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VALUE OF TIME SENSITIVITY TO MODEL SPECIFICATION†

MARC J. I. GAUDRY
Département de Sciences économiques et Centre de recherche sur les transports,
Université de Montréal, C. P. 6128, Succursale A. Montréal H3C 3J7, Canada

SERGIO R. JARA-DIAZ
Department of Civil Engineering, Universidad de Chile, Casilla 228-3, Santiago, Chile

and

JUAN DE DIOS ORTUZAR
Department of Transport Engineering, Pontificia Universidad Catolica de Chile, Casilla 6177,
Santiago 22, Chile

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Abstract—We compare values of time obtained with linear utility functions used in LOGIT and HIERARCHICAL LOGIT specifications of a nine-mode passenger choice model estimated with disaggregate data from Santiago, Chile, and find that they are sensitive to the specification used, unconvincingly high, and imply that the value of waiting time is a decreasing function of income. We then find that removing observations for individuals who face limiting values of price or service level yields partially improved results if the utility functions are still linear; but the most convincing results are obtained with explicitly nonlinear BOX–COX LOGIT specifications.

1. INTRODUCTION

In models of choice among discrete alternatives, the assumption is made that individual choices are based on perceptions of the relative charateristics of the alternative options; in this way, implicit equivalences are subjectively established. The best known of these in travel demand forecasting is the valuation of time in terms of monetary units, which can be empirically estimated from the results of the estimated models. This *subjective value of time* (SVOT) has concentrated the attention of researchers and policymakers within the industrialized world (see Bates and Roberts, 1986) due to the great importance of time savings in the economic appraisal of transport projects. Given this importance, one would like to achieve estimates of SVOT that are robust and ideally independent from the functional form of the models used to estimate them.

In this paper, we first study the SVOT values derived from linear LOGIT and HIERARCHICAL LOGIT specifications and find values (as a percentage of income) that vary with the specification, are uncommonly high, and, in the case of waiting time, appear to be inversely related to income.‡ These results are partly improved by removing observations that contain zero values for some of the explanatory variables; but explicitly nonlinear utility specifications of a BOX–COX LOGIT type yield results that are generally much more reasonable than those of the more constrained linear form and, for waiting time, values that increase with income both relatively to those of walking time and in absolute terms.

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‡ This is true only in percentage of income terms, but not in money (i.e. Chilean pesos per minute) terms.

2. MODELS AND DATA

2.1 *Standard specification*

In this paper, we first consider a variety of models of the LOGIT family (Domencich and McFadden, 1975; Williams, 1977) with linear utility functions

$$\overline{U}_i = \sum_k \Theta_k Z_{ki}, \tag{1}$$

where the vector of taste parameters $\Theta = \{\Theta_1, \ldots, \Theta_k\}$ is estimated from revealed preference data about individual choices among a set of discrete alternatives **A**, given attributes **Z** (socioeconomic characteristics of the individuals and level-of-service variables of the alternatives). We also consider BOX–COX LOGIT models (Gaudry and Wills, 1978) where one uses Box–Cox transformations defined for any strictly positive **Z** as:

$$Z^{(\lambda)} = \begin{cases} (Z^\lambda - 1)/\lambda, & \text{if } \lambda \neq 0, \\ \ln Z \quad , & \text{if } \lambda = 0. \end{cases} \tag{2}$$

The particular generalization of the utility function used here is:

$$\overline{U}_i = \sum_k \Theta_k Z_{ki}^{(\lambda_k)}, \tag{3}$$

which, except for the coefficient of the constant, clearly reduces to linear form (1) if all λ_k are constrained to 1.

The standard model specification includes travel time t_i and an attribute of the form (c_i/w), with c_i the cost of travelling by alternative A_i, and w the individual's wage rate (Train and McFadden, 1978). Usually w is approximated by the ratio I/w, where I is the individual's income and W his/her working hours in a given period of time. In this paper, w is net personal income per minute of work (\$/minute).

