban planning institute of the parisian region, in order to obtain a pseudo stated preferences (SP) datasets (see [Hensher, D.A., 1994] for an extensive review of such matchings). We refer the reader to [BenAkiva, M., Nesterov, Y. and A.de Palma 2000] for an understanding of these traffic simulation tools. The main characteristics of the IAURIF model follow a classical four steps approach (see [Quinet, E. 1998]), and cluster the geographical region in 488 zones. The road network is built using the land use planning and is defined with 6800 arcs and 4500 nodes. The transit network is including rail, bus and subways, and is constituted with 2900 arcs and 1000 nodes. Once calibrated, our sample has a size \(N = 865\). It contains individual trip which occurs in morning 7.00-10.00 PM weekdays. All sampled individuals are licensed drivers with at least a vehicle for his/her corresponding household. They also have access both to transit and road networks, and they have all carried a regular journey-to-work out, thus they are employed people.

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7 I greatfully thank the THEMA-TTR university research group for making me available this subsample.

8 Note the name of this syndicate has changed and is today STIF.
5.1 Models specifications and estimation results.

Clearly, the general econometric specification developed above allows for several joint tastes and behaviors econometric specification. We use the following variables for estimation purpose: Constant, Age, Sex for individual specific (IS) variables, which are associated to the private vehicle transportation mode for identification purpose. Constant and age parameters are denoting the difference of weights between mode, that is to which mode is corresponding the higher impact of these variables. The IS variable sex is built to introduce a difference between male and female. The corresponding parameter is characterizing the impact of being a female on the transportation mode choice and in which mode it is the most present. We also have attribute specific (AS) variables: travel times are split in times "in" and times "out". These former are for the transit mode the "on-board" travel time, and for the private vehicle (PV) mode the net from local access/egress times travel time. Times "out" are access/egress, wait and walk times for the transit mode, and local access/egress times for the PV mode. The transit fares are computed with the actual transit card fare grid and aggregate statistics given by the STP. It is crossed with the survey variable denoting the possessing or not of a transit cards to define the fare variable. Concerning the PV mode, the STP gives an using kilometer cost, accounting for gas and non gas costs. Finally, we define the net from travel cost income for transit and PV.

The data generating process for the latent system is assumed to be the one developed above. We expect for the net incomes weight parameters to have a positive effect on their corresponding choice probabilities, and for times weights to have negative effects. Concerning Box-Cox parameters, they are applied to attribute specific variables, and we expect all of them to be positive. Hence, four general surfaces can be set for the prices of times function, which are depending on the travel mode k, the time component type l, the amplitude of the twisting effects $\lambda_{k,l}$ and $\lambda_{k,inc}$, the attribute specific levels $p_{n,k,l}$ and $t_{n,k,l}$, the individual n income and are equal to

$$\forall l, \forall n, \forall k, \pi_{n,k,l} = -\frac{\theta_{k,l}}{\theta_{k,inc}} \lambda_{k,l}^{\lambda_{k,inc}-1} [R_n - p_{n,k,l}]^{1-\lambda_{k,inc}}.$$
According to different possible values for the Box-Cox parameters, the following three dimensional graphical representations of the consequent prices of times surfaces are giving quite intuitive results. Each of them is corresponding to a specific traveller’s behavior:

\[
\theta_{\text{inc}} > 0, 0 \leq \lambda_{\text{inc}} < 1, \theta_t < 0, \lambda_t > 1
\]

The left figure is characterizing an increasing with effective time and net income price of travel time. It is a situation where higher income people have a higher WTP, but whatever is their income, all the travellers are willing to pay more and more as travel time is increasing. Such responses are denoting an impatient behavior for the individuals: they are more and more willing to get rid partly of their travel times, thus a higher WTP, because they are perceiving more and more time resources losses in when travelling. The right figure is considering people can endogenize more and more leisure patterns as their travel time are increasing, thus they are perceiving fewer time losses. Because they can improve their satisfaction by doing some leisure during their trips, they are desiring to pay less and less for saving their travel times as those are during more and more.