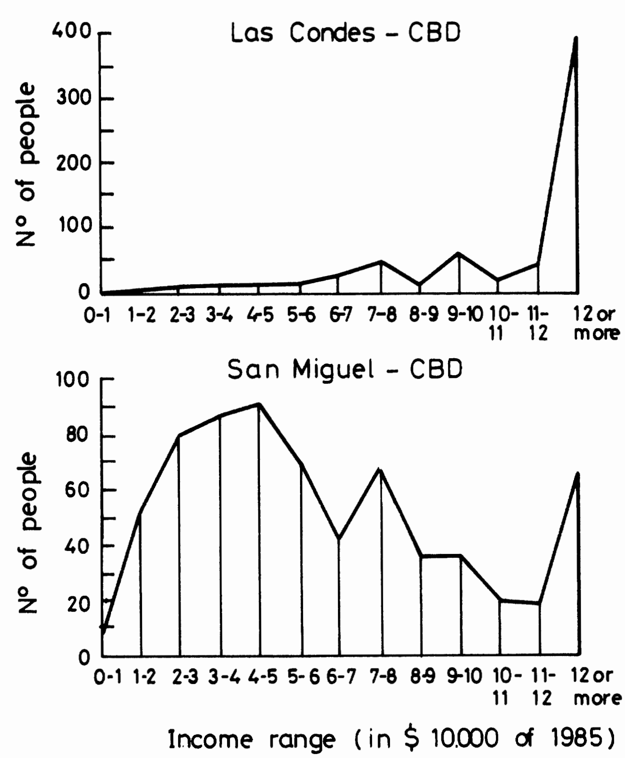

Fig. 1. Income distribution in each corridor.

Table 1. Modal split in the two corridors

Alternative	Modal split (%)	
	Las Condes CBD (1983)	San Miguel CBD (1985)
1. Car driver	17.4	7.8
2. Car passenger	5.9	4.3
3. Shared taxi	6.7	2.2
4. Metro	17.9	27.2
5. Bus	15.4	49.3
6. Car driver metro	16.0	0.9
7. Car passenger metro	5.6	1.3
8. Shared taxi metro	3.6	6.1
9. Bus metro	11.0	0.9

To calculate SVOT, it is necessary to find the rate of substitution between perceived times and costs of travelling with the i^{th} mode:

$$\left.\frac{-dc_i}{dt_i}\right|_{\bar{U}_i} = \frac{\partial \bar{U}_i/\partial t_i}{\partial \bar{U}_i/\partial c_i}. \tag{4}$$

For the linear case, this simply yields

$$\mathrm{SVOT} = w \cdot \frac{\Theta_t}{\Theta_c}, \tag{5}$$

where Θ_t is the coefficient of t_i and Θ_c the coefficient of the term c_i/w. From (5) one can easily see that the ratio Θ_t/Θ_c represents SVOT as percentage of income (SVOT/w).

For the Box–Cox case, SVOT/w is

$$\frac{\mathrm{SVOT}}{w} = \frac{\Theta_t t_i^{(\lambda_t - 1)}}{\Theta_c (c_i/w)^{(\lambda_c - 1)}}. \tag{6}$$

and will clearly vary across alternatives if $\lambda_k \neq 1$. This formula implies that, for modes that are more expensive per minute, the model will necessarily yield higher value of time estimates if both λ_c and λ_t are equal and smaller than one†; however, this may not be the case if the λs differ.

Table 2. Modal split by income stratum for the two corridors

Alternative	Modal split (%)		
	Low income	Middle income	High income
1. Car driver	3.3	12.1	22.4
2. Car passenger	2.4	6.1	6.7
3. Shared taxi	2.4	5.6	5.4
4. Metro	25.3	23.1	20.0
5. Bus	58.9	31.6	11.0
6. Car driver metro	0.9	3.6	17.7
7. Car passenger metro	0.5	4.6	4.3
8. Shared taxi metro	5.9	5.3	3.9
9. Bus metro	0.5	8.0	8.6

† The derivative of SVOT/w with respect to c_i/t_i when $\lambda_t = \lambda_c$ is given by:

$$\frac{\theta_t w^{\lambda - 1}}{\theta_c}(1 - \lambda)(c_i/t_i)^{-\lambda}.$$

It will therefore be positive for $\lambda < 1$.