The two following figures are situations we do not expect in our empirical results, since they do correspond to quite unintuitive framework for our theoretical framework. The left figure is contesting the natural income effect: it is stating people with higher budget resources have a lower WTP for saving the attributes of the boring travel activity, even if they have a higher implicit purchase power of time. The increasing of the WTP with the effective travel time is
characterizing the impatience behavior of the travellers. The right figure is corresponding to the same situation concerning the net income variable, but here the decreasing of the WTP with time is denoting a sort of lassitude: because it possible to endogenize higher and higher amount of leisure times when travelling, it is not necessary for the individuals to pay for saving these more and more "leisured" travel times.

\[ \theta_{\text{inc}} > 0, \lambda_{\text{inc}} > 1, \theta_t < 0, \lambda_t > 1 \]

\[ \theta_{\text{inc}} > 0, \lambda_{\text{inc}} > 1, \theta_t < 0, 0 \leq \lambda_t < 1 \]

When a Box-Cox parameters are equal to one, then the corresponding attribute impact on the associated WTP is null: the corresponding marginal utility (or cost) is constant whatever is the initial level. Then, other situations are possible, each defining a particular price of time surface. Hence, the (widely used) strict linear specification is defining for the WTP a parallel to ground plan, such that:

\[ \theta_{\text{inc}} > 0, \lambda_{\text{inc}} = 1, \theta_t < 0, \lambda_t = 1. \]
It appears as a very particular surface function, which is characterizing a total neutrality of the individual own’s behaviors on his/her WTP. Another important issue is the modal weights parameters: can we consider the decisional determinants have the same importance in each transportation mode? Or not? This lead to distinguish the several possible econometric specifications we can use to give an empirical application to our theoretical framework.

5.1.1 The generic specifications.

The generic specifications are considered when the linear weights (tastes) parameters are assumed to be the same whatever is the transportation mode. In addition, when there is no Box-Cox transformations on any variable, we fall back on a conditional logit analysis framework (see [McFadden, D. 1973]), where the decision process is depending only on the differences between the attribute specific variables. It is important to note it is a widely used logit specification in transportation analysis. We deviates slightly by introducing IS variables, meaning they have different weights according to the different transportation modes. It captures more detailed results.

<table>
<thead>
<tr>
<th>Estimates (M1)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[Student in brackets]</td>
<td></td>
</tr>
<tr>
<td>Constant (PV)</td>
<td>−0.5231 [−1.0396]</td>
</tr>
<tr>
<td>Age (PV)</td>
<td>0.0267 [2.9842]</td>
</tr>
<tr>
<td>Sex (PV)</td>
<td>−0.1390 [−0.7656]</td>
</tr>
<tr>
<td>Net Income difference</td>
<td>0.0523 [2.3418]</td>
</tr>
<tr>
<td>Travel time in difference</td>
<td>−0.0829 [−12.4189]</td>
</tr>
<tr>
<td>Travel time out difference</td>
<td>−0.0714 [−7.4872]</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>−372.318</td>
</tr>
<tr>
<td>LR stat (df=5)</td>
<td>453.785</td>
</tr>
<tr>
<td>% right</td>
<td>79.54</td>
</tr>
</tbody>
</table>
There is no significant difference between the reservation levels of utility corresponding to the two alternatives. Only the age seems to have a significant effect on the individual choice probabilities concerning the IS variables, and lead to the quite natural fact that older people have preferences for private vehicle trip.