2.2 *The Santiago data*

To derive the values presented in Section 3, we used a data bank about the choice of modes for the journey to work in the morning peak in two corridors of Santiago. Although the individuals work in the same institutions of the central business district (CBD), the corridors have important differences with respect to income (see Fig. 1) and car ownership, a frequent occurrence in developing countries.

Table 1 describes the alternatives available and the observed frequencies of choice in each corridor. The significant differences between them† are better explained by income than by spatial factors (Ortuzar and Espinosa, 1986). Table 2 shows the frequency of choice of three subsamples created by pooling both data sets and stratifying by family income. All modal shares increase or fall with income except for mode 7, which appears to combine the opposite trends of its component modes.

The model specification included price relative to net personal income, walking and in-vehicle time variables for all nine modes, a waiting time variable for all modes except car, a variable describing the number of cars per driving permit holder in the household for car driver modes 1 and 6, and, finally, a sex dummy variable for modes 2, 3, and 7 listed in Tables 1 and 2.

3. RESULTS WITH LINEAR UTILITIES IN LOGIT AND HIERARCHICAL LOGIT MODELS

In Table 3, we present estimates of SVOT/w for representation utilities of linear form (1) used in LOGIT (L) and HIERARCHICAL LOGIT (HL) structures defined in Fig. 2. The improvement in model fit associated with hierarchical structures occurs in both corridors but the values of time are more sensitive to model structures in the lower-income San Miguel corridor than in the higher-income Las Condes corridor. The most surprising result is that the value of waiting time as a percentage of income is higher in the poorer corridor.

To explain this unexpected result, one is tempted to appeal to misspecification error due to the absence of proper explanatory variables. In this case, one might conjecture that the existence of rigid work hours for some employees (presumably those with lower income) of CBD firms leads them to make sure that they arrive at the office on time (Jara-Diaz and Ortuzar, 1986). To support this conjecture, one could appeal to the results obtained if the data for the two corridors are merged and then broken down by income group. These are shown in Table 4. Surprisingly, the value of waiting time as a percentage of income first rises with income and then falls at the highest income level, thus casting doubt on our conjecture and pointing to a U-shaped (quadratic) form, or at least to a nonlinearity on the relationship between income and the value of time. In fact, this phenomenon occurs for all values of time shown in the table.

Table 3. Value of time as a percentage of net personal income with linear utilities

Corridor	Las Condes CBD (1983)			San Miguel CBD (1985)		
Model	L	HL-UC	HL-EST	L	HL-UC	HL-EST
Variable						
In-vehicle time (IVT)	353.5	314.7	308.1	155.9†	243.2	388.2
Walking time (WALK)	694.2	643.3†	566.6	182.4	393.0	423.5
Waiting time (WAIT)	637.2†	634.0†	574.4†	738.2	1271.1†	1324.4
$\bar{\rho}^2$	0.074	0.119	0.123	0.059	0.156	0.162
Sample size		622			677	
w (1985 $/minute)		8.69			3.66	

† Not significant at 95% level.

† There is no coordination between modes; therefore people choosing mixed-mode options like Bus–Metro have to pay a fare equal to the sum of the fares of each single option. This is particularly important in the San Miguel-CBD corridor since low income families spend over 17% of their income in mandatory trips (Jara-Diaz and Farah, 1986).