Using our general formula for the prices of times, we obtain for time "in" 95.11 $FF/Hr and for times "out" 81.91$FF/Hr. These are considered as standard results for the french parisian region (see for instance in [Quinet, E. 1998], and more recently [Boiteux, M,2001.] and [de Palma, A. and C. Fontan, 2001] for empirical results), thus we are satisfied to find a stylized fact as a starting point for the extension of these primal model. There is implicitly two ways of extending the above framework. First we can introduce twisting effects in the conditional framework, using Box-Cox transformations. Since it not a senseful assumption to allow for alternative differenced Box-Cox transformations, we assume there is one unique transformation for each type of travel component, whatever is the alternative. When we introduce Box-Cox transformation on prices and times attributes, two variants are possible: grouped transformation for time or not. This latter case leads to the fact that IS variables have the same interpretions as before. The major shiftings occur in the attribute specific weights, because of the Box-Cox transformation. The major difference with the first specification is there is now one price of time for each individual of the sample. Because there is a huge improvement in the output results of the model, it becomes more difficult to find a guardianship value. We use below graphical representations of the different distributions and surface functions of the several prices of times to characterize explicitely the sample willingnesses to pay. Aggregate statistics are provided to help in understanding the outputs.

From a general standpoint, the results gives regular signs for the AS variables. It appears the income effect is significantly present as long as the corresponding Box-Cox parameter is known to be not significantly different from zero, but is numerically a positive number.Implicitely, such effect imply a relatively more important weight for the net income in the decision process,
since people with lower budget resources are more sensitive to price variations, whatever is the transportation mode. Because the marginal utility is known to be varying with the initial level of income, the weight parameter will appear more significantly.

<table>
<thead>
<tr>
<th>Estimates (M2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[Students in brackets]</td>
<td></td>
</tr>
<tr>
<td>Constant (PV)</td>
<td>-0.4937</td>
</tr>
<tr>
<td></td>
<td>[-0.9995]</td>
</tr>
<tr>
<td>Age (PV)</td>
<td>0.0254</td>
</tr>
<tr>
<td></td>
<td>[2.8369]</td>
</tr>
<tr>
<td>Sex (PV)</td>
<td>-0.1279</td>
</tr>
<tr>
<td></td>
<td>[-0.7020]</td>
</tr>
<tr>
<td>Net Incomes difference</td>
<td>2.9827</td>
</tr>
<tr>
<td></td>
<td>[2.6046]</td>
</tr>
<tr>
<td>Travel times &quot;in&quot; difference</td>
<td>-0.0545</td>
</tr>
<tr>
<td></td>
<td>[-12.8551]</td>
</tr>
<tr>
<td>Travel times &quot;out&quot; difference</td>
<td>-0.0507</td>
</tr>
<tr>
<td></td>
<td>[-7.9272]</td>
</tr>
<tr>
<td>Box-Cox Net Income</td>
<td>0.0668</td>
</tr>
<tr>
<td></td>
<td>[0.1440]</td>
</tr>
<tr>
<td></td>
<td>[-2.0103]</td>
</tr>
<tr>
<td>Box-Cox Travel times</td>
<td>1.1122</td>
</tr>
<tr>
<td></td>
<td>[9.2741]</td>
</tr>
<tr>
<td></td>
<td>[0.9355]</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-369.648</td>
</tr>
<tr>
<td>LR stat (df=7)</td>
<td>459.125</td>
</tr>
<tr>
<td>% right</td>
<td>80.00</td>
</tr>
</tbody>
</table>

Concerning the travel times, two important remarks have to be noticed. First, they are significant variables, either conditionally or not to the Box-Cox parameters. Secondly, this latter is clearly different from zero, but is tested to be not significantly different from one, even if the numerical value is higher than 1. This imply the WTP is neutral to effective travel time variation. The perceived cost in travel time is not convex, but linear for the JTW framework with this specification, where we group the different types of travel times such that they are twisted the same way.
The empirical estimation of the hourly prices of travel times can be done with several approaches. The first step is to characterize the distributions of the prices of times, whatever are the effective level of alternative specific variables. For each individual, we have the prices of times of each time dimension of each transportation alternative, and we desire to characterize their distribution through the sample unconditionally to any variables.

Unconditional distribution of the prices of times

<table>
<thead>
<tr>
<th>POT FF/HR</th>
<th>PV in</th>
<th>PV out</th>
<th>POT FF/HR</th>
<th>PV in</th>
<th>PV out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>149.34</td>
<td>138.93</td>
<td>Kurtosis</td>
<td>5.76</td>
<td>5.76</td>
</tr>
<tr>
<td>Median</td>
<td>129.87</td>
<td>120.82</td>
<td>Asymmetry coeff.</td>
<td>1.90</td>
<td>1.90</td>
</tr>
<tr>
<td>Mode</td>
<td>158.46</td>
<td>147.41</td>
<td>Min</td>
<td>7.99</td>
<td>7.44</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>97.13</td>
<td>90.35</td>
<td>Max</td>
<td>728.85</td>
<td>678.03</td>
</tr>
</tbody>
</table>

Unconditional distribution of the prices of times
<table>
<thead>
<tr>
<th>POT FF/HR</th>
<th>Transit in</th>
<th>Transit out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>139.57</td>
<td>129.84</td>
</tr>
<tr>
<td>Median</td>
<td>118.71</td>
<td>110.43</td>
</tr>
<tr>
<td>Mode</td>
<td>213.87</td>
<td>198.96</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>97.14</td>
<td>90.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>POT FF/HR</th>
<th>Transit in</th>
<th>Transit out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>5.79</td>
<td>5.79</td>
</tr>
<tr>
<td>Asymetry coeft.</td>
<td>1.91</td>
<td>1.91</td>
</tr>
<tr>
<td>Min</td>
<td>7.13</td>
<td>7.56</td>
</tr>
<tr>
<td>Max</td>
<td>716.36</td>
<td>666.41</td>
</tr>
</tbody>
</table>

Such results are conform with the analysis of [Arduin, J.P., Ni, J. and O. Pick, 1994]: our distributions are more or less log-normal distributions. It is also a distribution assumption that is often made when using the modal split price-time model ([Abraham, C. and J.D. Blanchet, 1973]).

Because the time twisting effect is null, we focus on the response of the hourly prices of times according to the monthly income.
From an aggregated standpoint, it appears that times "in" are higher valued than times "out". It is denoted in average, median and modal distributional statistics. Graphically, the response functions show times "out" are lower than times "in" for a strict majority of the individuals. However, times "in" are more volatile, since their support is longer and their standard deviations higher. It is a strange result of the model, because we have expected the contrary, since access, egress, wait and walk times are certainly more penibles than "on-board" times.

Another remark is that the PV prices of times are higher than transit implicit prices. This is due to the fact the PV mode is a more constraining riding framework for endogenizing leisure patterns. Moreover, this model gives higher WTP for time attributes than the simple conditional logit model: this is due to the convexity of the indirect utility in travel cost, which imply the utility function is concave in consumption argument, thus an income effect improving the WTP (see [Varian, H., 1992]). These empirical response functions to an income increasing also show the individual with higher budget income has an implicit time purchase power that is higher than the individual with a low income, thus has a higher WTP, for all times types ("in" and "out"). Precisely, we notice there is different regimes according to specific income range. The sensitivity of the response function is increasing with the income class. This strange effect is maybe due to the fact we have available only very few observation for high income classes, thus it may be biasing the form of the response function. In middle classes, we observe a regular concave function for all prices of times.

When there is one transformation by travel attribute, that is there is no grouped twisting effects in the conditional logit framework, we obtain different results. This model $M_3$ is statistically justified, since the likelihood ratio (LR) test between this model and the above ones, which are constrained frameworks, state the following results are improving the above ones. The test statistics ($M_1 \subset M_3$ and $M_2 \subset M_3$) fails to reject the fact the following model is not improving the results, even if the %right appear to be lower or equal than the previous models ones. We allow for the differentiation of the twisting effects according the travel time types. Clearly, these
appear to be numerically different and do not imply the same WTP. Concerning IS variables, age has still the same sign and imply the same relative preference for PV choice when the individual is older. Moreover, there is a significant gap in favor of the transit mode between the reservation utility levels. It is difficult to justify the negative sign because generally the levels of comfort and trip schedule independency for the PV mode ensure an initial reservation utility level for the PV mode greater than the one of the transit mode.