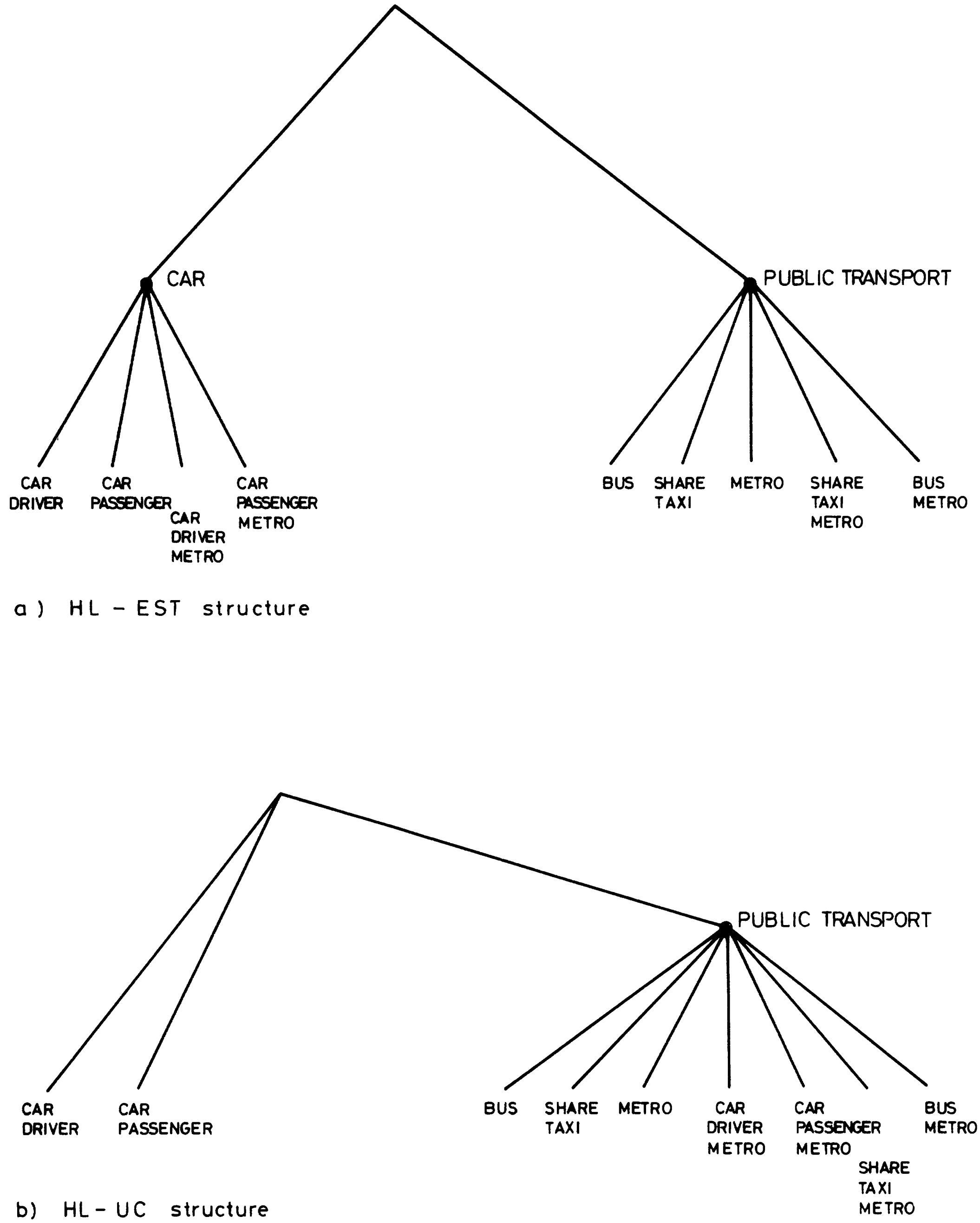

Fig. 2. Hierarchical logit structures.

4. RESULTS WITH BOX-COX LOGIT MODEL

To explore systematically the potential presence of nonlinearity, we use the BOX–COX LOGIT specification (Gaudry and Wills, 1978) on both the Las Condes CBD and full data sets. The results, shown in Table 5, are based on arithmetic averages of modal values.† Column (0) contains the same results as those found in column L of Table 3 and Column (0′) contains corresponding values for the full sample: in line with the results for the San Miguel corridor of Table 3, adding the San Miguel corridor data to the Las Condes data yields a higher value of waiting time than that obtained for the Las Condes corridor by itself—that is again the unexpected result of the previous section.