<table>
<thead>
<tr>
<th>Estimates (M3) [Students in brackets]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (PV)</td>
</tr>
<tr>
<td>Age (PV)</td>
</tr>
<tr>
<td>Sex (PV)</td>
</tr>
<tr>
<td>Net Incomes difference</td>
</tr>
<tr>
<td>Travel times &quot;in&quot; difference</td>
</tr>
<tr>
<td>Travel times &quot;out&quot; difference</td>
</tr>
<tr>
<td>Box-Cox Net Income</td>
</tr>
<tr>
<td>Box-Cox Time &quot;in&quot;</td>
</tr>
<tr>
<td>Box-Cox Time &quot;out&quot;</td>
</tr>
<tr>
<td>Log-likelihood</td>
</tr>
<tr>
<td>LR stat (df=8)</td>
</tr>
<tr>
<td>% right</td>
</tr>
</tbody>
</table>

It appears that times "out" are perceived as concave cost entering the utility function, since the numerical result is included in the [0, 1] segment. The corresponding student is significantly different from zero at the level of 1%, but significantly different from 1 at the level of 10%, meaning we have to reject the logarithmic transformation of this variable, but it is possible we should have to
apply a concave transformation, since the rejection probability for the $\lambda = 1$ test is not exhibiting a really significant statistic. However, we submit to the student test and admit this time type is twisted in the sense it occurs a lassitude effect, that is the travellers have sufficient schedulable leisure resources to be endogenized at an increasing speed during their trips. Implicitly, it is stating people are caring about these times types, and prevent from them by scheduling possible leisure activities to be used with different intensity levels when these formers occur. Concerning times "in"; even if the numerical results is clearly greater than one, it appears to be not significantly different from 1. It means it is a time type that is clearly identified by the traveller, but it also imply an unvarying response of the correspondiong WTP when the effective travel time "in" increase. We also observe a significant income effect through the Box-Cox transformation value, and its student statistics, of the net income variables, which lead to consider a logarithmic form for these latters. It is a result we already have proposed in the previous model. As stated by [McFadden, D., 2000], this effect should be considered in all possible econometric specifications of a discrete mode choice model. The corresponding unconditional to travel attributes levels distribution of the prices of times across the sample and their aggregate statistics are given below.

![Empirical distribution of the prices of times](image_url)
We observe again approximative log-normal forms for the unconditional distributions of the different prices of times. However, it devotes some comments, since they have completely different aggregate statistics. An important remark when comparing the distributions is here again, the private mode imply higher WTP than the transit mode for saving times amounts. This result is the same than the previous model. It may be due to the fact a transit trip scheduling is less flexible than a PV trip, thus the travellers have less WTP for attributes, since they are not really choosing their optimal portfolio, but the nearest satisfying one proposed by the transit supplier.
We also notice that travel times "out" are more valuated than the times "in". The explanation of this point relies on the statistical results we have obtained. The relative importance of the time "out" variable is higher than the times "in" one for both PV and transit modes, and are both highly significant (1% level), whereas there are not significantly twisted (or only at a poor statistical significance level). Then, it is quite evident the most important variable is stronger valuated than the others. It is an intuitive result, since the more penible will be the travel time component, the less it will be comfortable during its duration, and the more the individual will be willing to pay a higher price for saving it in order to avoid boring time spendings. This model is then correcting the strange corresponding result of the model M2.

5.1.2 Asymmetric tastes specifications.

On an other hand, we also can allow for tastes differentiation among the transportation alternative. The Multinomial Logit approach allow for different weights for AS variables according to their corresponding alternatives (see [Rousseau, J. and C. Saut, 1997] for an application to the mode choice issue and its implementation in the RATP traffic assignment model). We propose here three models that are allowing for such assumption. The model M4 is the classical multinomial logit model (see for instance [Gourieroux, C., 1989] or [Maddala, G.S., 1983]). The model M5 is a constrained framework of M4. It argues one price unit is perceived the same whatever are transportation alternative. Finally, the model M6 is an extension of M5, allowing for twisting effects affecting prices and times. Because we have attempted to estimate many other models, and because some of them are not estimable, we have reduced our approach to consider the twisting effects are specific to each attribute type, but the same across the alternatives. This latter gives quite intuitive economical results.

We observe under weights asymmetry assumption we recover intuitive signs for the reservation levels, with a reservation preference for the PV mode, because of its more flexibility and comfort. This difference appears to be significant in the model M6 (at the level 10%), but for model M4 and
M5, it seems to be the age and travel attributes which are really absorbing the determinants of the traveller’s decision. Here again, the age effect has a positive significant sign, of the same order for all our models (M1 to M6), meaning the older travellers will prefer the PV transportation mode.

<table>
<thead>
<tr>
<th></th>
<th>Estimates (M4, M5, M6)</th>
<th>[Students in brackets]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (PV)</td>
<td>0.1118</td>
<td>0.3941</td>
</tr>
<tr>
<td></td>
<td>[0.1773]</td>
<td>[0.6340]</td>
</tr>
<tr>
<td>Age (PV)</td>
<td>0.0236</td>
<td>0.0266</td>
</tr>
<tr>
<td></td>
<td>[2.5744]</td>
<td>[2.9353]</td>
</tr>
<tr>
<td>Sex (PV)</td>
<td>−0.0953</td>
<td>−0.1160</td>
</tr>
<tr>
<td></td>
<td>[−0.5158]</td>
<td>[−0.6324]</td>
</tr>
<tr>
<td>Net Incomes difference</td>
<td>0.0233</td>
<td>2.9342</td>
</tr>
<tr>
<td></td>
<td>[0.7498]</td>
<td>[2.0671]</td>
</tr>
<tr>
<td>Net Income PV</td>
<td>0.0251</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.8023]</td>
<td></td>
</tr>
<tr>
<td>Travel time &quot;in&quot; PV</td>
<td>−0.0771</td>
<td>−0.0758</td>
</tr>
<tr>
<td></td>
<td>[−10.6164]</td>
<td>[−10.6310]</td>
</tr>
<tr>
<td>Travel time &quot;out&quot; PV</td>
<td>−0.1597</td>
<td>−0.1447</td>
</tr>
<tr>
<td></td>
<td>[−3.6680]</td>
<td>[−3.3737]</td>
</tr>
<tr>
<td>Net Income transit</td>
<td>0.0219</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.6094]</td>
<td></td>
</tr>
<tr>
<td>Travel time &quot;in&quot; transit</td>
<td>−0.0585</td>
<td>−0.0561</td>
</tr>
<tr>
<td></td>
<td>[−4.7403]</td>
<td>[−4.5094]</td>
</tr>
<tr>
<td>Travel time &quot;out&quot; transit</td>
<td>−0.0572</td>
<td>−0.0570</td>
</tr>
<tr>
<td></td>
<td>[−4.7248]</td>
<td>[−4.7309]</td>
</tr>
<tr>
<td>Box-Cox Net Income</td>
<td>0.0471</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0891]</td>
<td>[1.8025]</td>
</tr>
<tr>
<td>Box-Cox travel times &quot;in&quot;</td>
<td>1.1681</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.2006]</td>
<td>[0.7486]</td>
</tr>
<tr>
<td>Box-Cox travel times &quot;out&quot;</td>
<td>1.8419</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.0113]</td>
<td>[2.9688]</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>−363.074</td>
<td>−367.681</td>
</tr>
<tr>
<td>LR stat (df in brackets)</td>
<td>472.275</td>
<td>463.060</td>
</tr>
<tr>
<td></td>
<td>[9]</td>
<td>[7]</td>
</tr>
<tr>
<td>% right</td>
<td>80.12</td>
<td>80.00</td>
</tr>
</tbody>
</table>

The model M4 leads to constant values of prices of times. They are different for each time type and each choice alternative. It is important to note that even if weights are asymmetric, the model M4 is a very particular case from a theoretical viewpoint. Moreover, in our empirical framework, it
gives unfeasible values for the prices of times. For time "in" PV, 184.3$FF/HR$, for time "out" VP, 381.75$FF/HR$, for time "in" transit, 139.84$FF/HR$, and for time "out" transit, 136.73$FF/HR$. It is important to note these results are computed with a non significant parameter for the net incomes variables. This is explained by the nature of our problem: remind we are interesting in the mode choice issue, that is only a part of the trip schedule. We assume here all other trip related choice are solved, such that the departure hour, the destination,... If the traveller has an optimal scheduled trip in all its components, then a marginal variation around the equilibrium can lead to excessive reponse of the individual. Another drawback of this model is the WTP for time "out" of the transit mode is lower than the WTP for transit time "in", whereas they are certainly the most penible times during a typical trip.