† Table A1 of the Appendix contains modal values for the most general case of column (3′). All derivatives are evaluated at the mean value of the available observations.

Table 4. Value of time as a percentage of net personal income, by income stratum in the full sample, with linear utilities

Income stratum	Low	Middle	High
Variable			
In-vehicle time (IVT)	269.1	80.6†	229.9
Walking time (WALK)	171.7	452.3	391.9
Waiting time (WAIT)	560.0	1093.6	433.3
$\bar{\rho}^2$	0.048	0.060	0.098
Sample size	423	412	464
w (1985 $/minute)	2.25	4.88	10.61

† The variance of the estimator of Θ_i was higher than its mean.

In Columns (1) and (1′), the samples have been reduced to exclude observations with zero values for the explanatory variables but the utilities are still linear. This procedure is appropriate if one wants to use Box–Cox transformations that can be applied only to strictly positive observations.† The unexpected result of Columns (0) and (0′) vanishes, which confirms our feeling that the form of the utility function varies with the level of the explanatory variables: removing the lowest values yields expected results both in the sense that waiting time is valued more highly than walking time but also in the sense that adding individuals of lower average income reduces the value of waiting time (from 1197.2 to 790.8) expressed as a percentage of income. However, the value of waiting time *relative* to that of walking time is much larger in the full sample than in the Las Condes subsample (3.36 against 2.15), thus lending residual support to the conjecture of the previous section concerning the greater importance of arriving on time for lower income workers, at least in the sense of the relative values of level-of-service characteristics, if not relatively to income.

The application of a single Box–Cox transformation to the price relative to income variable and to all three time variables yields optimal values of λ (equal to 0.36 for Column (2) and equal to 0.66 for Column (2′)) that differ significantly from both the linear case ($\lambda = 1$) and the logarithmic case ($\lambda = 0$) and improves model fit. Moreover, the resulting values of time behave as expected in the sense that adding lower income individuals reduces the value of waiting time both as a percentage of income (it falls from 693.5 to 592.1) and relatively to that of walking time (it falls from 3.29 times to 3.09 times that of walking time).‡

Column (3′) presents results for the full sample when three distinct λs are used for the price to income variable, the in-vehicle time variable, and the remaining time variables.§ The resulting orders of magnitude are not very different from those of Column

Table 5. Value of time as a percentage of net personal income, with linear and nonlinear utilities

Corridor	Las Condes			Las Condes and San Miguel			
Model	0 LIN	1 LIN+	2 BC(1)	0′ LIN	1′ LIN+	2′ BC(1)	3′ BC(3)
Variable							
In-vehicle time (IVT)	353.5	277.6	66.7	215.9	213.5	140.7	121.1
Walking time (WALK)	694.2	557.1	210.7	294.9	235.1	191.2	194.3
Waiting time (WAIT)	637.2	1197.2	693.5	689.5	790.8	592.1	525.9
Ratio WAIT/WALK	0.92	2.15	3.29	2.33	3.36	3.09	2.71
$\bar{\rho}^2$	0.074	0.066	0.073	0.093	0.095	0.099	0.103
Log-likelihood	−855.00	−657.27	−652.38	−1472.00	−988.88	−984.07	−979.81
Sample size	622	470	470	1299	931	931	931
w (1985 $/minute)	8.69	9.25	9.25	6.07	6.34	6.34	6.34

† An alternative procedure would consist in compensating zero-valued observations with dummy variables but this amounts to a change of specification. For this reason, we simply excluded these observations.

‡ This is true for the average but is not systematically the case for modes with less reliable frequencies. The Appendix explains what could be done to explore our conjecture.

§ The optimal valules were 0.60, 1.57, and 0.33, respectively. Values of 0.23, 1.56, and 0.44 were found for the Las Condes subsample alone but the gain in log-likelihood was only 2.38, so the results are not shown in Table 5.