The model M5 is a constrained framework of the model M4. It follows from the non significant difference between the net income weight, thus we assume it is the same for the two alternatives. The LR test between M4 and M5 ($M_5 \subseteq M_4$) shows that M4 is improving M5. In fact, we also observe a non significant net income weight. This latter model is giving for the prices of times 195.19$FF/HR$ for PV time "in", 372.62$FF/HR$ for PV time "out", 150.64$FF/HR$ for transit time "in", 147.3$FF/HR$ for transit time "out". Because the weight asymetry of the net incomes variables is not the source of unsignificancy, it is maybe the variable itself that does not enter in its original form in the utility function. The model M6 is introducing such considerations.

The aggregate statistics of the sample distributions of the different prices of travel times shows unfeasible theoretical values for the prices of travel times. The high level they all have is due to the conditional to known other trip scheduling components framework. We do not account for substitution patterns between the different dimensions of the trip schedule.
However, here again, we observe forms of log-normal distributions for these prices. We do not have represented the corresponding figures, because they are graphically similar to the above ones. The main features of the model M6 is about the results concerning the twisting effects. We observe the times "out" are significantly perceived as convex cost entering the utility function. It is the most intuitive result, since for the most penible time component, the corresponding WTP are increasing with the effective duration, thus we denote an impatient behavior of the traveller during these times (see figure below, where the WTP for transit time "out" is increasing erratically but with an underlying concave form). The times "in" effect is numerically but not significantly greater than 1, and the associated prices of times are unchanging with a variation of the time "in" duration. Only the income effect is significant in the WTP for these prices. Moreover, the net income weight is found to be significant and to have a relative higher importance than other attributes in the decisional process. Finally, we note all parameters signs are regular with their associated choice probability.
6 Concluding remarks.

Our results have refound some important results concerning the concept of the price of travel time. The standard conditional logit specification lead to prices of times that correspond to the tutel values cited in [Boiteux, M, 2001.]. When we allow for Box-Cox transformations, it appears an income effect which improve the significancy of the net income weight parameter in the decisional process. This result conforms with [McFadden, D., 2000]: the WTP for saving travel time increase with the budget resources, since an implicit higher time purchase power. It is not evidently stated there are increasing prices of times with the trip duration. However, numerical results lead to mainly consider for time components Box-Cox transformations with values greater than 1 and one model find significant convex costs for the most penible time type. An intuitive age effect with a stable and significant weight accross the different models is also found to be significant, respecting the fact that older active people have a higher preference for private vehicle trip, since their (assumed) better socioeconomical conditions. In addition, Times "out" are higher valued than times "in", because the higher penibilities they imply, except for M2. The issue concerning the preordering of the reservation utility levels is not solved: there are trade-offs between comfort and indepedency, and more flexible leisure patterns to be realized during trips.
We note two main limits to our approach that could justify some of the strange results we have obtained. First, the model M6 we have found reproducing an intuitive framework for the prices of times concepts is not a feasible model from a practical viewpoint, since the since the unrealistic prices of travel times we have estimated. This is mainly due to the second limit which is we have reduced the trip scheduling to the mode choice issue, by assuming departure hour, destination and route already chosen from the origin point, thus we can not account for substitution patterns that could emerge in joint decisional processes concerning the trip schedule, constituted by different types of discretes choices to be made (see [Bhat, R.C. 1998]), then we can obtain excessive WTP when the individual is on his optimal schedule, and when we make vary only one component of the global decision process.

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