(2′). The tendency of the BOX–COX LOGIT values of time to be smaller, and consequently more convincing,† than those of the linear LOGIT is clear. In the Las Condes subsample, for instance, linear LOGIT values of time are twice as large as BOX–COX LOGIT values; in the full sample, the decrease is not proportionately as important, but it is very significant.

5. CONCLUSION

In this paper, we have used a reliable disaggregate database on the mode choice of individuals travelling to the CBD in Santiago, Chile. We showed that the subjective values of time obtained with linear representative utilities in LOGIT and HIERARCHICAL LOGIT structures were not robust, were uncommonly high, and yielded the unexpected result that the value of waiting time relative to income was inversely related to income. We showed that removing a few observations in a linear LOGIT specification corrected this result but still implied that the value of waiting time relative to that of walking time was inversely related to income. Only explicitly nonlinear BOX–COX LOGIT estimates yielded "expected" values of waiting time, as well as subjective values of time that were generally reasonable.

The important nonlinearities involved, and the more convincing results found, are consistent with the theoretical conclusions of Jara-Diaz and Farah (1987), where higher-order terms of cost and time‡ are interpreted as capturing the role of income in developing countries.

Acknowledgements—The HIERARCHICAL LOGIT structures were estimated using the program ALOGIT by its author, Andrew Daly, as part of some tests performed for the Great Santiago Strategic Transport Study (ESTRAUS). The program P-2 (Liem and Gaudry, 1987) was used to obtain BOX–COX LOGIT results with the assistance of Abdelkrim Nadir. Luis Willumsen made useful comments on an earlier draft of the paper.

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†However, it is important to note that values of time as a percentage of income exceeding 300% are not a unique feature of these data or models. McFadden *et al.* (1977) found values for IVT of 227% and 338% for WALK in linear LOGIT models estimated for the famous Berkeley project; more recently, Swait and Ben-Akiva (1987) reported values of 429% for IVT in the case of car users in linear LOGIT models and values twice as high for more general functions. Our results suggest that linear specifications may be partly responsible for such values.

‡ In that paper, the authors also give a theoretical justification for the replacement of the c_i/w variable by c_i/g, where g is an expenditure rate given by the ratio of income to time spent out of work. Also, one should be careful not to equate Box–Cox estimates automatically with quadratic estimates, obtained for instance with a combination of first- and second-order terms: Box–Cox estimates are monotonic and imply diminishing returns when $\lambda < 1$ and increasing returns when $\lambda > 1$, but quadratic forms are U-shaped and require the analyst to check whether the estimates, given the range of available positive observations, are reasonable in the sense that they fall in the expected part of the U.

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APPENDIX

In the BOX–COX LOGIT model, the values of time are mode-dependent, as noted in our presentation of eq (6). It is therefore interesting to examine the values obtained for the most general of the models estimated for which only average values were used in Table 3, Column (3′). The detailed values are shown in Table A1. Although modes that involve the subway (metro) tend to exhibit values of time somewhat smaller than the mean, this is not true of the car driver–metro combination 6 that has a value even larger than that of the bus. To establish the conjecture that values of wait time are systematically related to the reliablity of each mode, and both to income, one would require an explicit measure of reliability, such as the variance of wait time, and formal statistical testing of the model with the added variable.

Table A1. Model values of time as a percentage of personal income for the most general Box–Cox logit specification (3′) of Table 5

Mode	1	2	3	4	5	6	7	8	9	
Time component	Car D.	Car P.	S. Taxi	Metro	Bus	Car D.-M.	Car P.-M.	S. Taxi-M.	Bus-M.	Mean
IVT	144.4	81.5	153.6	67.5	142.0	116.6	81.5	146.6	156.4	121.1
WALK	271.9	157.5	237.3	96.3	195.4	210.5	146.6	219.8	215.0	193.3
WAIT	—	—	963.4	368.2	562.7	571.2	390.9	415.5	410.5	525.9
Ratio WAIT/WALK	—	—	4.05	3.82	2.88	2.71	2.67	1.89	1.91	2.71